Bose-Einstein Condensation and Superfluidity of Dirty Bose Gas

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Motivation

- How does disorder affect Bose-Einstein condensation and its Superfluidity?
- What is the relation between BEC and superfluidity?
- By adjusting the disorder, we may divide BEC from superfluidity and understand this relation!
- Mathematical technique is more difficult than the Fermi system with disorder because of the existence of BEC

Experiment

Liquid ^{4}He in porous Vycor glass

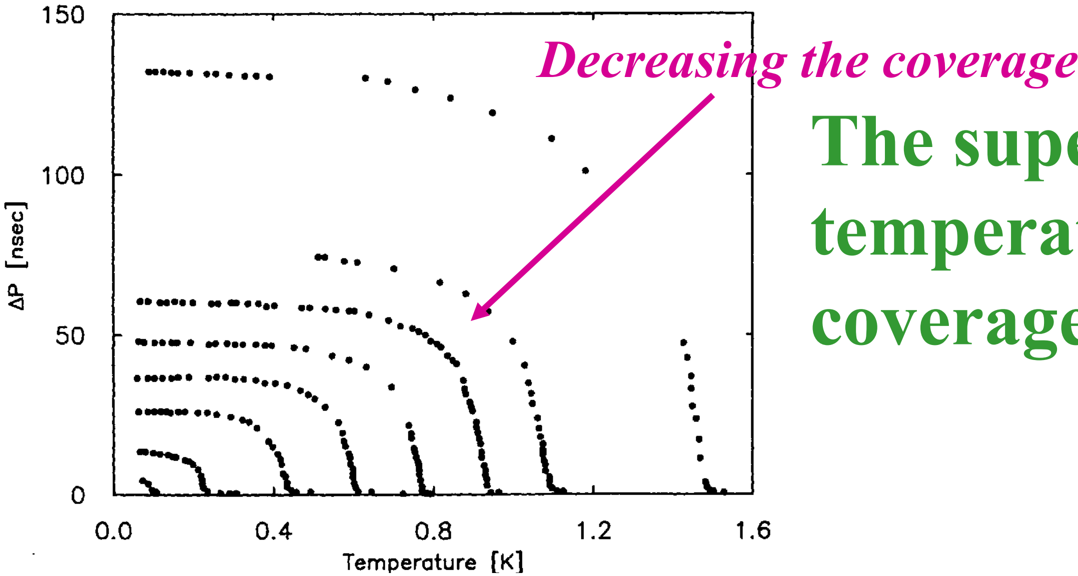
Porous Vycor glass

Average pore size is from 30Å to 80 Å

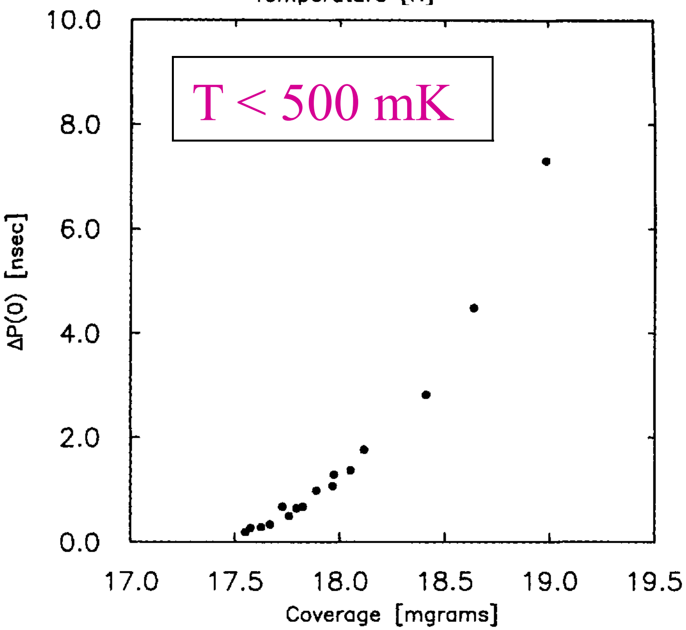
The density of He can be changed by adjusting the adsorbed He coverage.

Measurement of Superfluidity.

J. D. Reppy, J. Low Temp. Phys. 87(1992)205



The superfluid critical temperature decreases with the coverage.



The superfluid density can no longer exist, even near 0 K, below a certain critical coverage (~17.5 mg).

Calculation

Out first aim is to build the model that can express these experiment quantitatively!

- We think the situation where the coverage is very small: the density of He is very small.
- It is said that the coherence length of BEC(~500Å) is much longer than the average pore size (~50Å) in Vycor: ^{4}He has three-dimensional behavior

Our theoretical model
"Dilute Bose Gas in a Random Potential"

Hamiltonian

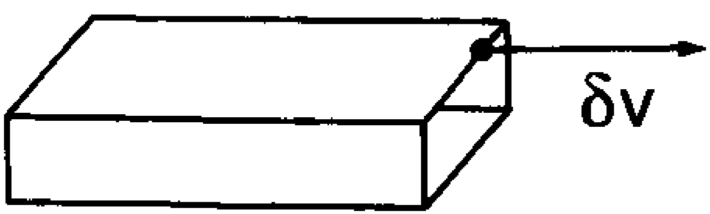
$$\hat{H} - \mu\hat{N} \equiv \hat{K} = \hat{K}_0 + \hat{K}_1 + \hat{K}_2$$

$$\hat{K}_0 = \int d\vec{x}\,\hat{\Psi}^\dagger(\vec{x})\left[-\frac{\hbar^2}{2m}\nabla^2 - \mu\right]\hat{\Psi}(\vec{x})$$ Kinetic Energy

$$\hat{K}_1 = \frac{2\pi a\hbar^2}{m}\int d\vec{x}\,\hat{\Psi}^\dagger(\vec{x})\hat{\Psi}^\dagger(\vec{x})\hat{\Psi}(\vec{x})\hat{\Psi}(\vec{x})$$ Interaction Between Two Particles

$$\hat{K}_2 = \int d\vec{x}\,\hat{\Psi}^\dagger(\vec{x})U(\vec{x})\hat{\Psi}(\vec{x})$$ Random potential

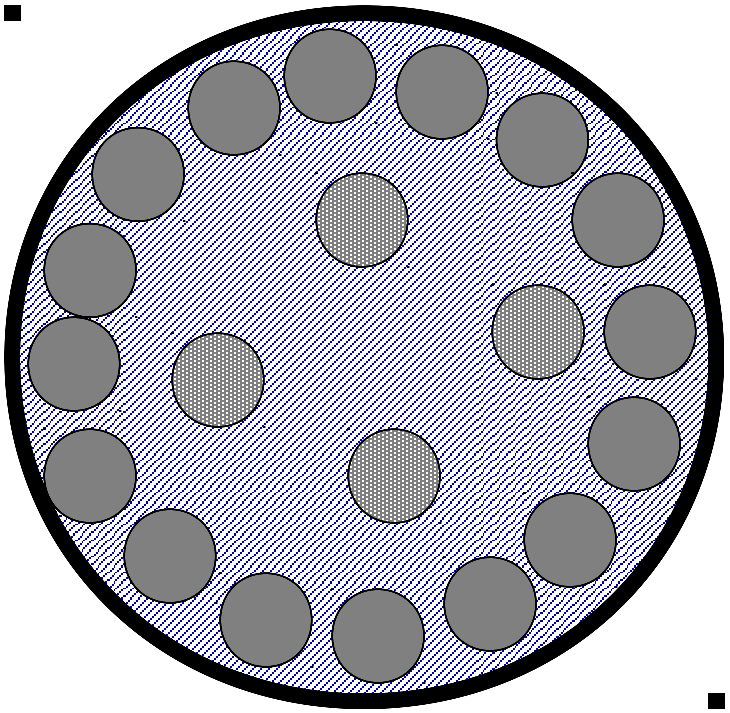
We use linear response theory for superfluidity



Only the normal fluid density which have viscosity responds to dragging such a pipe.

Preparation of Calculation

We estimate the density of He from the behavior of He atoms in Vycor glass



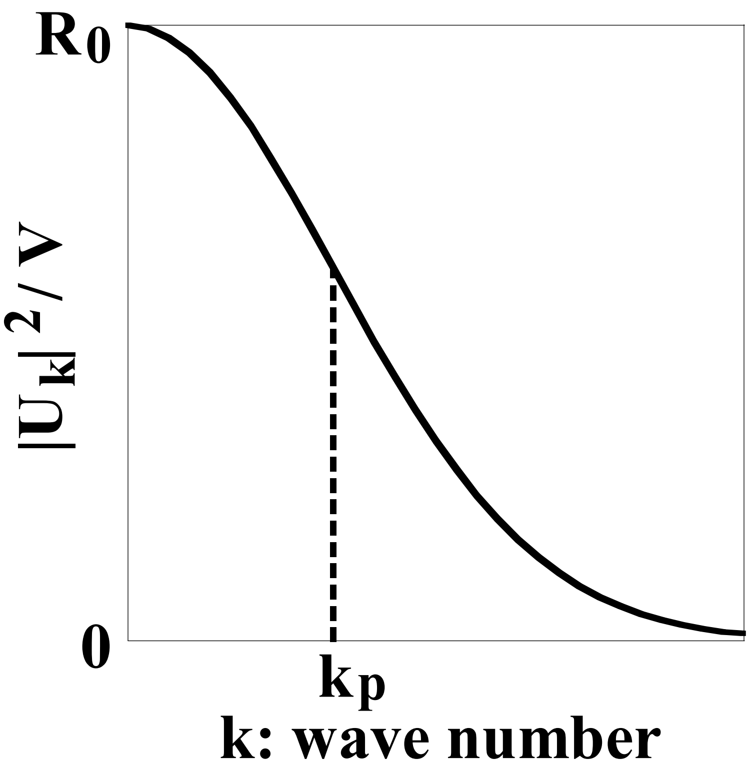
The atoms are adsorbed and fully cover the surfaces of pores. The rest of atoms do not participate in the first layer solid and behave as a gas!

Pore of Vycor glass

Random potential

We take an ensemble average and quench the random potential

U_k: Fourier transformation of <U(x)>



We assume that the quenched random potential U_k decays above k_p

$$|U_k|^2/V = R_0 \,\mathrm{Exp}[-k^2/2k_p^2] \text{ (Gaussian distribution)}$$

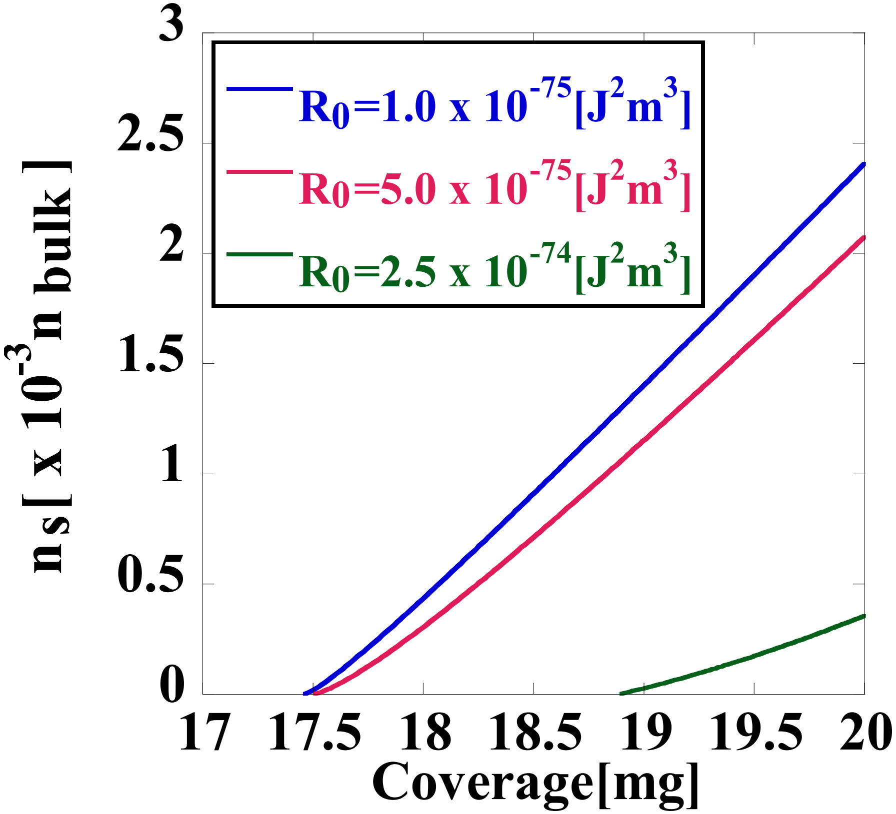
$k_p = 2\pi/\lambda_p$

λ_p : Average pore size of Vycor

R_0 : characteristic strength of the random potential

R_0 can be fixed by the comparison of the critical coverage between our calculation and experiment

Calculation of the superfluid density at T = 0 K



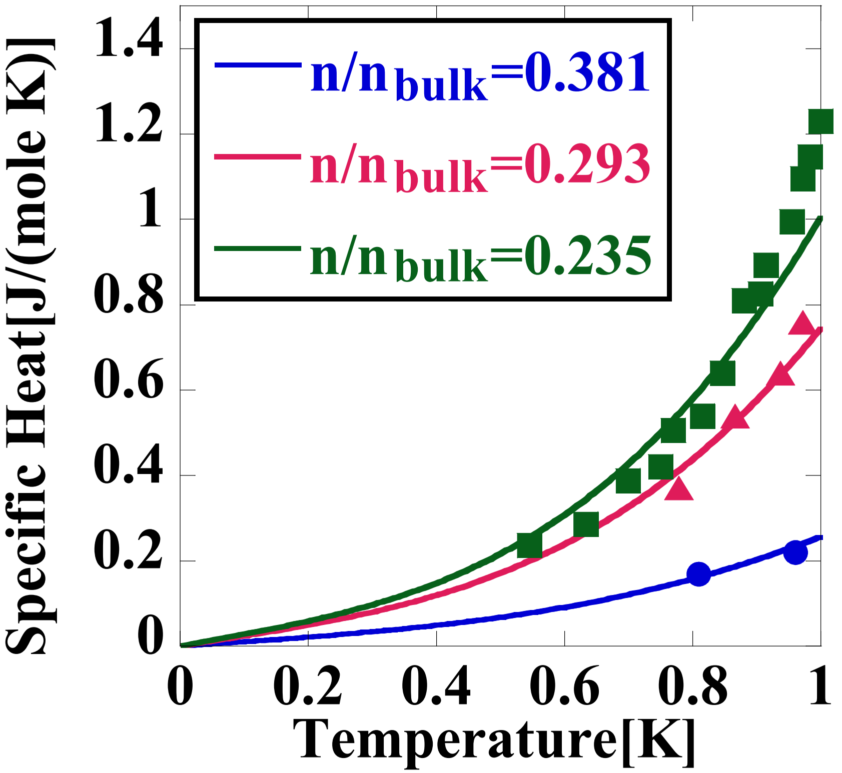
The critical coverage depend on R_0.
From experiment, we can get

$$R_0 = 5.0 \times 10^{-75}\ J^2\ m^3$$

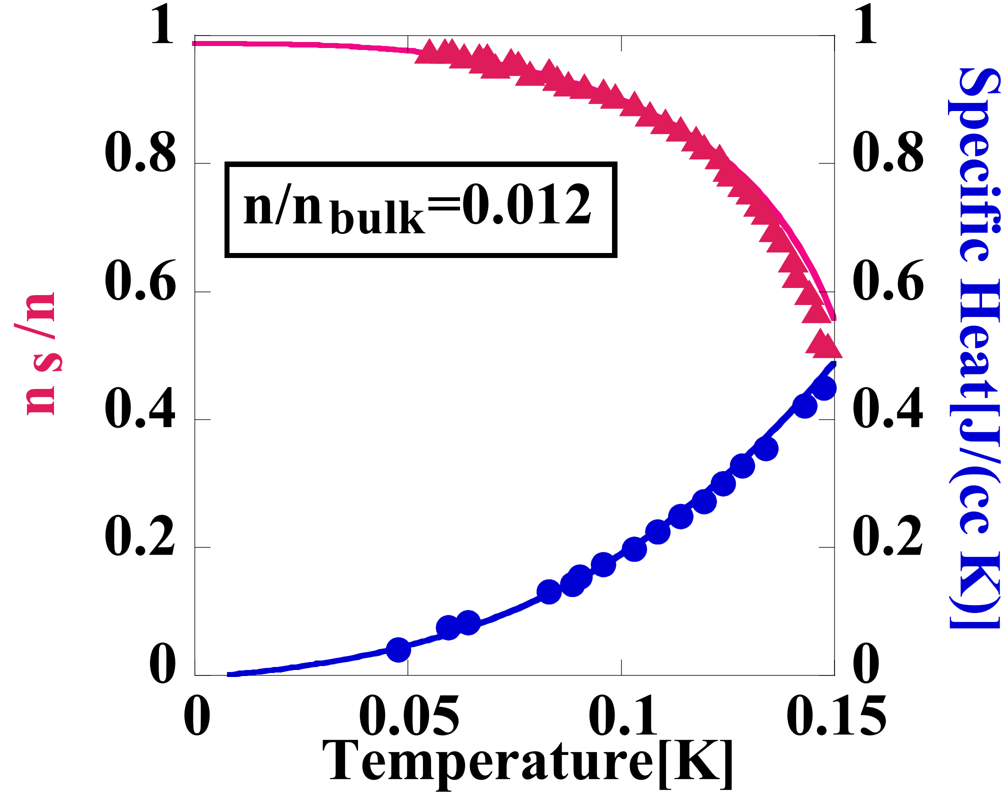
Finally, our model has no free parameters!!

Comparison with experiments

Comparison of the specific heat (high density)



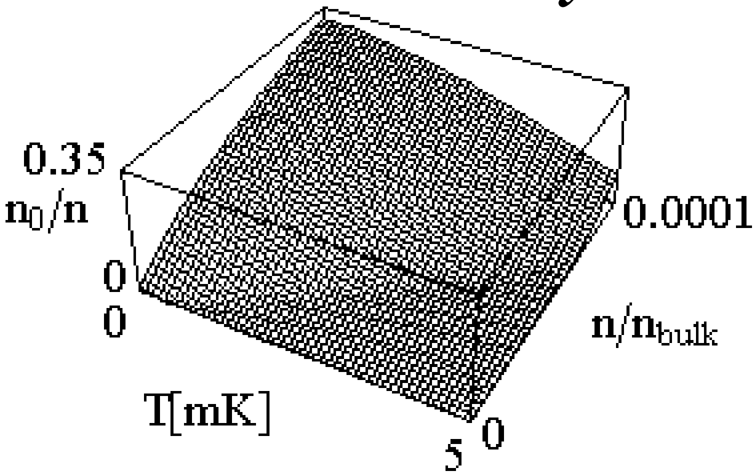
Conparison of the specific heat and superfluid density (low density)



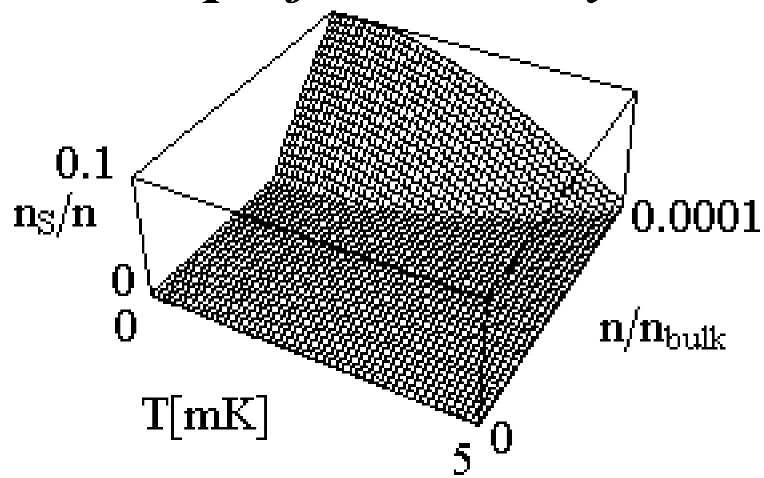
Our result agrees with the experiment quantitatively!!

Condensate density and superfluid density

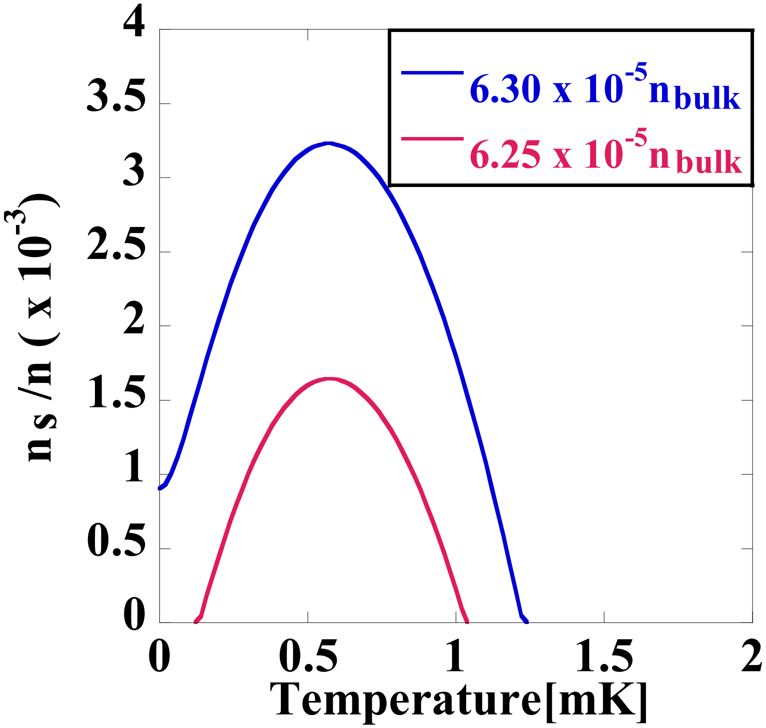
Condensate density



Superfluid density



The condensate density can remain without the superfluid density



The superfluid density at low densities and low temperatures

The effect of the random potential causes this superfluid reentrant transition!

Conclusion

- We develop the model, “dilute Bose gas in a random potential”.
- We can fix all parameters of our model by comparison with experiment.
- The specific heat agrees quantitatively with experimental data at low temperatures without free parameters.
- Our model can predict other interesting phenomena of the BEC and superfluidity.
- We must improve the more general model explaining phenomena at high temperatures.