Superfluid-insulator transition of a Bose-Einstein condensation in a periodic potential and its interference pattern

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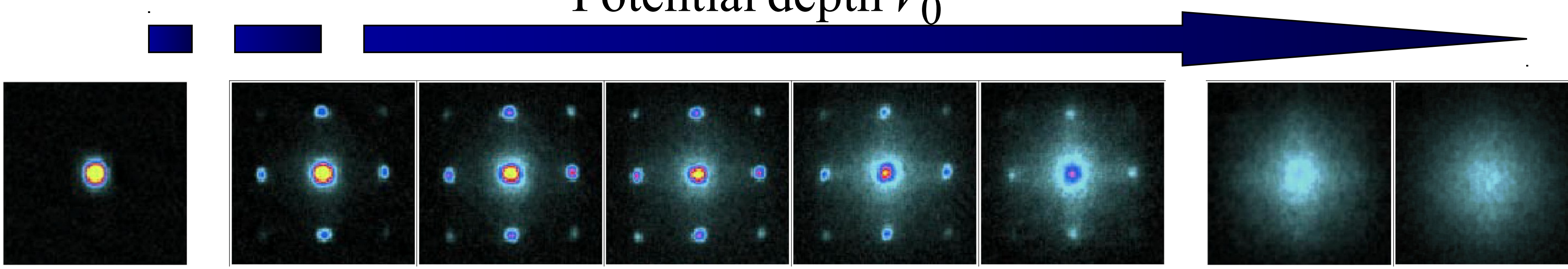
- Introduction
- Model: Gross-Pitaevskii equation
- Pure periodic potential
- Periodic and trapping potential

1, Introduction

Superfluid-Mott insulator transition of trapped alkali atomic BEC in an optical lattice potential

Greiner *et. al.* Nature **415** 39 (2002)

Potential depth V_0



$V_0 = 0$

Appearance of the interference pattern by the periodic potential

Disappearance of the pattern

Disappearance of the long-range coherence by the deep periodic potential: Superfluid-Mott insulator transition

Summary of this work

- We discuss this system by using the Gross-Pitaevskii (GP) equation with a periodic potential.
- Since the GP equation assumes the BEC, it is impossible to discuss the Mott insulator phase.
- However the GP equation gives the detailed structure of the amplitude and the phase of the BEC.
- Changing the potential depth, we investigate what happens to the BEC.

2, Model: the GP equation

$$i\hbar \frac{\partial}{\partial t}\Phi(\boldsymbol{x},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 - \mu + V(\boldsymbol{x}) + g\left|\Phi(\boldsymbol{x},t)\right|^2 \right]\Phi(\boldsymbol{x},t)$$

$\Phi(\boldsymbol{x},t)$: Macroscopic wave function of BEC

$V(\boldsymbol{x})$: External potential

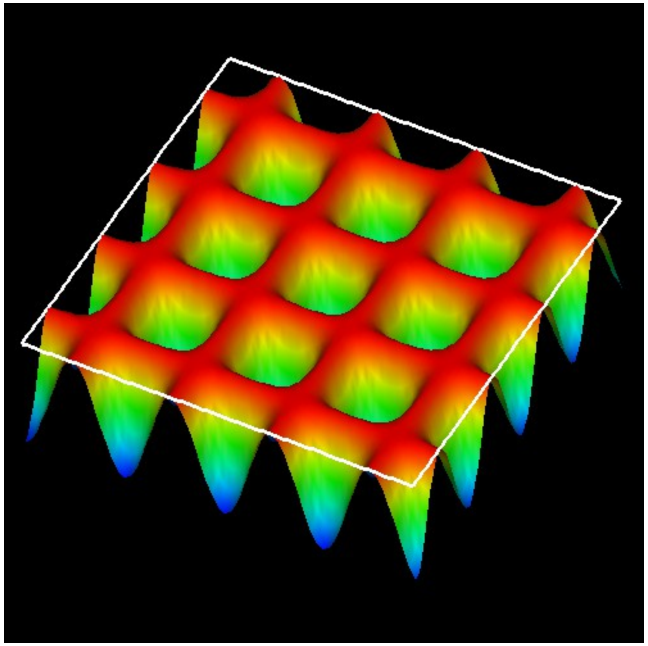
μ : Chemical potential

g : Coupling constant

Numerical calculation of this equation about two-dimensional system

3,Pure periodic potential

$$V(\boldsymbol{x}) = -V_0 \cos^2(Kx)\cos^2(Ky)$$



We look for the ground state by introducing the dissipative term.

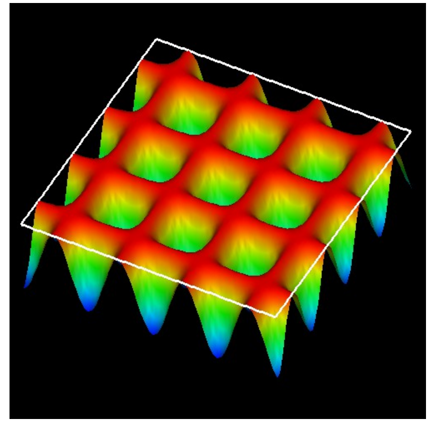
$$(\mathrm{i}-\gamma)\hbar\frac{\partial}{\partial t}\Phi(\boldsymbol{x},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 - \mu + V(\boldsymbol{x}) + g\left|\Phi(\boldsymbol{x},t)\right|^2\right]\Phi(\boldsymbol{x},t)$$

Ground state

$$E_R = \frac{\hbar^2 K^2}{\pi^2 m}$$

$$gK^2 / \pi^2 E_R = 1$$

$$\iint_{\text{1-site}} |\Phi(\boldsymbol{x})|^2 \, \mathrm{d}\boldsymbol{x} = 1$$

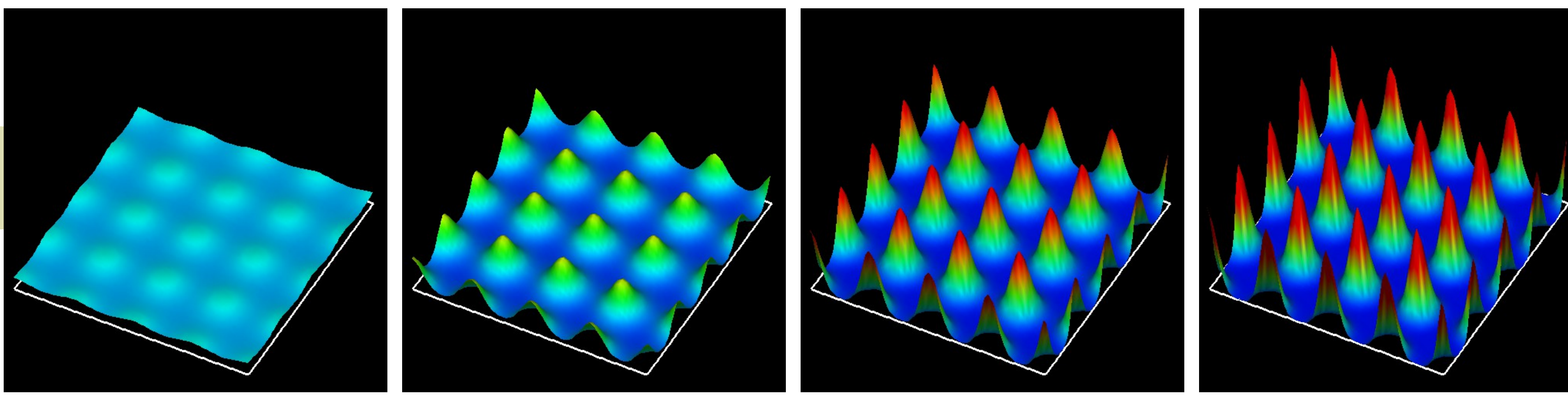


Potential

$$V(\boldsymbol{x}) = -V_0 \cos^2(Kx)\cos^2(Ky)$$

V_0

$|\Phi(\boldsymbol{x})|^2$



$V_0 / E_R = 5$ | $V_0 / E_R = 25$ | $V_0 / E_R = 50$ | $V_0 / E_R = 75$

Localization of the amplitude

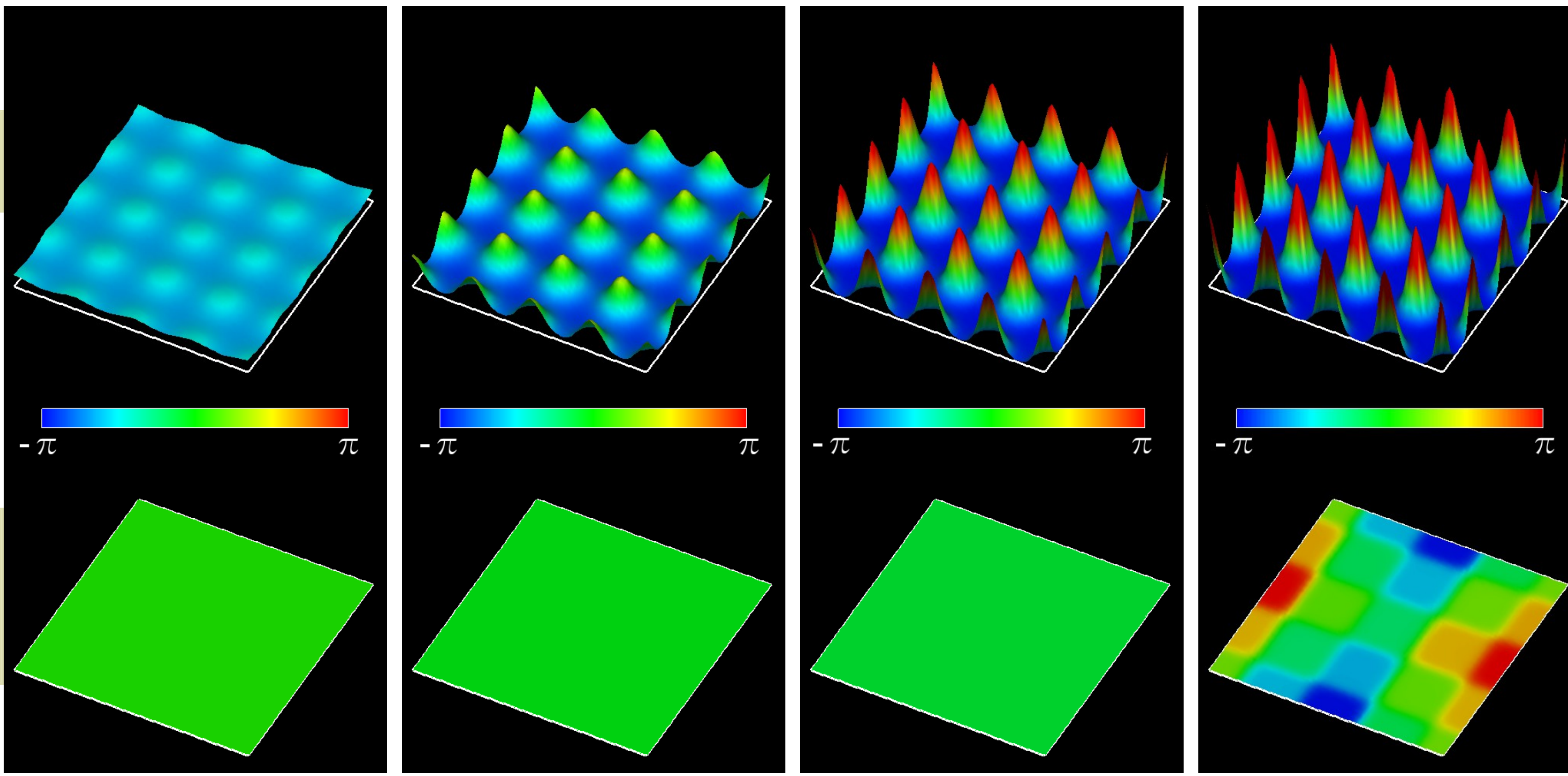
The phase of the ground state

$$\Phi(\boldsymbol{x}) = |\Phi(\boldsymbol{x})| \exp[i\theta(\boldsymbol{x})]$$

V_0

$|\Phi(\boldsymbol{x})|^2$

Phase $\theta(\boldsymbol{x})$



$V_0 / E_R = 5$ $V_0 / E_R = 25$ $V_0 / E_R = 50$ $V_0 / E_R = 75$

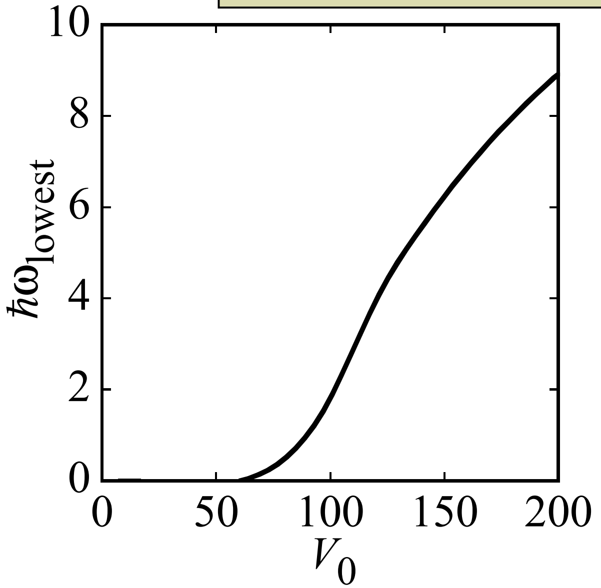
Localization of the phase: breaking of the long-range correlation

Lowest excitation

Hartree-Fock-Bogoliubov equation

$$\Phi(\boldsymbol{x}) \to \Phi(\boldsymbol{x}) + \varphi(x),\ \ \varphi(\mathrm{x}) = u(x)\mathrm{e}^{-\mathrm{i}\omega t} + v^*(x)\mathrm{e}^{\mathrm{i}\omega t}$$

$$\begin{bmatrix} -\dfrac{\hbar^2}{2m}\nabla^2 - \mu + V(\boldsymbol{x}) & g\Phi(x)^2 \\ -g\Phi^*(x)^2 & \dfrac{\hbar^2}{2m}\nabla^2 + \mu - V(\boldsymbol{x}) \end{bmatrix} \begin{bmatrix} u(x) \\ v(x) \end{bmatrix} = \hbar\omega \begin{bmatrix} u(x) \\ v(x) \end{bmatrix}$$

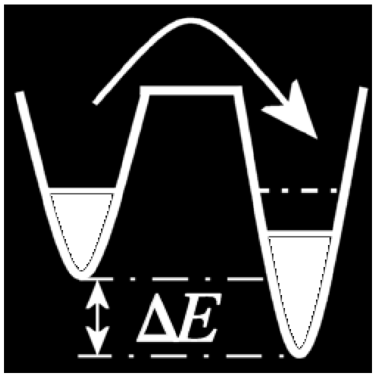


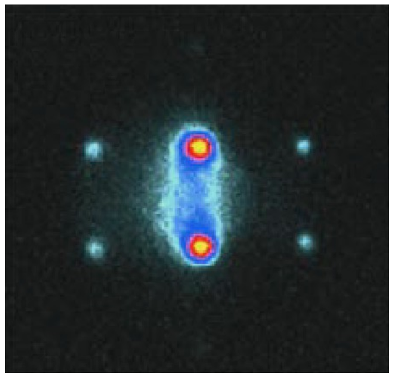
Localization of the phase ⇒ Finite excitation energy: breaking superfluidity

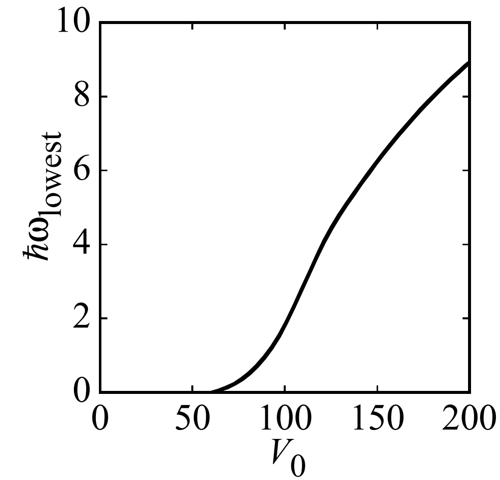
Energy gap of Mott-insulator

A local interference pattern by the potential gradient

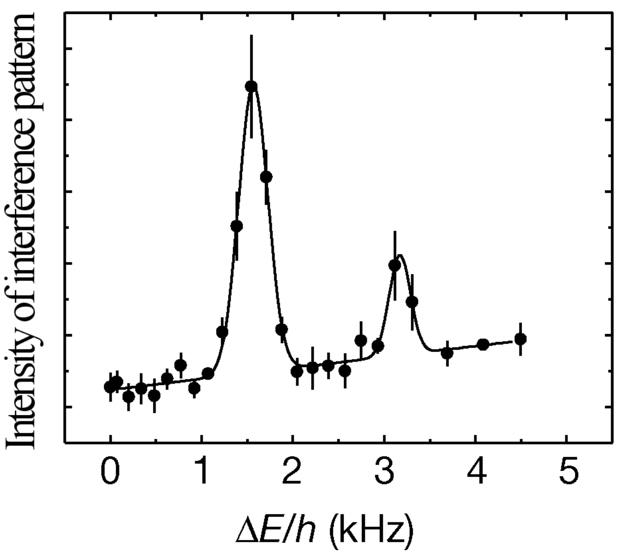
Greiner *et. al.* Nature **415** 39 (2002)







ΔE : Energy difference



A energy gap is observed in Mott insulator phase ⇒Is there any relation to the excitation energy gap given by Hartree-Fock-Bogoliubov equation?

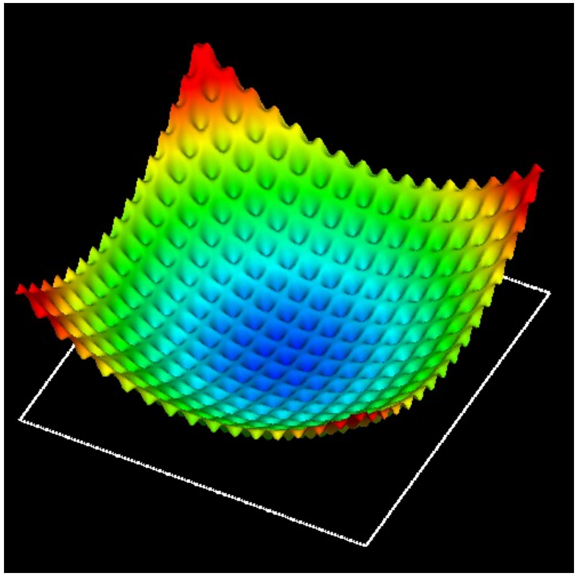
4, Periodic and trapping potential

$$V(\boldsymbol{x}) = -V_0 \cos^2(Kx)\cos^2(Ky) + \alpha_T(x^2+y^2)$$

$$E_R = \frac{\hbar^2 K^2}{\pi^2 m}$$
$$gK^2/\pi^2 E_R = 1$$

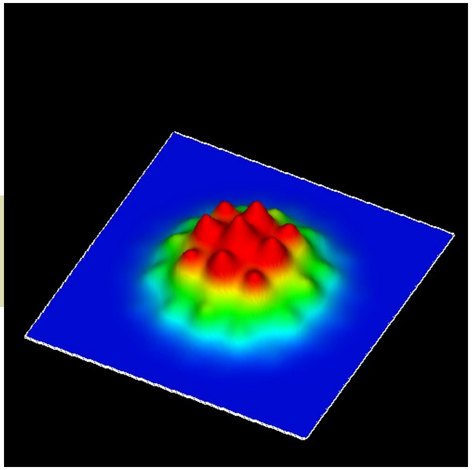
$V_0/E_R = 5$

$$\frac{\pi^2 \alpha_T}{K^2 E_R} = 1$$

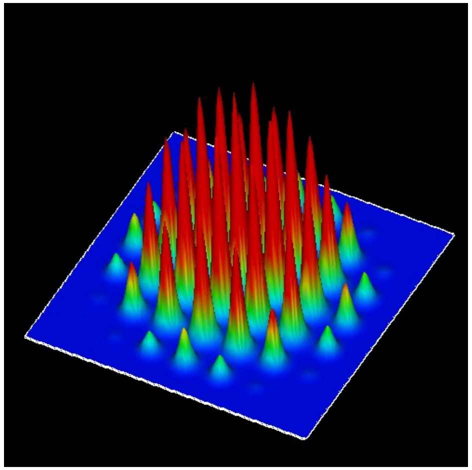


Ground state

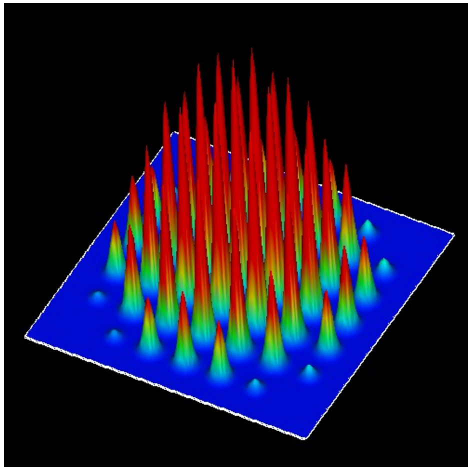
$|\Phi(\boldsymbol{x})|^2$



$V_0/E_R = 5$



$V_0/E_R = 50$



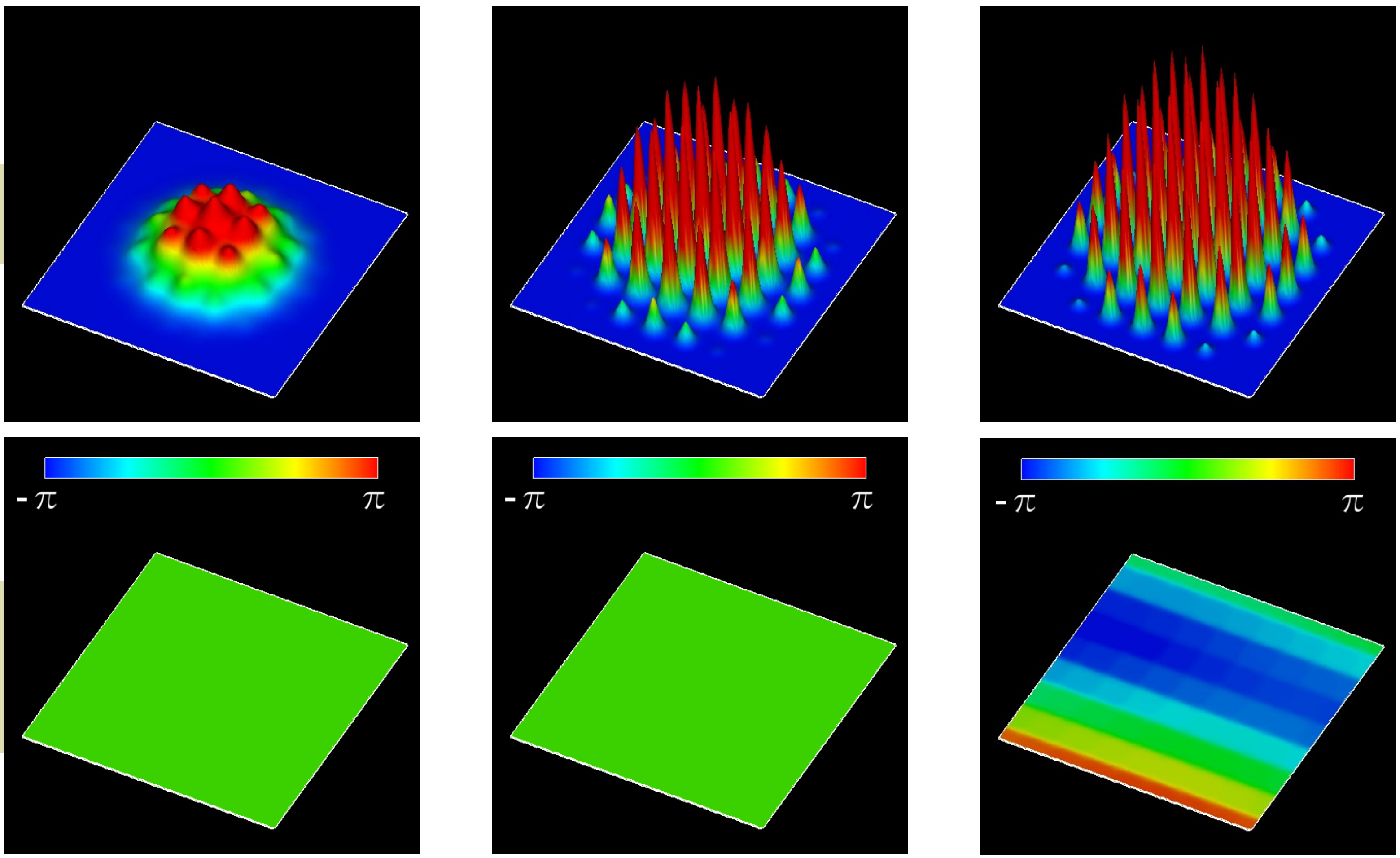
$V_0/E_R = 75$

The phase of the ground state

The localization of the phase

$|\Phi(\boldsymbol{x})|^2$

Phase $\theta(\boldsymbol{x})$

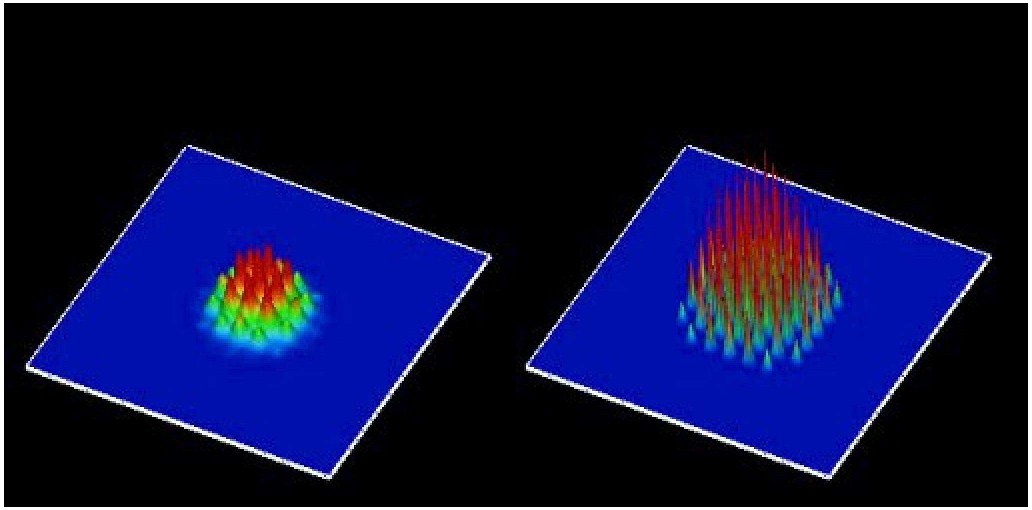


$V_0 / E_R = 5$ $V_0 / E_R = 50$ $V_0 / E_R = 75$

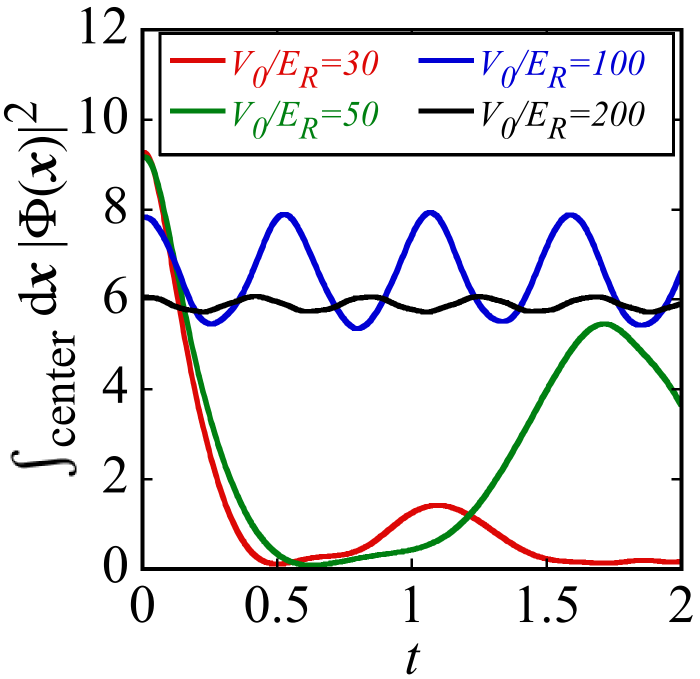
Removing only the trapping potential

$$E_R = \frac{\hbar^2 K^2}{\pi^2 m}$$

$$gK^2 / \pi^2 E_R = 1$$



$V_0 / E_R = 50$ $\quad$ $V_0 / E_R = 75$

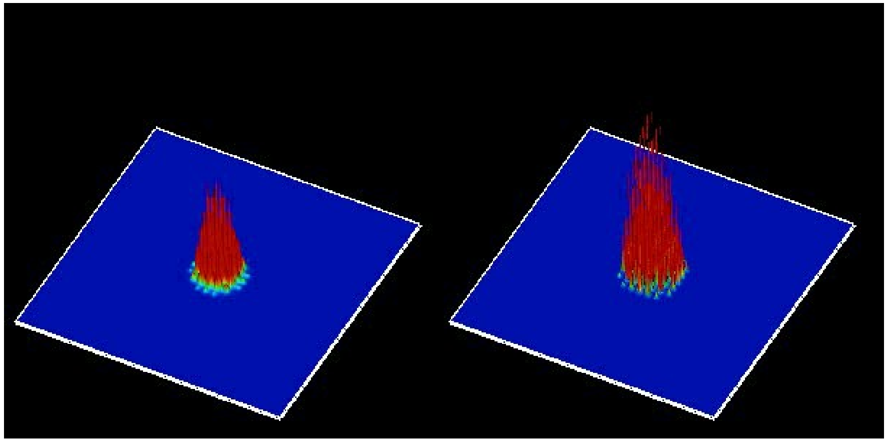


Even after removing the trapping potential, the localized wave function does not expand but oscillate.

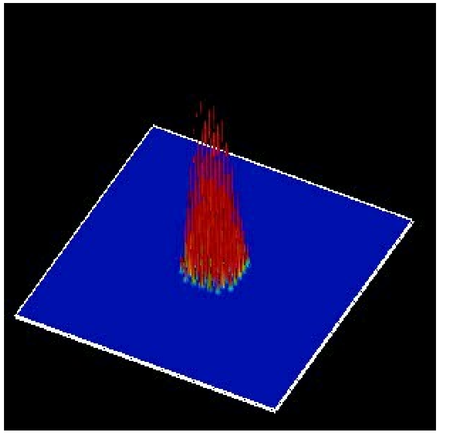
Removing the combined potential

$$E_R = \frac{\hbar^2 K^2}{\pi^2 m}$$

$$gK^2 / \pi^2 E_R = 1$$



$V_0 / E_R = 50$ $V_0 / E_R = 75$



$V_0 / E_R = 120$

At the deep periodic potential, the interference pattern disappears.

Conclusions

Using the GP equation, we find the signals concerned with the superfluid–insulator transition.

- **In the periodic potential, the phase of ground state localizes in each site and the energy gap appears in the lowest excitation.**
- **After removing only the trapping potential, the localized wave function does not expand but oscillate in each site.**
- **After removing the combined potential, the localized wave function does not make interference pattern.**