Bose-Einstein Condensation and Superfluidity of Strongly Correlated Bose Fluid in a Random Potential

Michikazu Kobayashi and Makoto Tsubota
Osaka-City-University, Japan

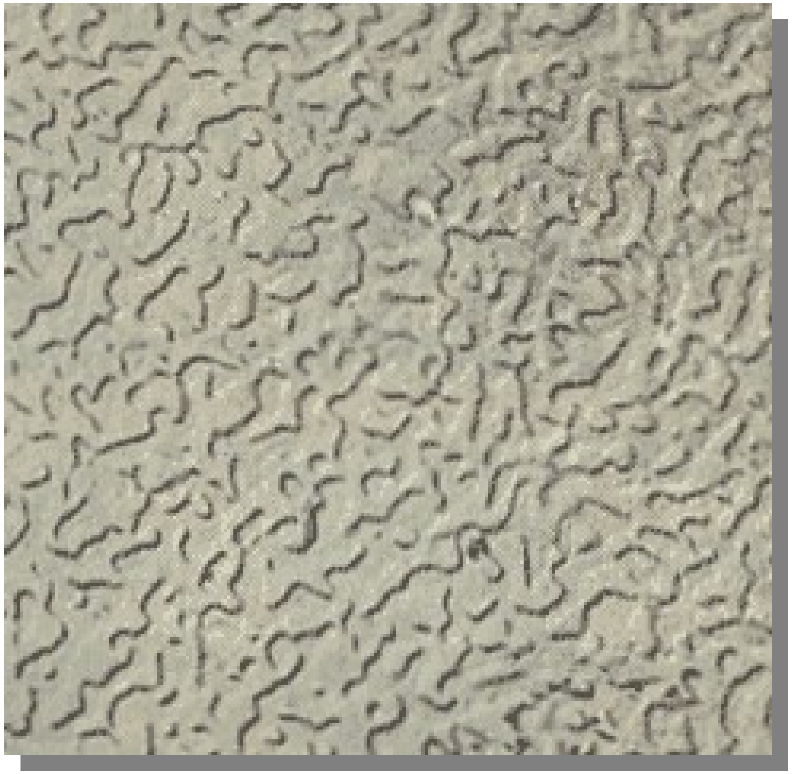
Experiment

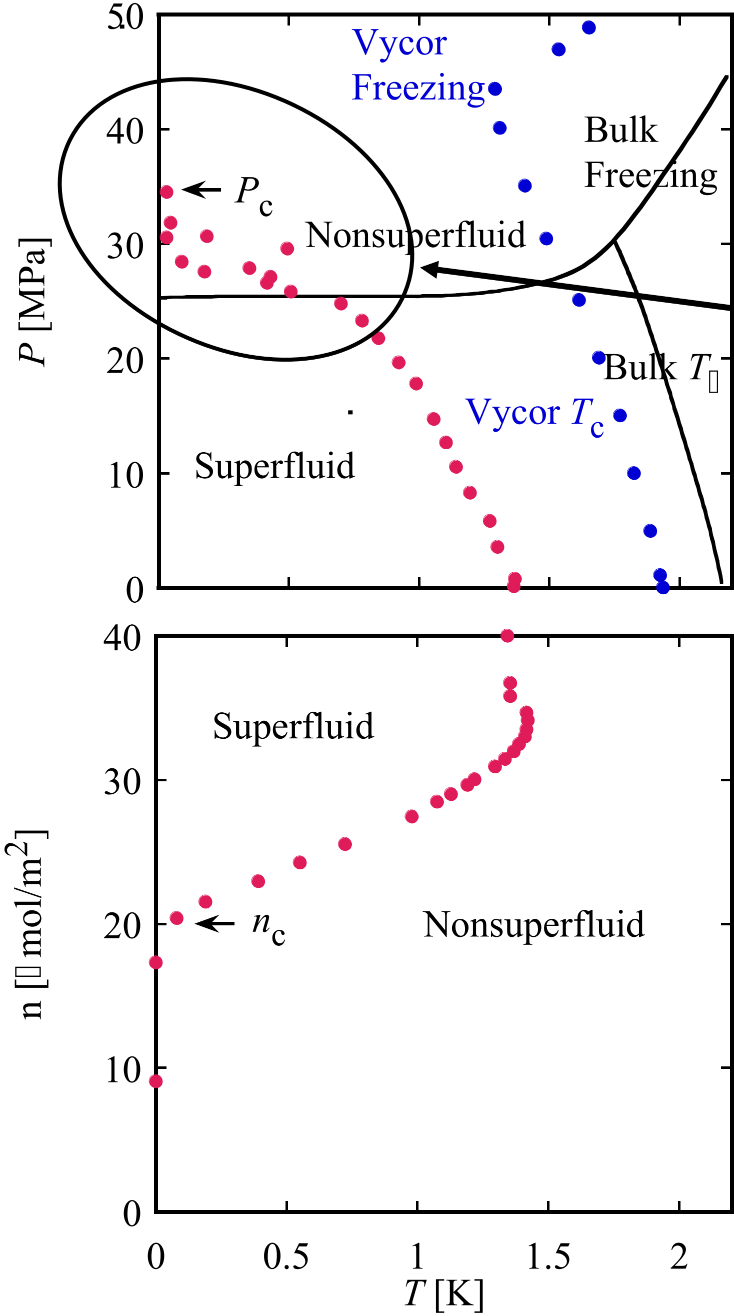
K. Yamamoto, H. Nakashima, Y. Shibayama and K. Shirahama,
cond-mat 0310375

Liquid 4He in porous Gelsil glass

Pore size : ~ 25Å
Filling rate : ~ 30%
Pore area : ~ 130m^2/cm^3

Porous glass have
Wormhole-like structure





Measurement of Superfluidity by Torsional Oscillator

Vanishing of superfluidity at high pressures** P > ~35 MPa **without freezing

The effect of particle correlation and randomness

This phenomenon can be described analytically?

Model

3-dimensional Bose Fluid in a Random Potential

$$\hat{H} - \mu \hat{N} = \hat{K}$$

$$= \int d^3x\, \hat{\Psi}^\dagger(\boldsymbol{x}) \left[-\frac{\hbar}{2m}\nabla^2 - \mu \right] \hat{\Psi}(\boldsymbol{x})$$: Kinetic energy $\hat{K}_0$

$$+ \frac{1}{2} \iint d^3x d^3x'\, \hat{\Psi}^\dagger(\boldsymbol{x}) \hat{\Psi}^\dagger(\boldsymbol{x}') g(\boldsymbol{x} - \boldsymbol{x}') \hat{\Psi}(\boldsymbol{x}') \hat{\Psi}(\boldsymbol{x})$$: Repulsion $\hat{K}_{\mathrm{I}}$

$$+ \int d^3x\, \hat{\Psi}^\dagger(\boldsymbol{x}) U(\boldsymbol{x}) \hat{\Psi}(\boldsymbol{x})$$: Random Potential $\hat{K}_{\mathrm{R}}$

$\hat{\Psi}$: Field operator of Boson
μ : Chemical potential
g : Coupling
U : Random potential

Perturbation of K_{I}

Calculation of Green Function and Self-energy

$$\hbar\mathscr{G}(\mathrm{i}\omega_l,k)^{-1} = \mathrm{i}\hbar\omega_l - (\varepsilon_k^0-\mu) - \hbar\Sigma(\mathrm{i}\omega_l,k)$$

$$\omega_l = \frac{2\pi l}{\beta\hbar}, \varepsilon_k^0 = \frac{\hbar^2k^2}{2m}$$

$$n = -\frac{1}{\beta\hbar(2\pi)^3}\sum_l\!\!\!\!\!\!\int \mathrm{d}^3k\,\mathscr{G}(\mathrm{i}\omega_l,k)$$

Near the critical temperature T_{c},

$$\mu = \hbar\Sigma(0,0)$$

$$\mathscr{G}(0,k)^{-1} \simeq -\frac{\hbar^2}{2m}k^{2-\eta}k_{\mathrm{c}}^{\eta}$$

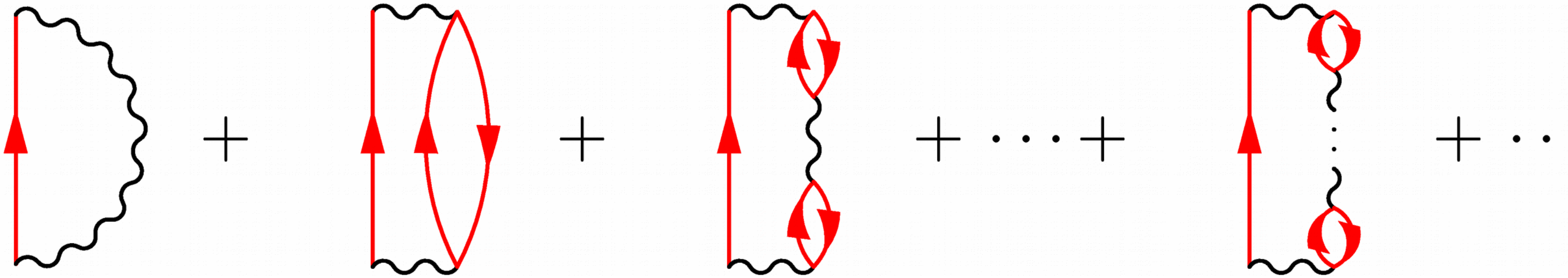
for $k/k_{\mathrm{c}} \ll 1$

Avoidance of divergence due to one-bubble

η and k_{c} are given by the self-consistent equation

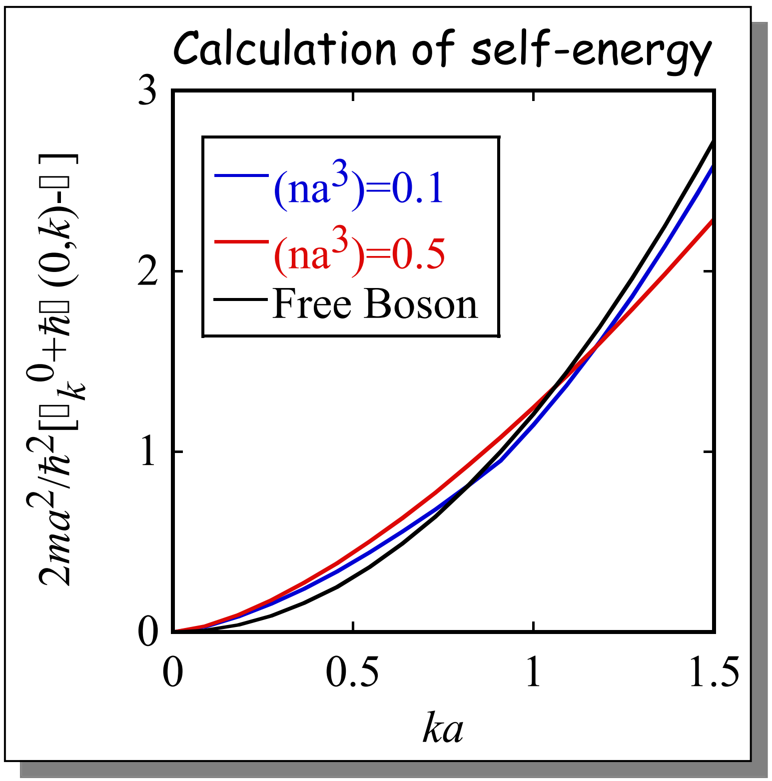
$$\hbar\mathscr{G}(\mathrm{i}\omega_l,k)^{-1} = \mathrm{i}\hbar\omega_l - (\varepsilon_k^0-\mu) - \hbar\Sigma[\mathscr{G}(\mathrm{i}\omega_l,k)]$$

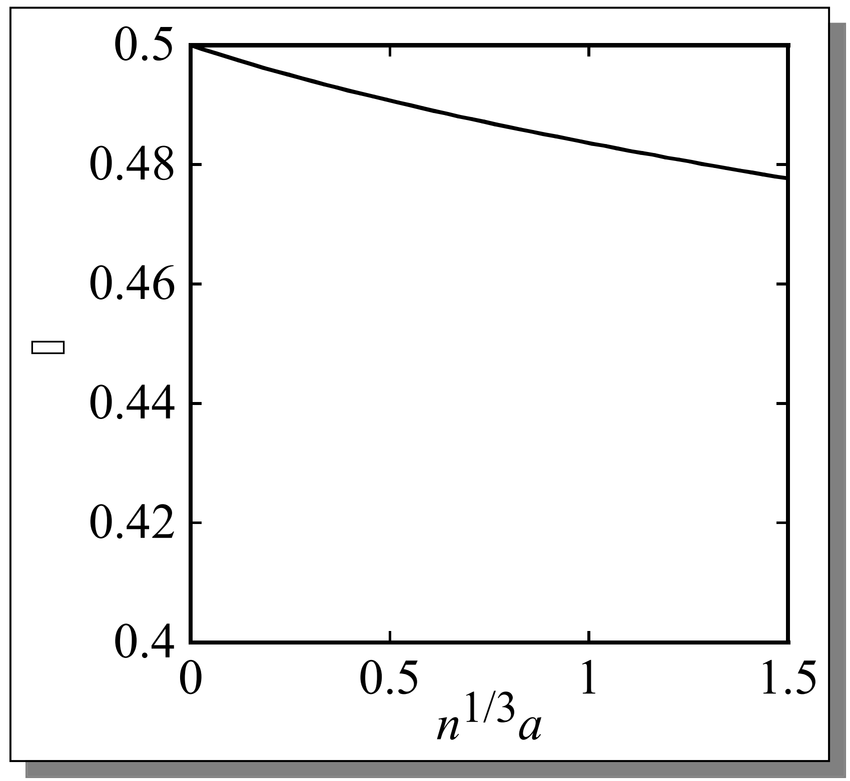
Self-energy : bubble approximation

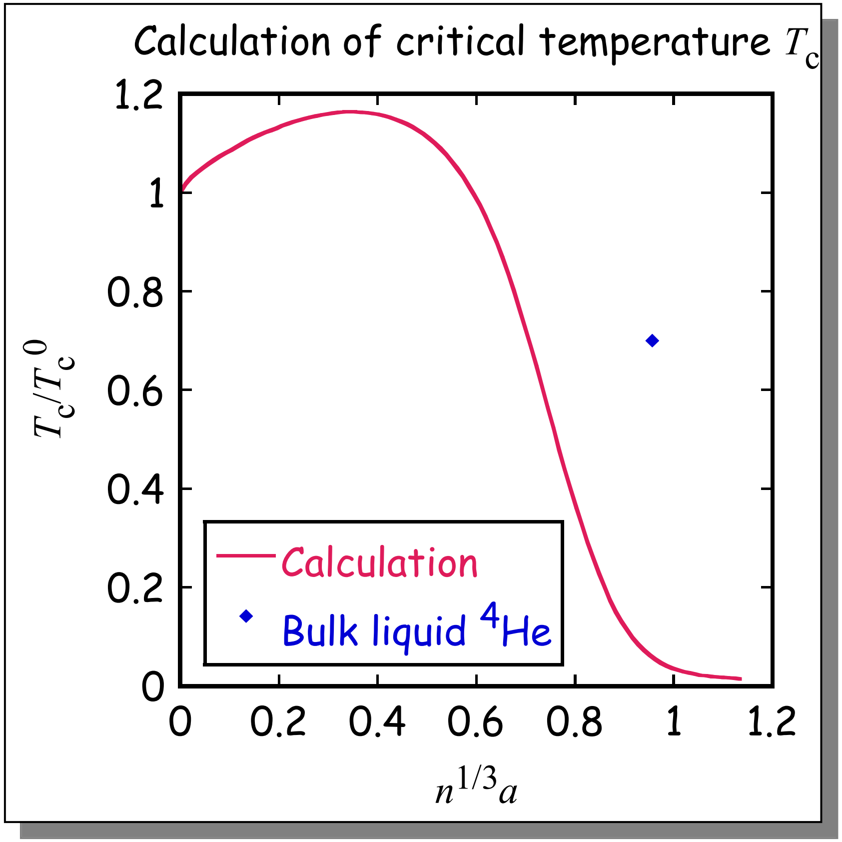


Particle interaction : hard sphere

$$g(\boldsymbol{x}-\boldsymbol{x}') = 3\hbar^2/ma^2\theta(a-|\boldsymbol{x}-\boldsymbol{x}'|) \xrightarrow{a\to 0} 4\pi a\hbar^2/m\delta(\boldsymbol{x}-\boldsymbol{x}')$$







About weak repulsion

$$T_c/T_c^0 \simeq 1 + 1.2 n^{1/3} a$$

For small a : increase of T_c

→increase of the excitation

For large a : decrease of T_c

→increase of the effective mass

Difference between the calculation and liquid ^{4}He

→This may be caused by the long-range attraction of liquid ^{4}He.

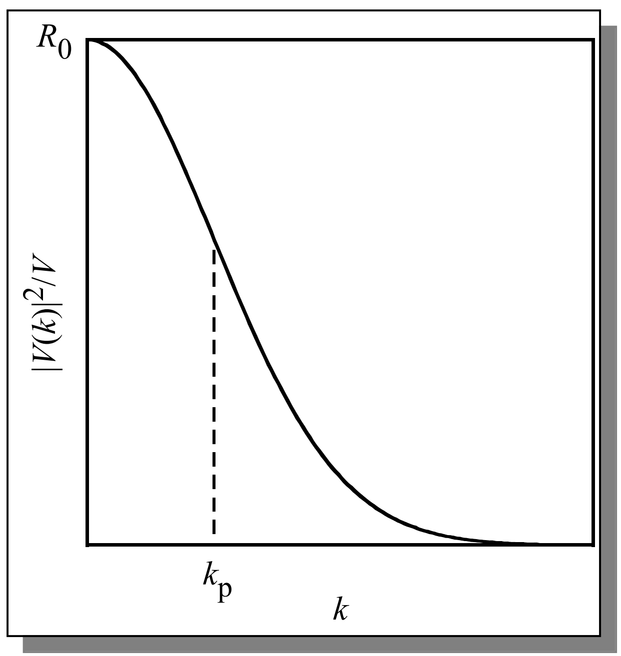
Perturbation of K_R

Random potential : taking ensemble average

$$\frac{<|U(\boldsymbol{k})|^2>_{\rm av}}{V} = R_0 \exp\left[-\frac{k^2}{2k_{\rm p}}\right]$$

$U(\boldsymbol{k})$	:	Fourier transformation of $U(\boldsymbol{x})$
$r_{\rm p} = 2\pi/k_{\rm p}$	:	average pore size
R_0	:	characteristic strength of the random potential

We assume that th e quenched random potential U_k decays above k_p



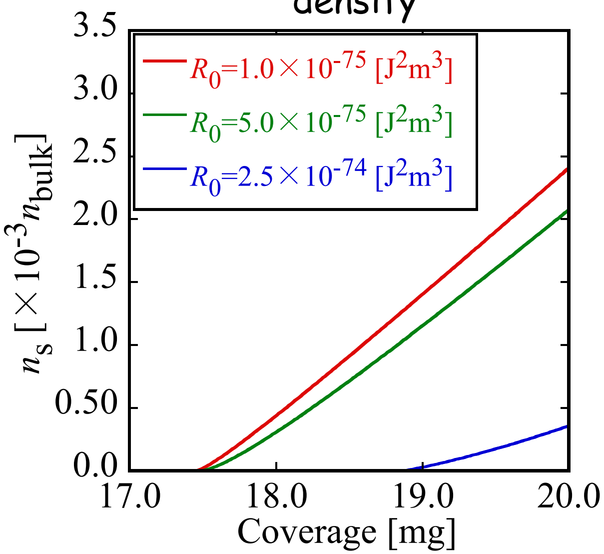
Perturbation
: Second-order

Determination of Strength of the Random Potential R_0

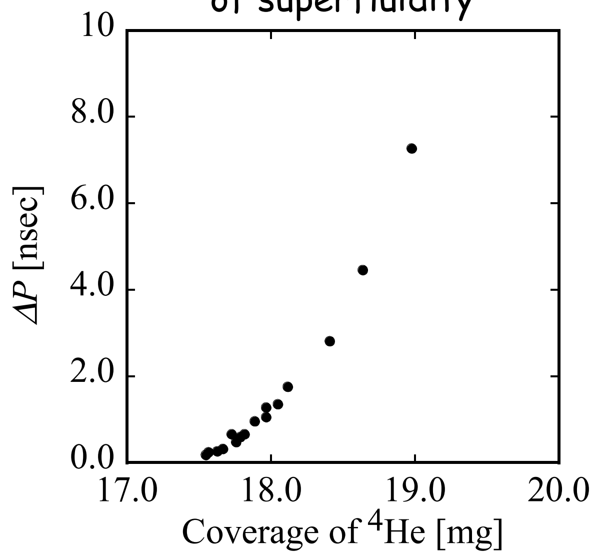
Quantitative comparison of the critical adsorbed coverage with an experiment of dilute ^{4}He in porous glass

M. Kobayashi and M. Tsubota, Phys. Rev. B66 174516 (2002)

Calculation of superfluid density



Experimental measurement of superfluidity



By the comparison, we can obtain $R_0=5.0\times10^{-75}$

Other quantitative parameters

$m = 6.6\times10^{-27}$ kg : mass of ^{4}He

$a = 5$ Å : s-wave scattering length of ^{4}He

$n_{\mathrm{bulk}} = 2.1\times10^{26}\ \mathrm{m}^{-3}$: density of bulk liquid ^{4}He

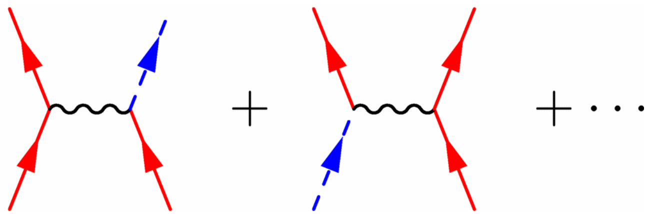
$V = 1\ \mathrm{cm}^3$: volume of porous glass

$k_{\mathrm{p}} = 25$ Å : pore size of porous glass

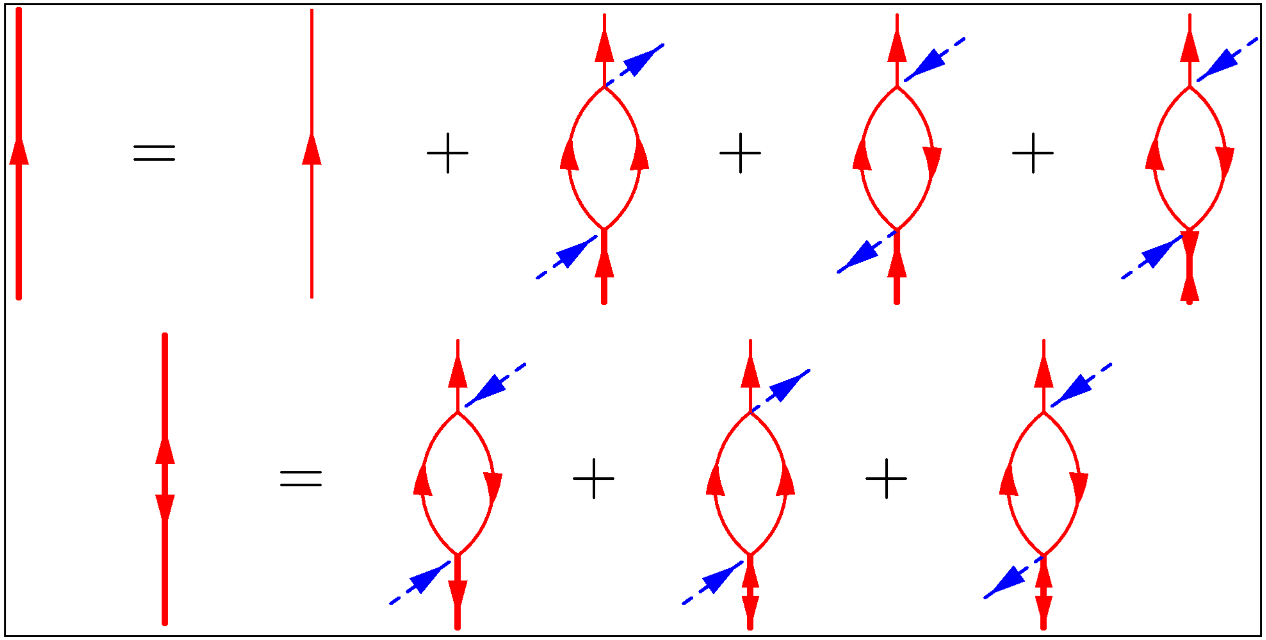
Calculation of BEC and Superfluidity

Near T_c

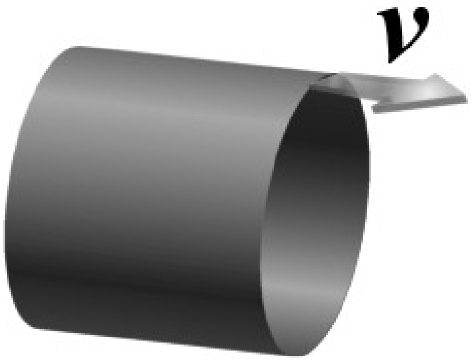
main scattering :



Dyson-equation : one-bubble



Superfluidity : Linear response theory



Only the normal fluid density which have viscosity responds to dragging this pipe.

$n = n_s + n_n$

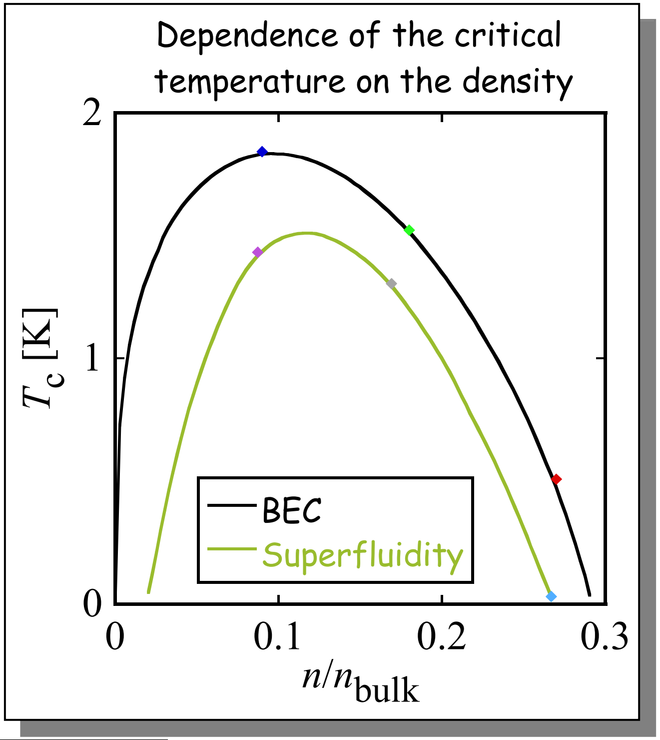
n_{s} : superfluid density

n_{n} : normal fluid density

Superfluid critical temperature are given by

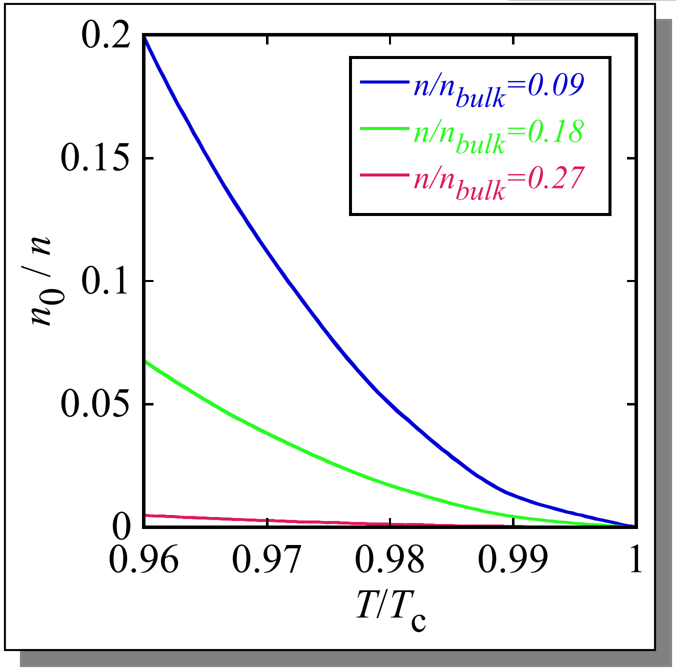
$$n_{\mathrm{n}}(T_{\mathrm{c}}) = n$$

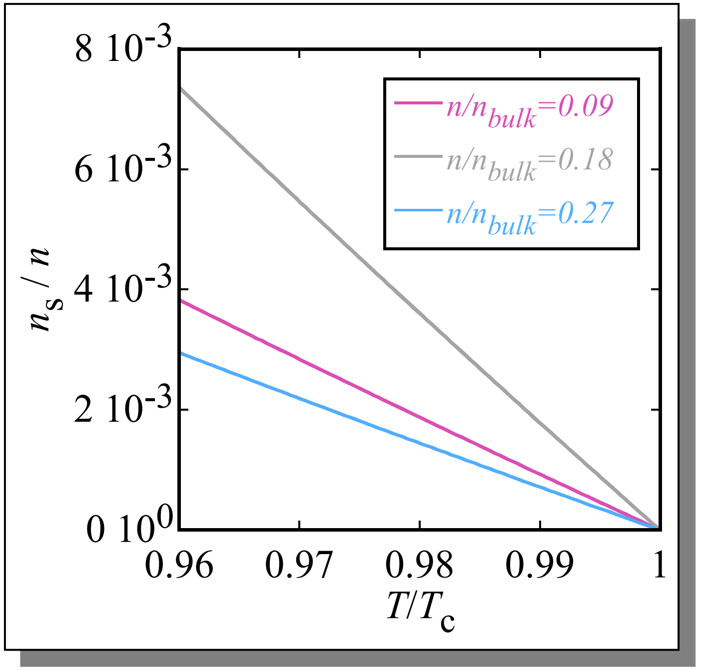
Results



Disappearance of BEC and superfluidity at high densities

: Qualitative agreement with the experimental result





Summary

1. We compare the model of 3-dimensional Bose fluid in a random potential with the recent experiment by Yamamoto et. al.
2. By using the perturbation of repulsive interaction and the random potential, we can obtain the BEC and superfluid critical temperatures.
3. BEC and superfluidity disappear at high densities. This is qualitatively consistent with the experimental result.