International Workshop on Photosynthetic Antennae and Coherent Phenomena. 16 December, 2007

Realization of Quantum Turbulence in Atomic Bose-Einstein Condensation

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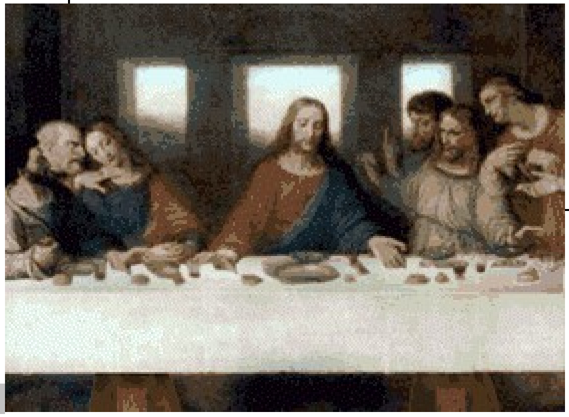
ELEMENTARY EXCITATION PHYSICS LABORATORY
THEORY OF CONDENSED MATTER



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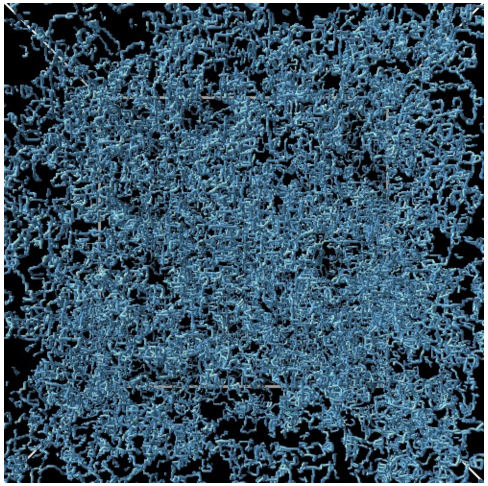
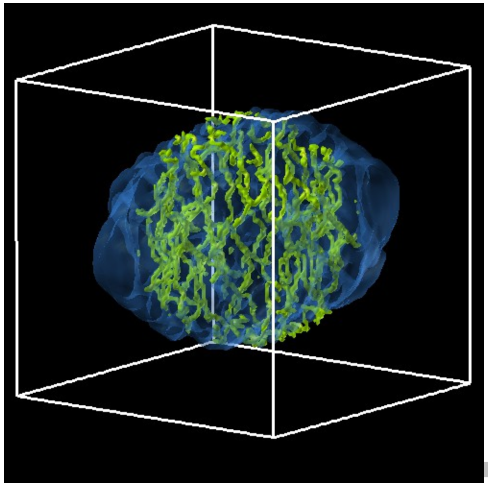




Quantum Fluid and Quantum Turbulence

System of quantum fluid and quantum turbulence

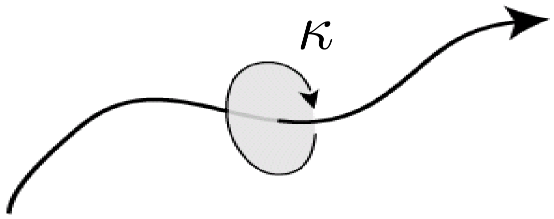
- Superfluid ^4He and ^3He
- Magnetically or optically trapped ultra-cold Atoms

→ At low temperatures, these systems show inviscid superfluid with Bose-Einstein condensation (or BCS) transition

Quantized Vortex

In quantum fluid, all vortices are quantized with quantum circulation $\kappa = h/m$

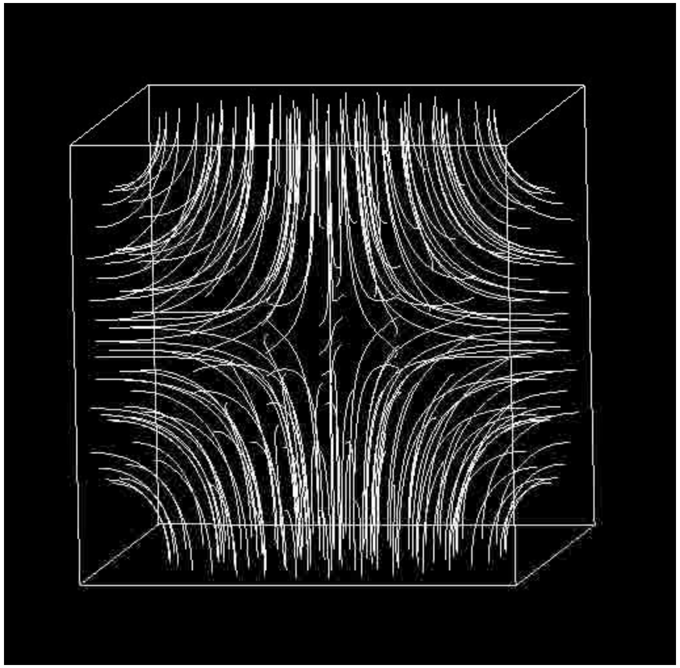
- All vortices have same circulation $\kappa = \oint \boldsymbol{v}_s \bullet \mathrm{d}\boldsymbol{s} = h\ /\ m$ around vortex cores.
- Vortex core is very thin ($\sim$Å : ^{4}He, ~10nm : ^{3}He, ~100nm BEC of cold atoms) : Vortex filament model becomes realistic



Quantum Turbulence From Quantized Vortices

Quantum turbulence can be realized as tangled quantized vortices

Simulation of quantum turbulence by vortex filament model

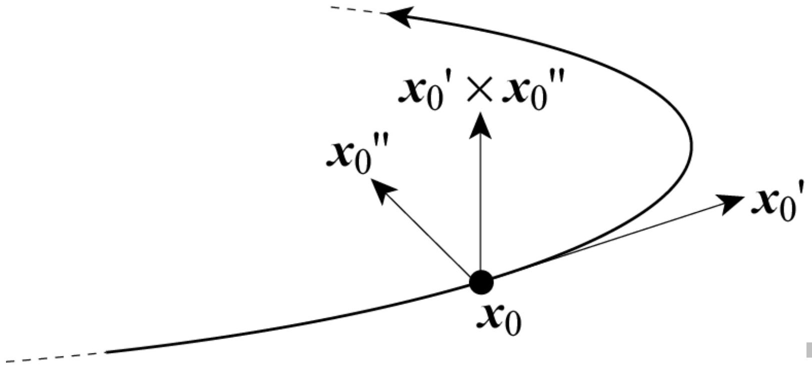


T. Araki, M. Tsubota and S. K. Nemirovskii, Phys. Rev. Lett. **89**, 145301 (2002)

$$\frac{\partial \boldsymbol{x}_0(t)}{\partial t} = \boldsymbol{v}_{\mathrm{s}}(\boldsymbol{x}_0)$$

$$\boldsymbol{v}_{\mathrm{s}}(\boldsymbol{x}) = \boldsymbol{v}_{\mathrm{ind}}(\boldsymbol{x}) + \boldsymbol{v}_{\mathrm{sa}}(\boldsymbol{x})$$

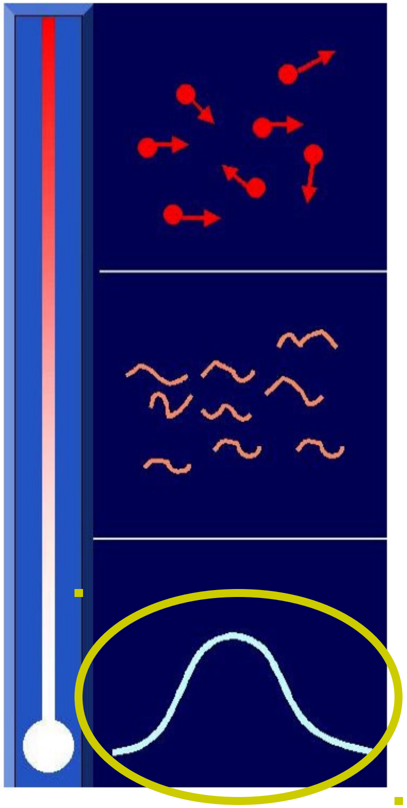
$$\boldsymbol{v}_{\mathrm{ind}}(\boldsymbol{x}) = \frac{\kappa}{4\pi}\int \frac{[\boldsymbol{x}_0(t) - \boldsymbol{x}] \times \mathrm{d}\boldsymbol{x}_0(t)}{|\boldsymbol{x}_0(t) - \boldsymbol{x}|^3}$$



Numerical Simulation of the Gross-Pitaevskii Equation

Gross-Pitaevskii equation

$$\hbar[\mathrm{i} - \gamma(x)]\frac{\partial}{\partial t}\Phi(x) = \left[-\frac{\hbar^2}{2m}\nabla^2 - \mu + \frac{4\pi\hbar^2 a}{m}|\Phi(x)|^2\right]\Phi(x)$$



a : Scattering length

$\gamma(x)$: Dissipation term for elementary excitations

Equation for dynamics of order parameter in BEC

Numerical Simulation of the Gross-Pitaevskii Equation

Gross-Pitaevskii equation

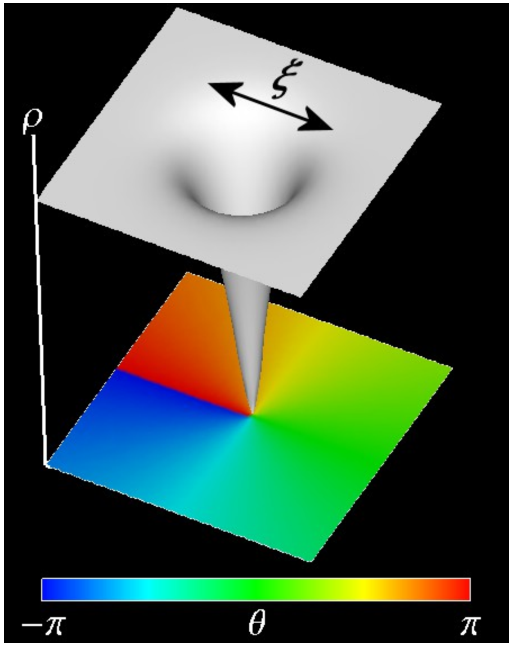
$$\hbar[\mathrm{i}-\gamma(x)]\frac{\partial}{\partial t}\Phi(x)=\left[-\frac{\hbar^2}{2m}\nabla^2-\mu+\frac{4\pi\hbar^2 a}{m}|\Phi(x)|^2\right]\Phi(x)$$

$$\Phi(\boldsymbol{x})=|\Phi(\boldsymbol{x})|\exp[\mathrm{i}\theta(\boldsymbol{x})]$$

$$\rho(\boldsymbol{x})=|\Phi(\boldsymbol{x})|^2 \text{ : Density}$$

$$\boldsymbol{v}(\boldsymbol{x})=(\hbar/m)\nabla\theta(\boldsymbol{x}) \text{ : Velocity field}$$

$$\xi=1/\sqrt{8\pi a\bar{\rho}} \text{ : Vortex core size}$$

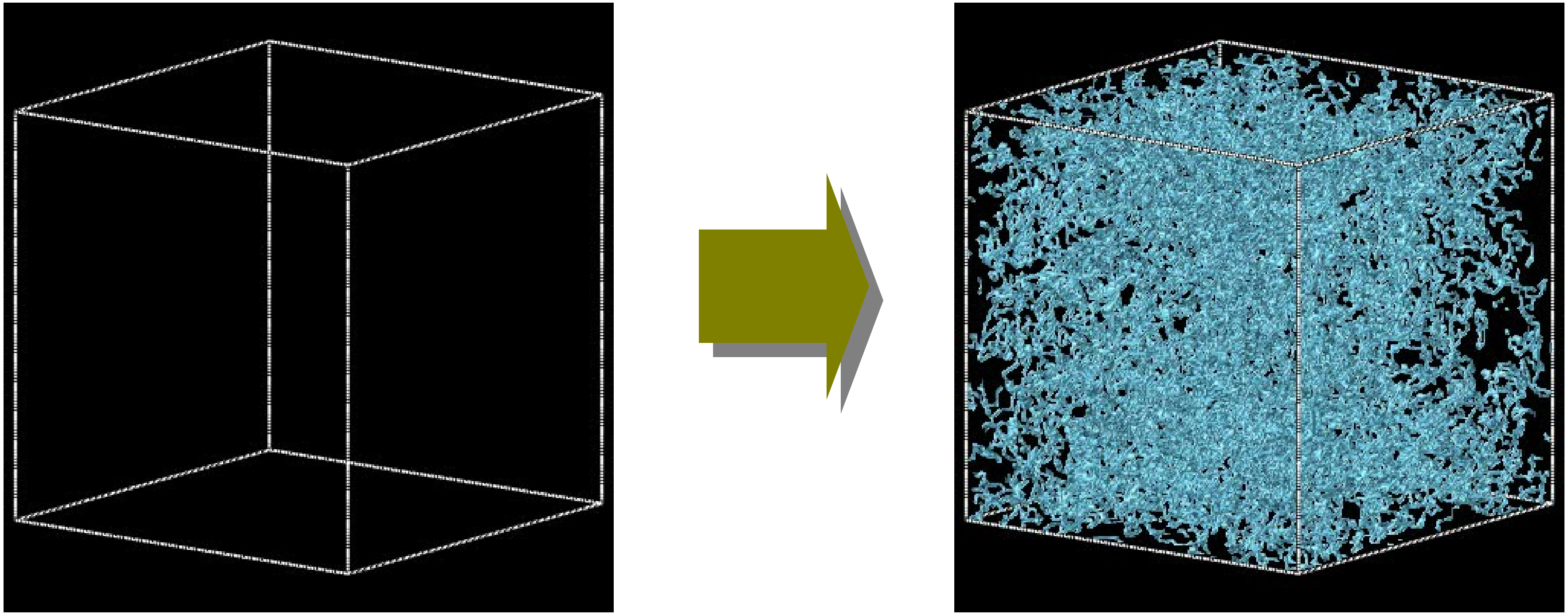


Vortex

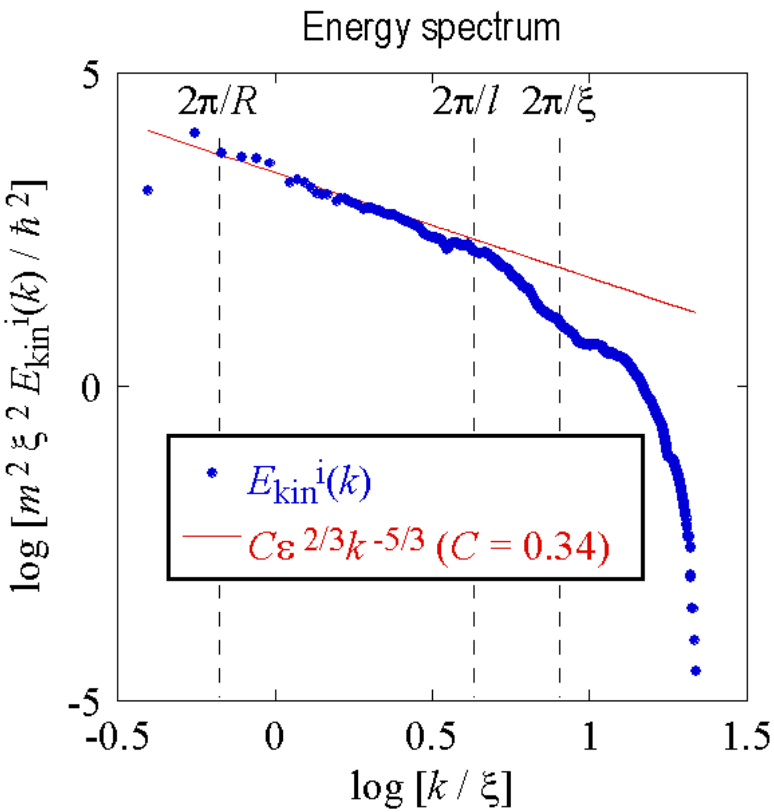
Quantum Turbulence From Quantized Vortices

Quantum turbulence can be realized as tangled quantized vortices

Simulation of quantum turbulence by Gross-Pitaevskii equation



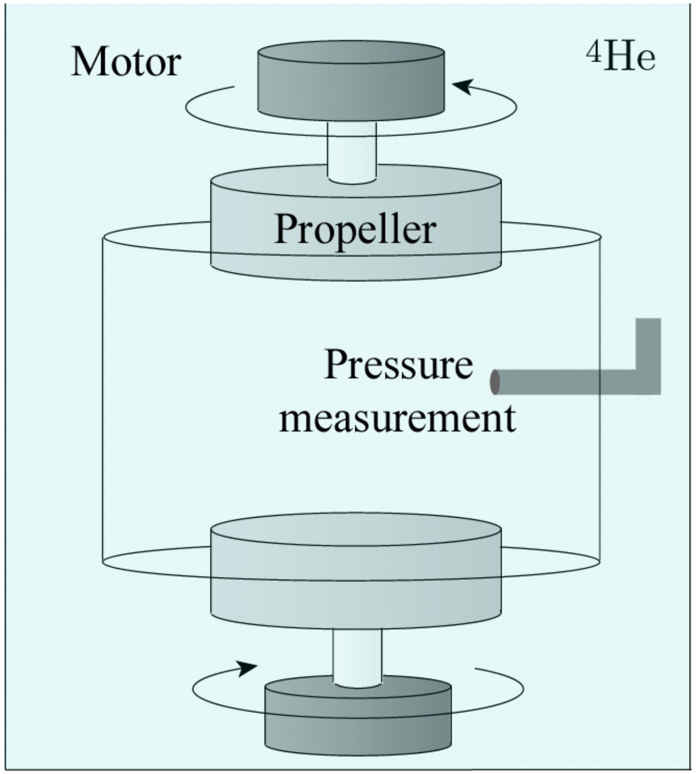
Energy Spectrum of the Gross-Pitaevskii Turbulence



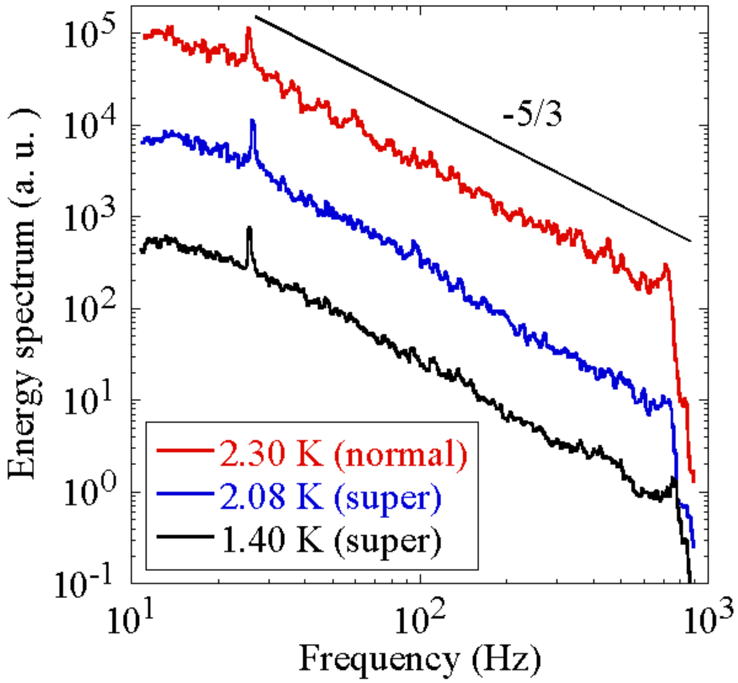
We observed the Kolmogorov law : $E(k) \propto k^{-5/3}$ between scale of injected vortex ring R and the vortex core size ξ.

Quantum Turbulence From Quantized Vortices

Quantum turbulence can be realized as tangled quantized vortices

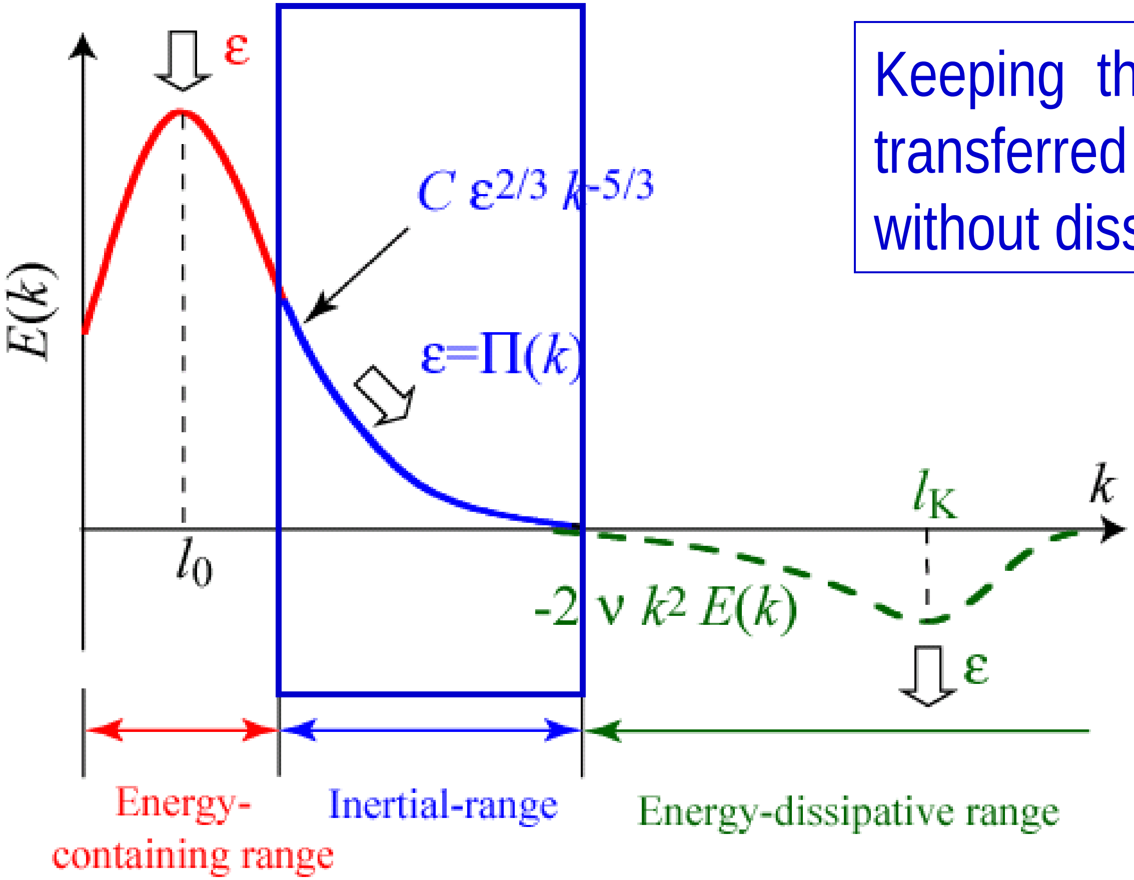


J. Maurer and P. Tabeling,
Europhys. Lett. **43** (1), 29 (1998)



There are some similarities between classical and quantum turbulence

Kolmogorov Law for Fully Developed Steady Turbulence

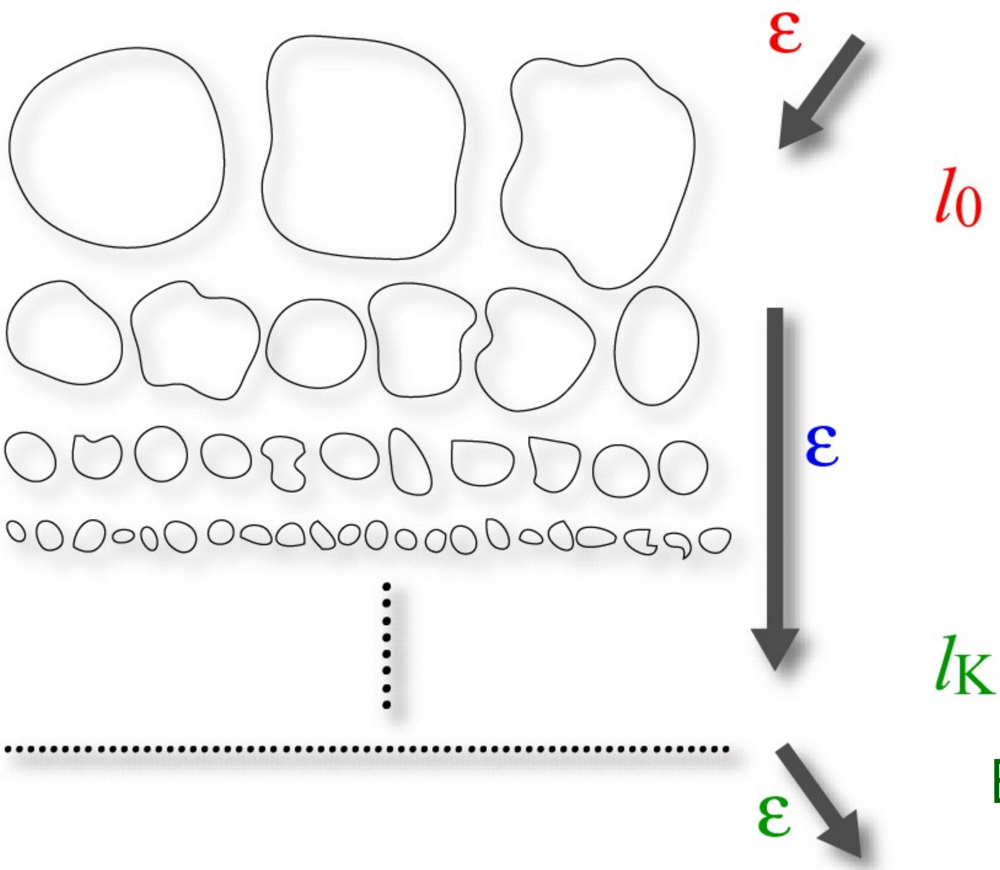


Keeping the self-similarity, Energy is transferred from large to small scales without dissipation →Kolmogorov law

$$E(k) = C\varepsilon^{2/3}k^{-5/3}$$

C : Kolmogorov constant

Richardson Cascade of Vortices



Energy-containing range : Large eddies are nucleated

Inertial range : Eddies are broken up to small ones

Energy-dissipative range : Small eddies are dissipated

Richardson Cascade of Vortices

WEATHER PREDICTION

BY

NUMERICAL PROCESS

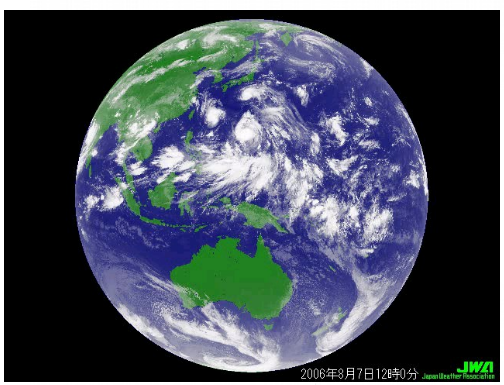
BY

LEWIS F. RICHARDSON, B.A., F.R.Met.Soc., F.Inst.P.

FORMERLY SUPERINTENDENT OF ESKDALEMUIR OBSERVATORY

LECTURER ON PHYSICS AT WESTMINSTER TRAINING COLLEGE

"big whirls have little whirls which feed on their velocity, and little whirls have lesser whirls and so on to viscosity —"



CAMBRIDGE
AT THE UNIVERSITY PRESS
1922

Exceptionally low diffusivities have been measured at night by L. F. Richardson (32) in the cold air near the earth. Airmen are very familiar with the increased bumpiness of the wind caused by sun shining on the ground below them. All these facts show that the production of eddies in the wind is greatly facilitated when the thermal equilibrium becomes less stable, although we may not suppose that actual thermal instability is reached in the majority of cases, because such an event is unusual among the collected observations made either by registering balloons or from aeroplanes.

A quantitative theory of the criterion of turbulence has been given by L. F. Richardson (32).

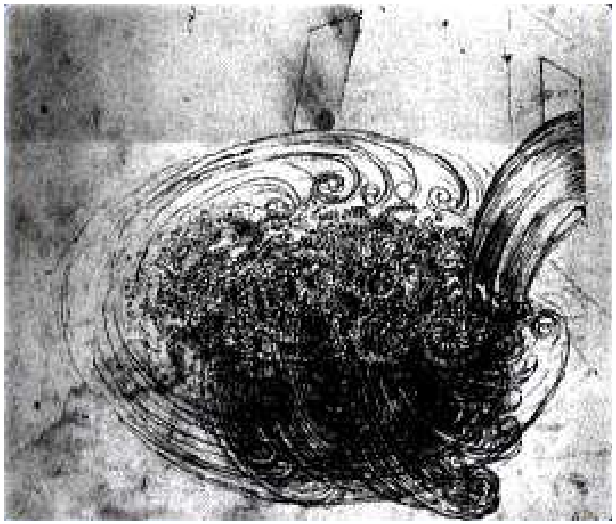
On the other hand we find that convectional motions are hindered by the formation of small eddies resembling those due to dynamical instability. Thus C. K. M. Douglas writing of observations from aeroplanes remarks: "The upward currents of large cumuli give rise to much turbulence within, below, and around the clouds, and the structure of the clouds is often very complex." One gets a similar impression when making a drawing of a rising cumulus from a fixed point; the details change before the sketch can be completed. We realize thus that: big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity—in the molecular sense.

Thus, because it is not possible to separate eddies into clearly defined classes according to the source of their energy; and as there is no object, for present purposes, in making a distinction based on size between cumulus eddies and eddies a few metres in diameter (since both are small compared with our coordinate chequer), therefore a single coefficient is used to represent the effect produced by eddies of all sizes and descriptions. We have then to study the variations of this coefficient. But first we must consider the differential equation. In doing so the aim has been to lay down theoretically only so much as can be determined with strictness, leaving all uncertainties to be decided by observation.

In hydrodynamics or aerodynamics it is customary to speak of the motions of "definite portions" of the fluid, portions which may be marked by a dot of milk in water or of smoke in air. The capital D in D/Dt is commonly used to denote a time differentiation following such a definite element. It is customary to ignore the fact that molecules are constantly passing in and out of the element called "definite." When we have to deal with eddies, the interchanges are more conspicuous, for boundaries marked by smoke would rapidly fade and disperse. Yet some way must be found of specifying an element which follows the *mean* motion. The fundamental idea seems to be the following. When there are no eddies we are accustomed to compute the flow of entropy or water across a plane from the flow of mass across the plane. As the effect of eddies is to be treated as additional, it should not include any flow due to the mean motion of mass across a plane. Accordingly we should adopt some such definition as the following:

Draw a sphere in the fluid. Let the radius be as large as is necessary to include a considerable number of eddies, but no larger. Let the sphere move so that the whole momentum of the fluid inside it is equal to the mass of the same fluid multiplied

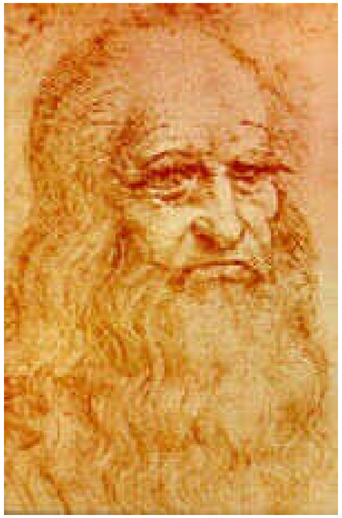
Leonardo da Vinci Already Had Same Image



Sketch of eddies in turbulence made by water pipe

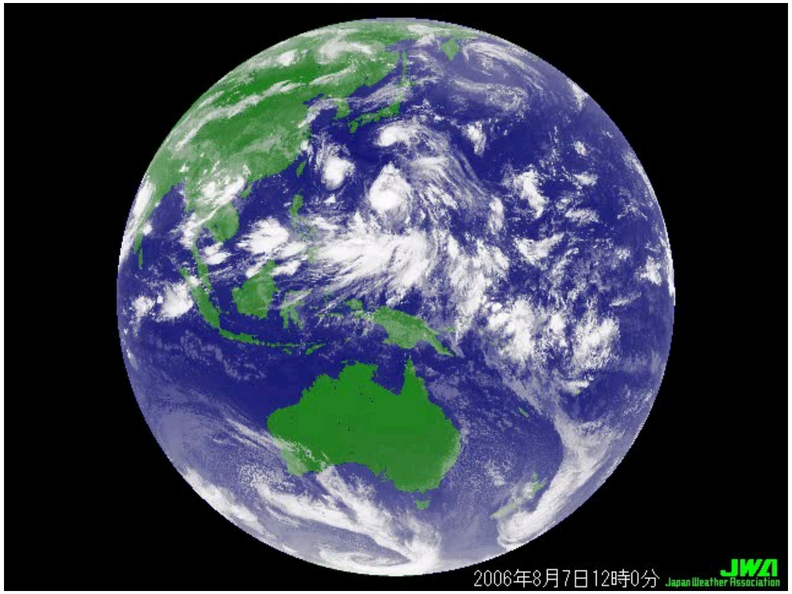
Leonardo da Vinci

- Turbulence is constituted by eddies.
- Turbulence classify eddies into size.
- Eddies with same class interact each other.

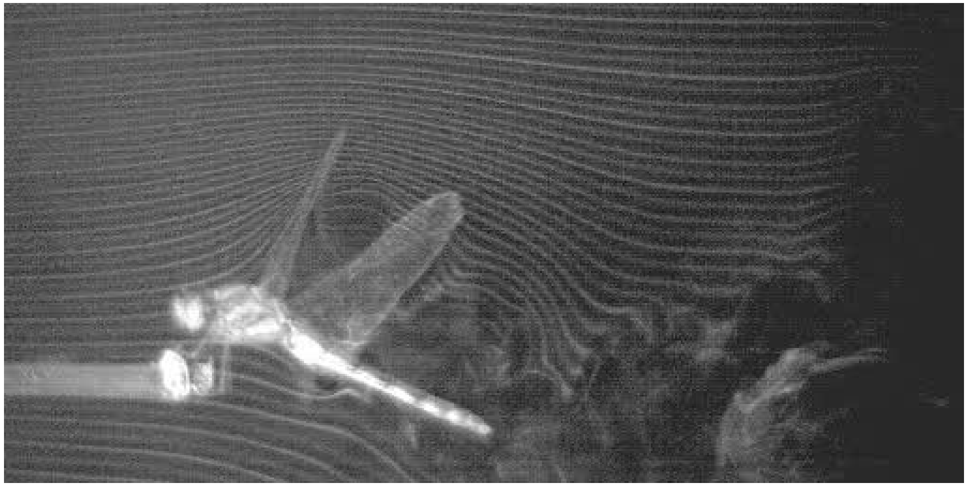


Eddies in Classical Turbulence

Earth turbulence



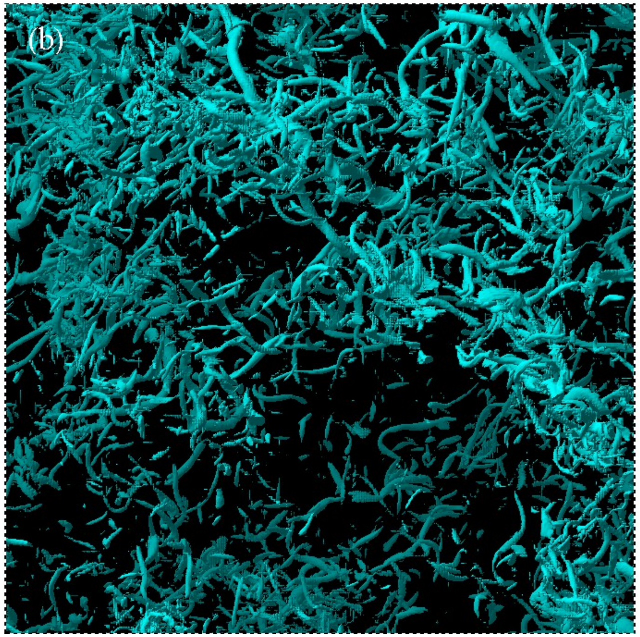
Dragonfly turbulence

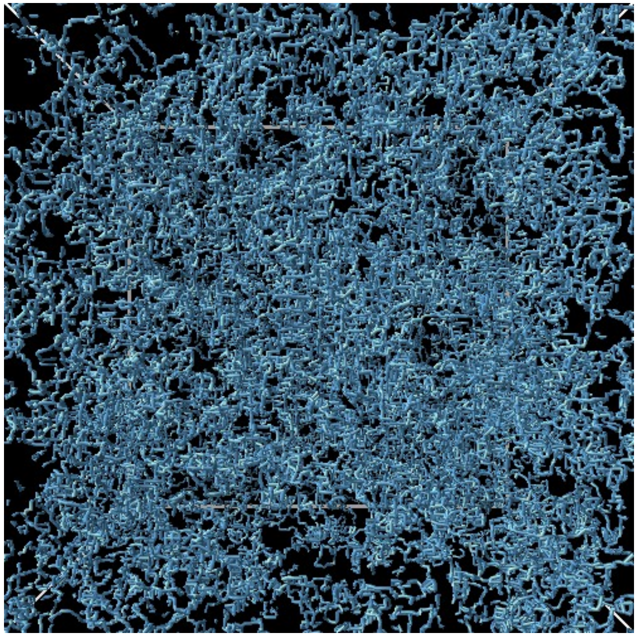


It is very difficult to identify eddies and the Richardson cascade (Eddies are diffused by the viscosity)

Identification of Vortices

Y. Kaneda, *et al*, Phys. Fluids. **15**, L21 (2003)



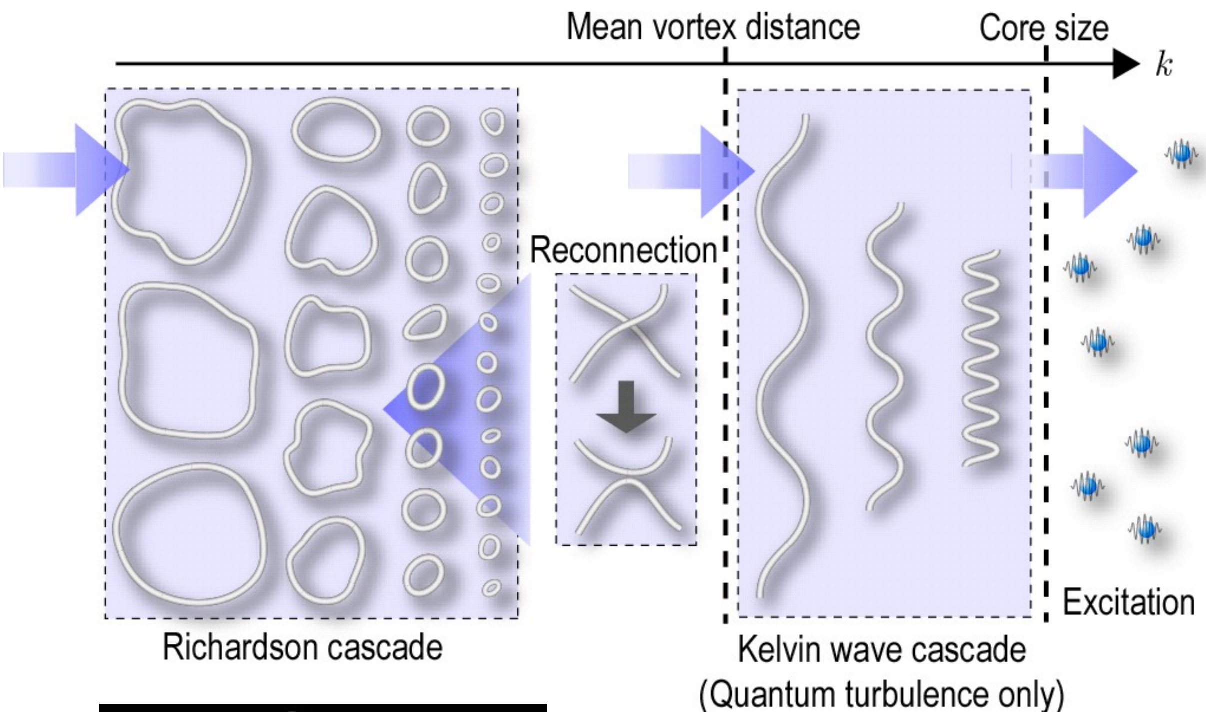


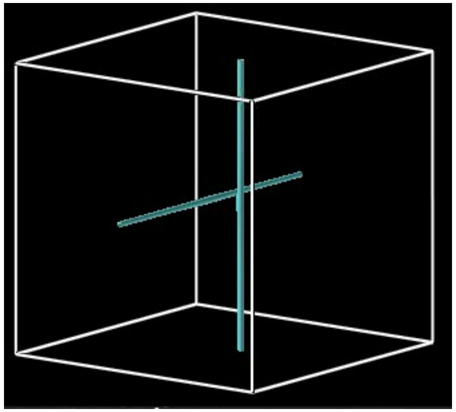
Classical turbulence : difficult

Quantum turbulence: already defined as topological defects

Richardson Cascade : Quantum Turbulence Version

W. F. Vinen and R. Donnelly, Physics Today **60**, 43 (2007)





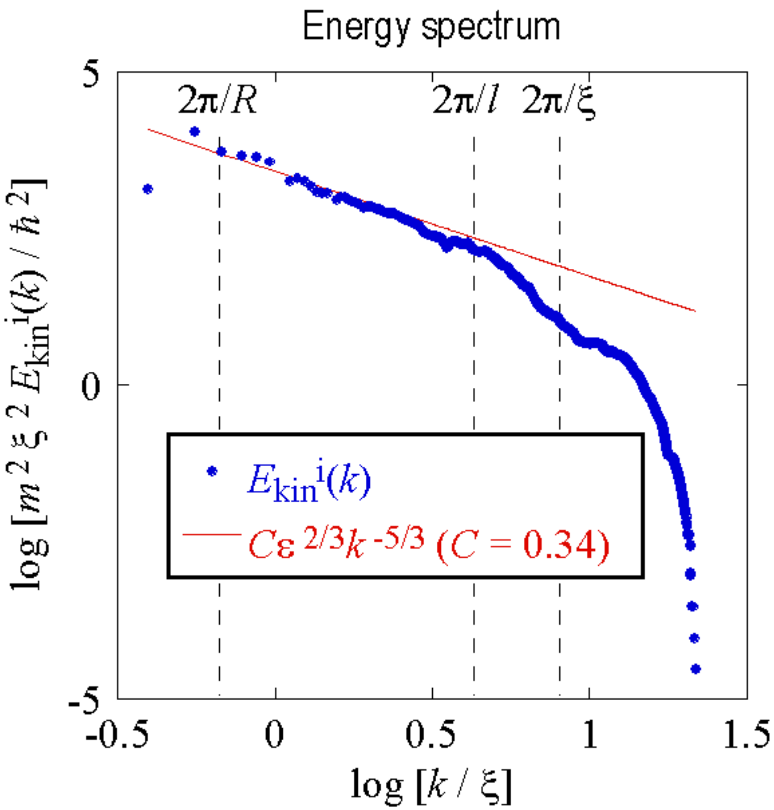
Reconnection : Elementary process of turbulence

Cascade of quantized vortices can be expected in quantum turbulence.

Not only Richardson cascade, but also Kelvin wave cascade is also expected in quantum turbulence

Vortex dissipates to elementary excitations (This effect is not included in Gross-Pitaevskii equation)

Energy Spectrum of the Gross-Pitaevskii Turbulence



R : Size of injected vortex rings

$E(k) \propto k^{-5/3}$: Kolmogorov law

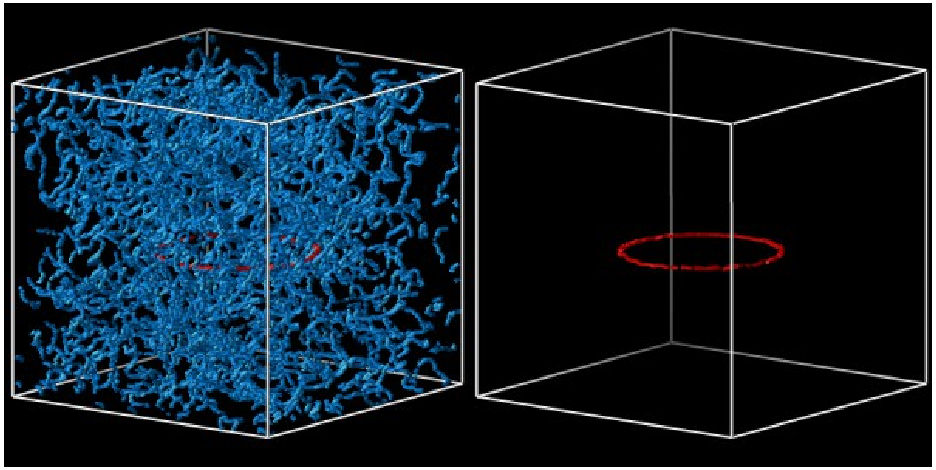
$l = (V/L)^{1/2}$: Vortex mean distance

$E(k) \propto k^{-6}$: Different scaling from the Kolmogorov law (Kelvin wave turbulence : intrinsic phenomenon of quantum turbulence?)

ξ : Vortex core size

The Study of Quantum Turbulence in the Viewpoint of Quantized Vortices

Quantized vortices give the real Richardson cascade in turbulence



Cascade of 1 vortex ring in turbulence

What is the relation between cascades in wave number space and real space?

Enstrophy and its spectrum

$$Q = \int \mathrm{d}\boldsymbol{x}\, |\nabla \times v(\boldsymbol{x})|^2 = \int \mathrm{d}k\, k^2 E(k) = \int \mathrm{d}k\, Q(k)$$

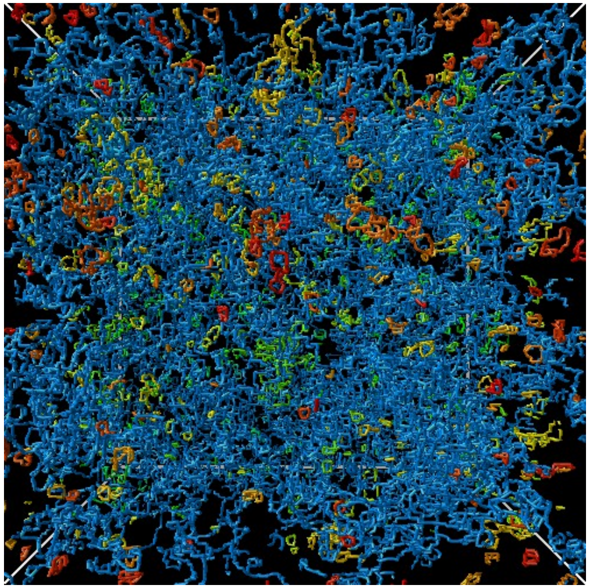
Relation Between Wave Number Space and Real Space

In quantum turbulence, enstrophy is directly related to vortex line length

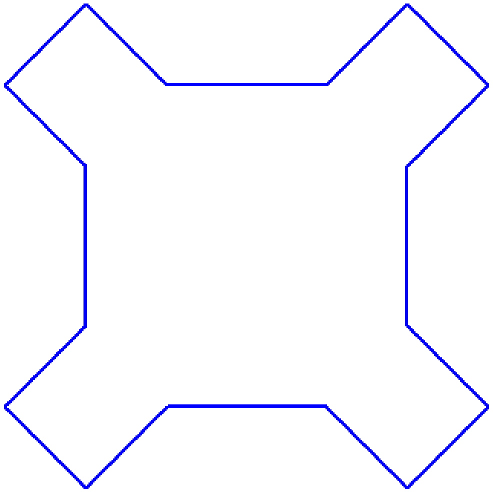
$$Q = \int \mathrm{d}\boldsymbol{x}\, |\nabla \times v(\boldsymbol{x})|^2 = \int \mathrm{d}k\, Q(k) \propto \kappa L \text{ (Quantum turbulence)}$$

Vortex line length spectrum : $E(k) \propto k^{-5/3} \rightarrow Q(k) \propto L(k) \propto k^{1/3}$

1, Vortex length by the size of vortex ring

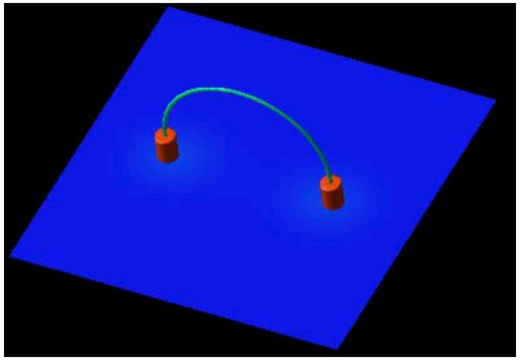


2, Fractal length



The Study of Quantum Turbulence by Superfluid Helium

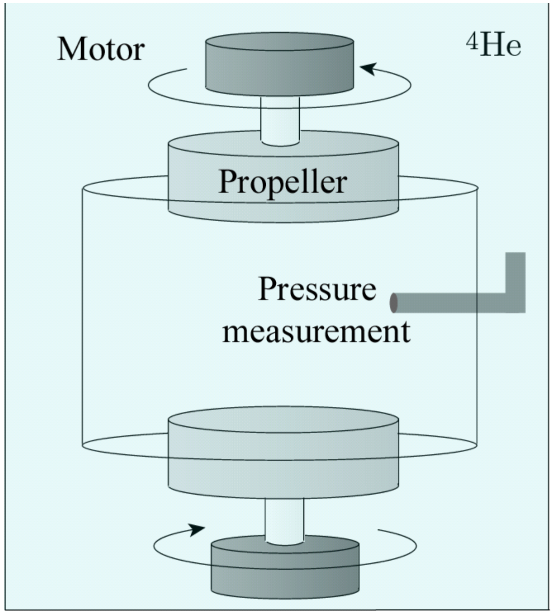
Quantum turbulence has been realized only in the system of superfluid helium



Vibrating wire

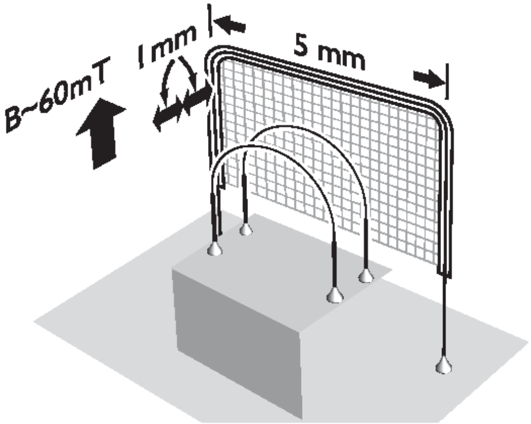
H. Yano *et al*. Phys. Rev. B **75**, 012502 (2007)

Two-counter rotating disks (Paris)



J. Maurer and P. Tabeling, Europhys. Lett. **43** (1), 29 (1998)

Oscillating grid (Lancaster)

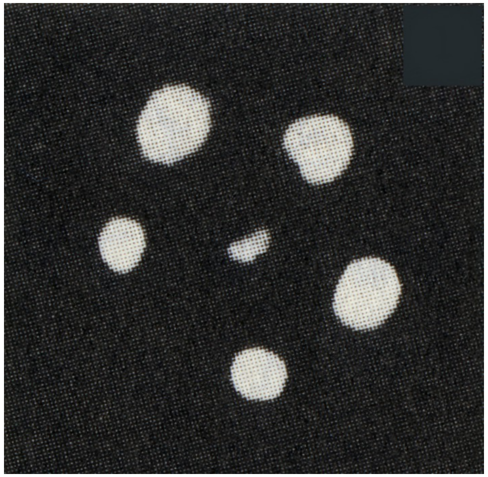


D. L. Bradley *et al*. Phys. Rev. Lett. **96**, 035301 (2-6)

Observation of Quantized Vortices

- (Second) sound
- Vibrating wire
- NMR second peak

→ Only total vortex line length can be measured



Visualization of vortex lattice under the rotation

E. J. Yarmchuk and R. E. Packard, J. Low Temp. Phys. **46**, 479 (1982).

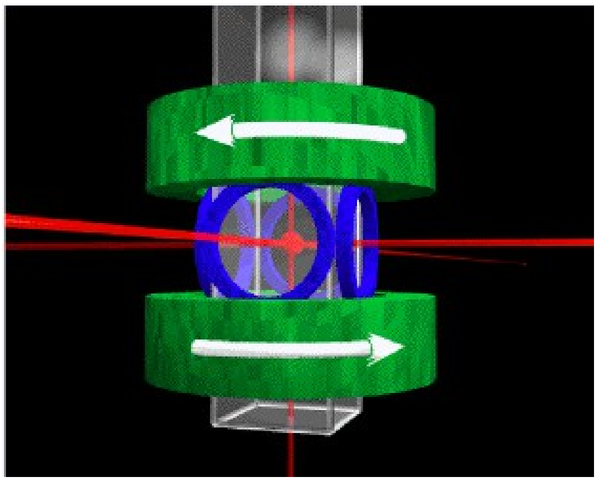
It is very difficult to measure the spatial distribution of quantized vortices

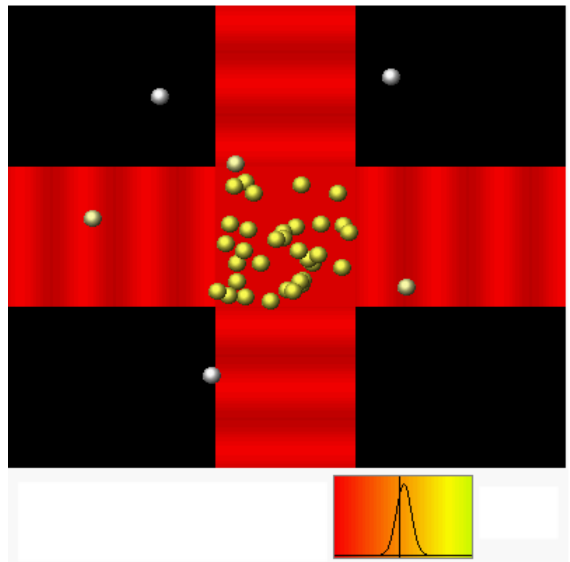
Atomic Bose-Einstein Condensates and Quantized Vortices

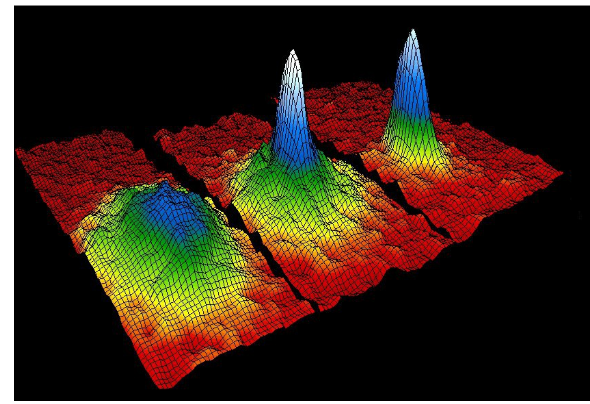
Laser cooling

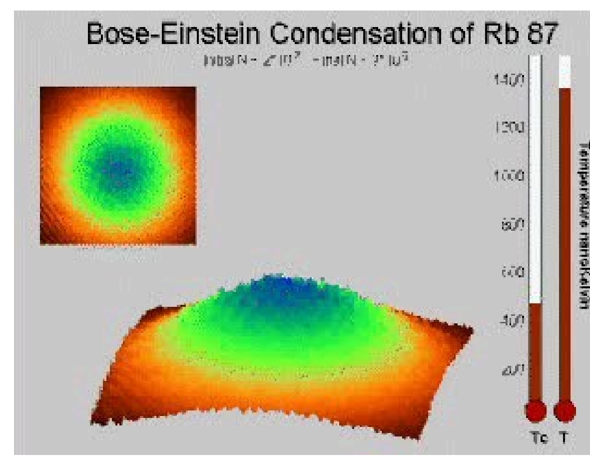
Trapped atomic gas

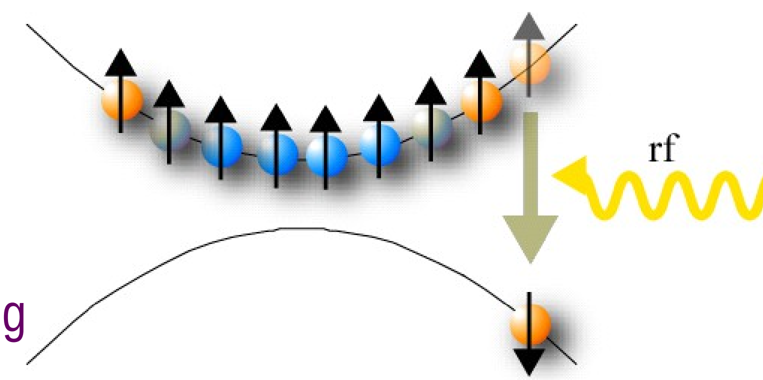
BEC







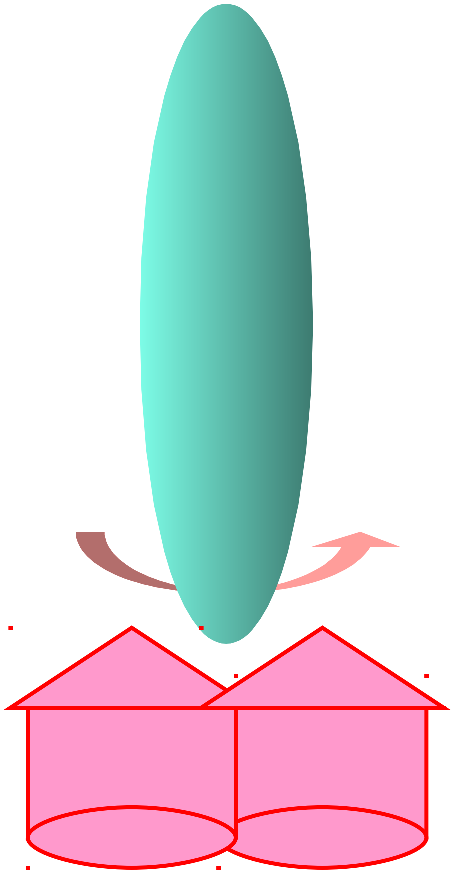




Evaporation cooling

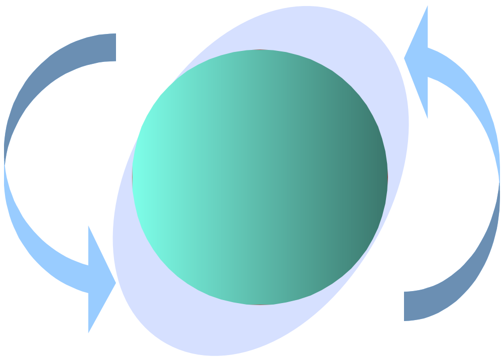
Observation of Vortex Lattice Under the Rotation

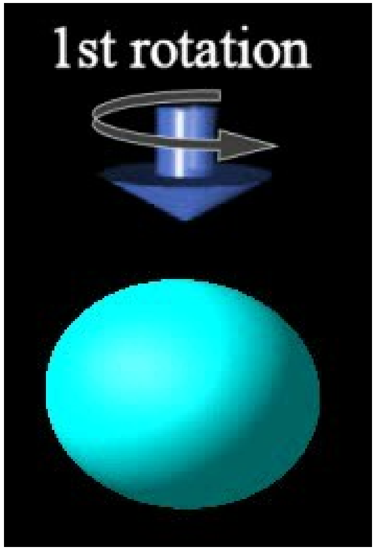
Rotation of BEC



Optical spoon

Rotation of anisotropic potential

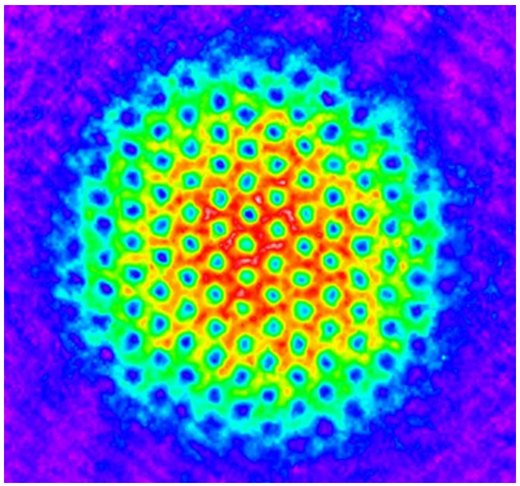




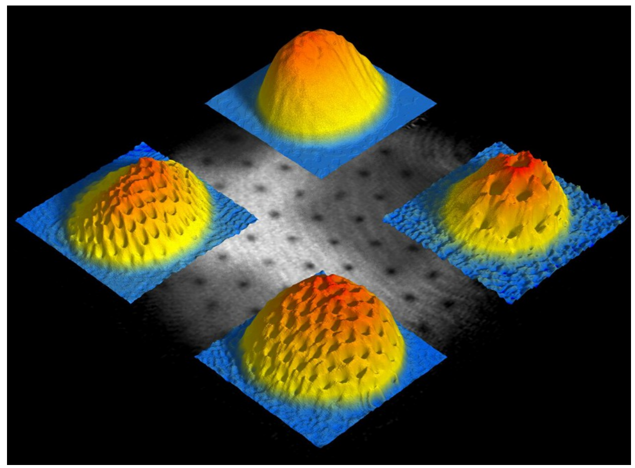
K.W.Madison et.al Phys.Rev Lett **84**, 806 (2000)

Observation of Vortex Lattice Under the Rotation

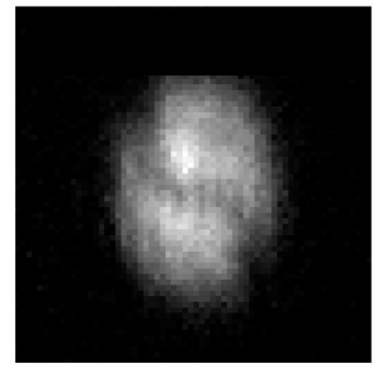
P. Engels, et.al
PRL **87**, 210403 (2001)



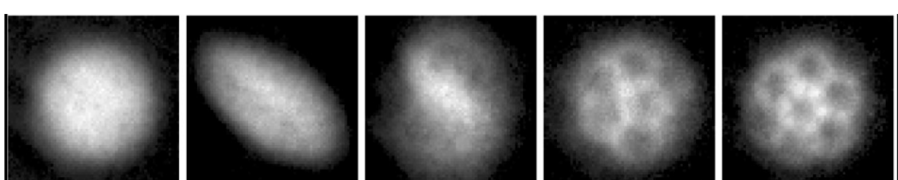
J.R. Abo-Shaeer, et.al
Science **292**, 476 (2001)



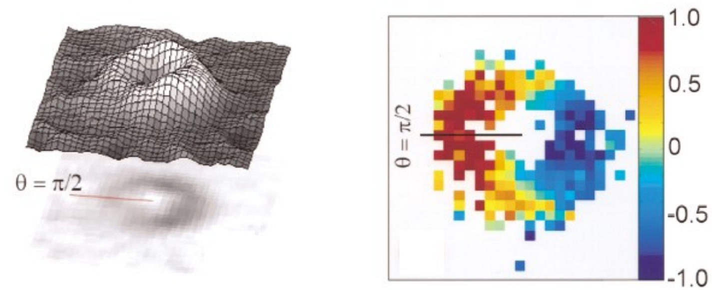
V. Bretin et al. PRL **90**, 100403(2003)



K. W. Madison et al. PRL **86**, 4443(2001)

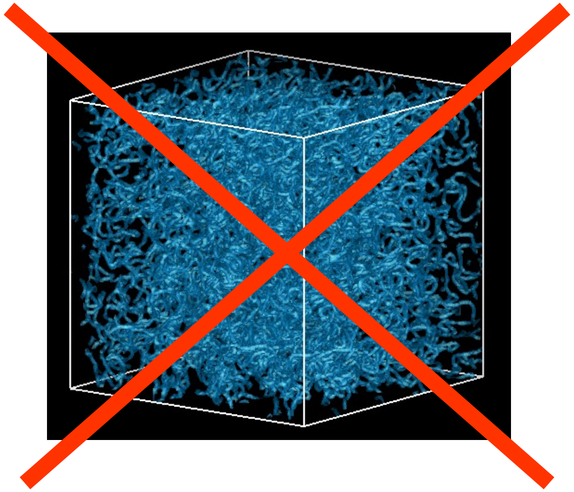


M. R. Matthews et al.
PRL **83**, 2498(1999)



The Study of Quantum Turbulence in Atomic BEC

There has been no research of quantum turbulence in this field



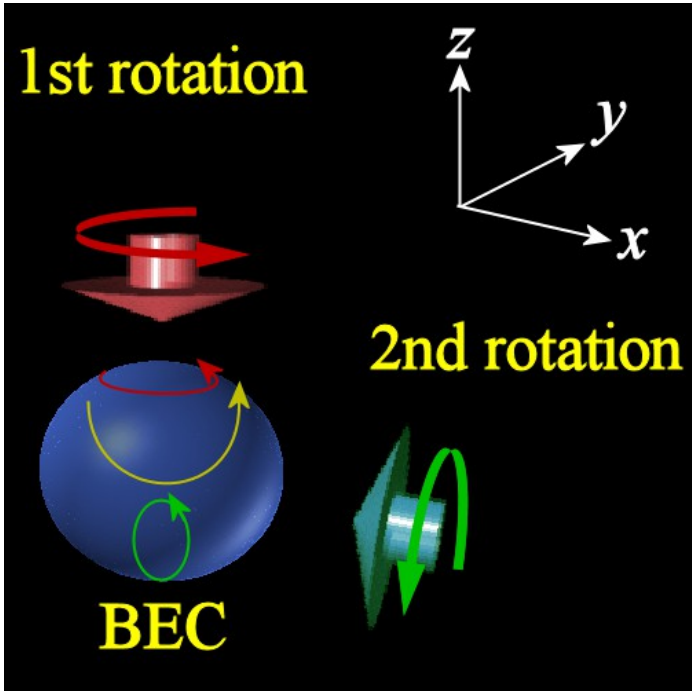
The merit of Atomic BEC

- Almost all physical parameters can be controllable such as the total number of particles, the temperature, the density, and even inter-particle interaction.
- Quantized vortices can be observed as holes of the density

Atomic BEC can be a good candidate to study quantum turbulence (Human being can get controllable turbulence!)

Toward the Realization of Quantum Turbulence

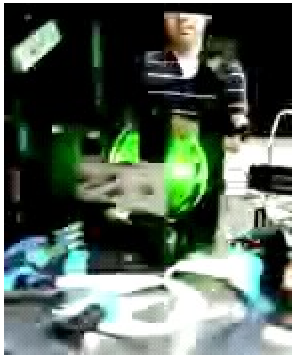
It is difficult to apply the velocity field to atomic BEC
→Effective tool : precession rotation



•Single rotation along one axis is realized without rotation along the other axis.

•Rotating vortex lattice can be realized when second rotation is weak.

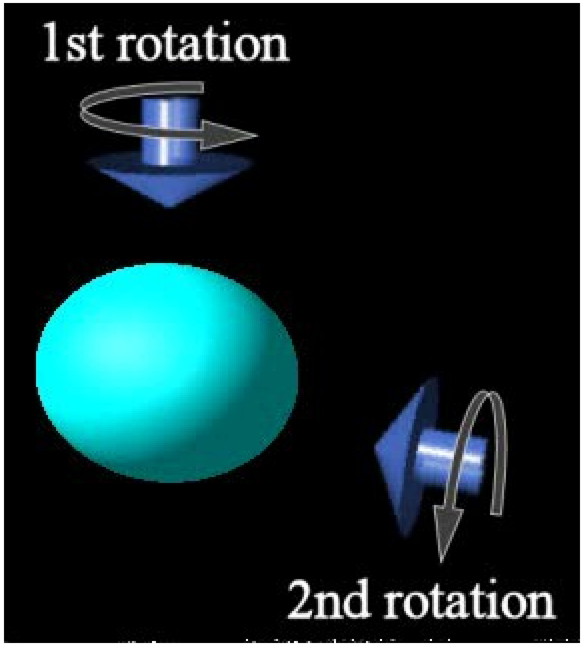
•Rotating lattice becomes unstable and enter turbulence when second rotation is strong.



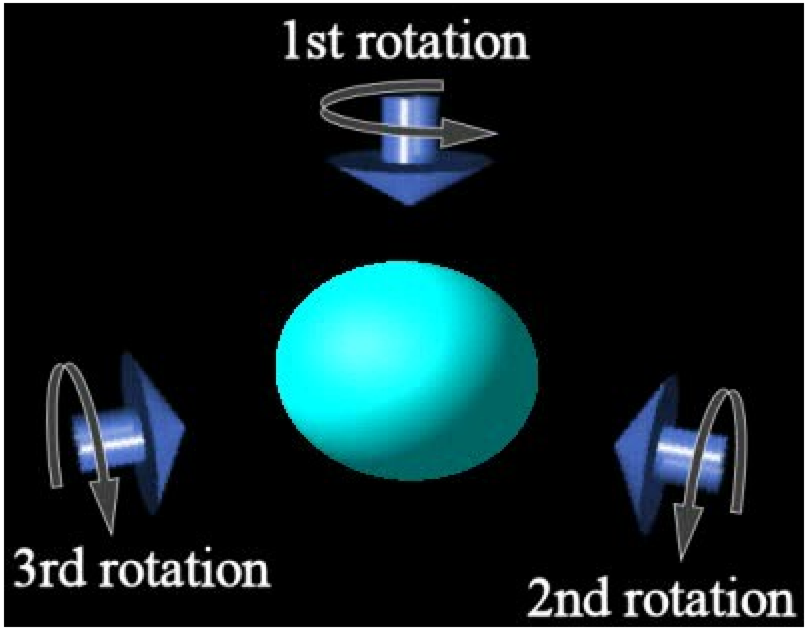
S. Goto, N. Ishii, S. Kida, and M. Nishioka Phys. Fluids **19**, 061705 (2007)

Precession Rotation in Atomic BEC

It is no need to rotate the experimental system itself for the case of atomic BEC



Precession rotation of optical spoon

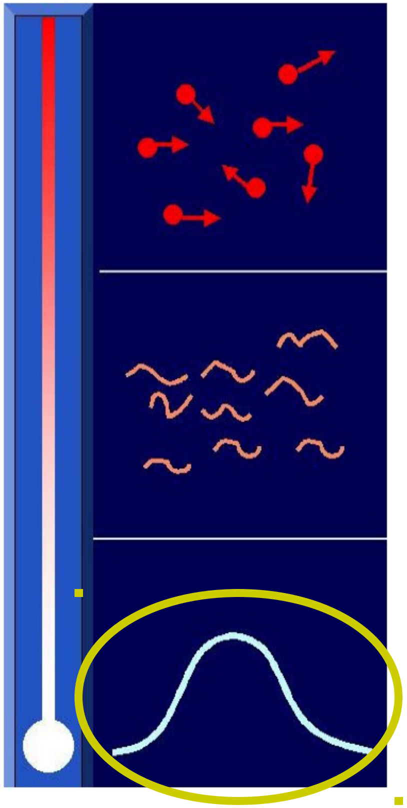


It is even possible to realize three axes rotation (more isotropic)

Numerical Simulation of the Gross-Pitaevskii Equation

Gross-Pitaevskii equation

$$\hbar[\mathrm{i}-\gamma(x)]\frac{\partial}{\partial t}\Phi(x)=\left[-\frac{\hbar^2}{2m}\nabla^2-\mu+U(\boldsymbol{x})+\frac{4\pi\hbar^2 a}{m}|\Phi(x)|^2-\boldsymbol{\Omega}(t)\cdot\boldsymbol{L}(\boldsymbol{x})\right]\Phi(x)$$



$U(\boldsymbol{x})$: Magnetic trapping potential

$\boldsymbol{\Omega}(t)$: Angular velocity of rotation

$L(\boldsymbol{x})$: Angular momentum operator

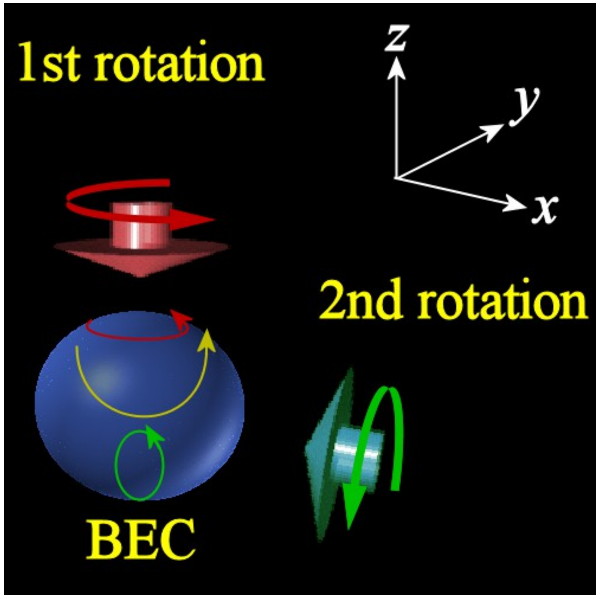
$\gamma(x)$: Dissipation term for elementary excitations

Equation for dynamics of order parameter in BEC

Numerical Simulation of the Gross-Pitaevskii Equation

Gross-Pitaevskii equation

$$\hbar[\mathrm{i}-\gamma(x)]\frac{\partial}{\partial t}\Phi(x) = \left[-\frac{\hbar^2}{2m}\nabla^2 - \mu + U(\boldsymbol{x}) + \frac{4\pi\hbar^2 a}{m}|\Phi(x)|^2 - \boldsymbol{\Omega}(t)\cdot\boldsymbol{L}(\boldsymbol{x})\right]\Phi(x)$$



Precession rotation

$$\boldsymbol{\Omega}(t) = (\Omega_x, \Omega_z \sin\Omega_x t, \Omega_z \cos\Omega_x t)$$

Numerical Simulation of the Gross-Pitaevskii Equation

Gross-Pitaevskii equation

$$\hbar[\mathrm{i}-\gamma(x)]\frac{\partial}{\partial t}\Phi(x)=\left[-\frac{\hbar^2}{2m}\nabla^2-\mu+U(\boldsymbol{x})+\frac{4\pi\hbar^2 a}{m}|\Phi(x)|^2-\boldsymbol{\Omega}(t)\cdot\boldsymbol{L}(\boldsymbol{x})\right]\Phi(x)$$

Anisotropic trapping potential

$$U(\boldsymbol{x})=\frac{m\omega^2}{2}[(1-\epsilon_1)(1-\epsilon_2)x^2+(1+\epsilon_1)(1-\epsilon_2)y^2+(1+\epsilon_2)z^2]$$

Numerical Simulation of the Gross-Pitaevskii Equation

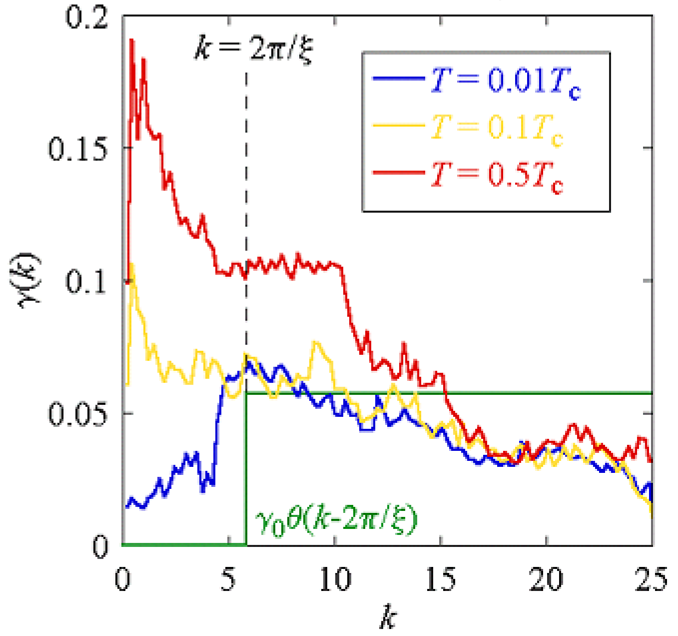
Gross-Pitaevskii equation

$$\hbar[\mathrm{i}-\gamma(x)]\frac{\partial}{\partial t}\Phi(x)=\left[-\frac{\hbar^2}{2m}\nabla^2-\mu+U(\boldsymbol{x})+\frac{4\pi\hbar^2 a}{m}|\Phi(x)|^2-\boldsymbol{\Omega}(t)\cdot\boldsymbol{L}(\boldsymbol{x})\right]\Phi(x)$$

Dissipation by the elementary excitation

$$\gamma(k)=\gamma_0\theta(k-2\pi/\xi_0)$$

: Effective in the scales smaller than the vortex core



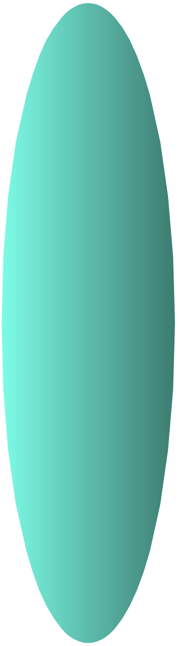
MK & MT, PRL. **97**, 145301 (2006)

Vortex Lattice Simulation

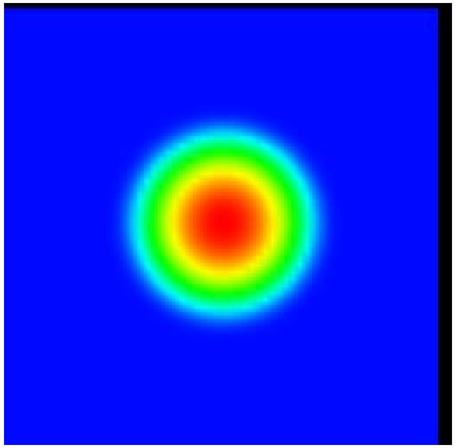
$^{87}\text{Rb atoms}: \quad m = 1.46 \times 10^{-25}\ \text{kg},\ a = 5.61\ \text{nm}$

$N = 2.50 \times 10^{5},\ \epsilon_1 = 0.05$

$\omega_x = \omega_y = 120 \times 2\pi\ \text{Hz},\ \omega_z = 20 \times 2\pi\ \text{Hz}$

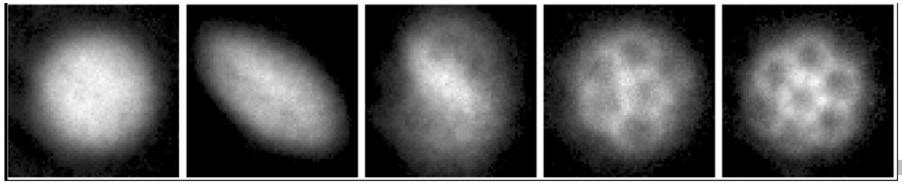


2D analysis for long BEC



$\Omega = 0.75\ \omega_x$

K. Kasamatsu, M. Tsubota and M. Ueda, PRA. **67**, 033610 (2003)



Quantum Turbulence Simulation

^{87}Rb atoms : $m = 1.46 \times 10^{-25}$ kg, $a = 5.61$ nm

$N = 2.50 \times 10^{5}$, $\omega = 150 \times 2\pi$ Hz

$\Omega_z = \Omega_x = 0.6\omega$, $\epsilon_1 = \epsilon_2 = 0.025$

Numerics : Space : Grid 512^3 with Dirichlet boundary

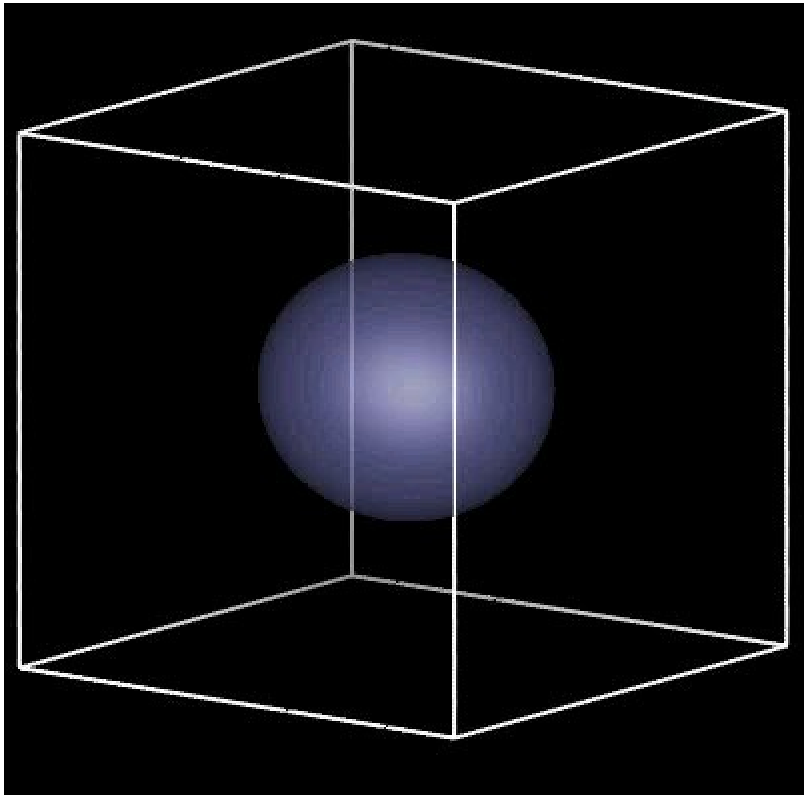
(Chebyshev+tau)^3, $V = 14.0^3$ μm Volume

Time : 4th ordered Runge-Kutta

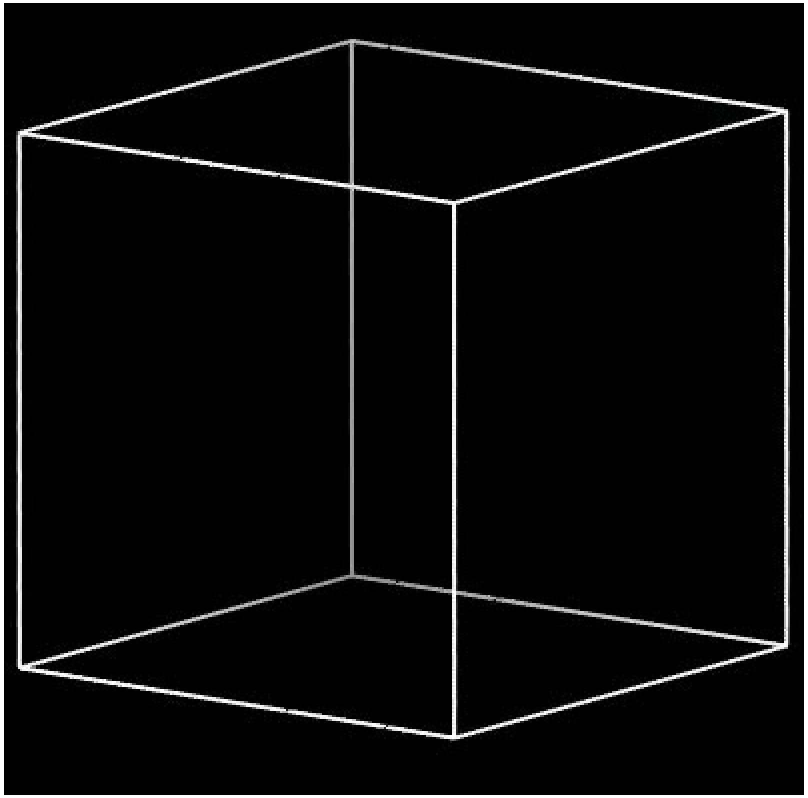
Initial condition : No rotation and anisotropy

Quantum Turbulence Simulation

Density



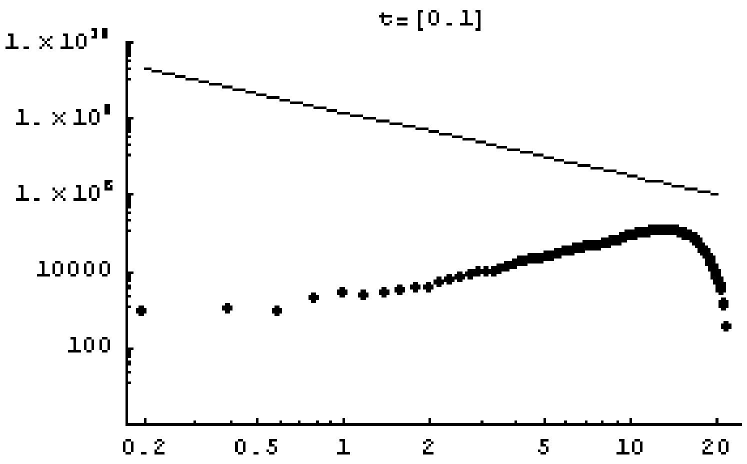
Vortex



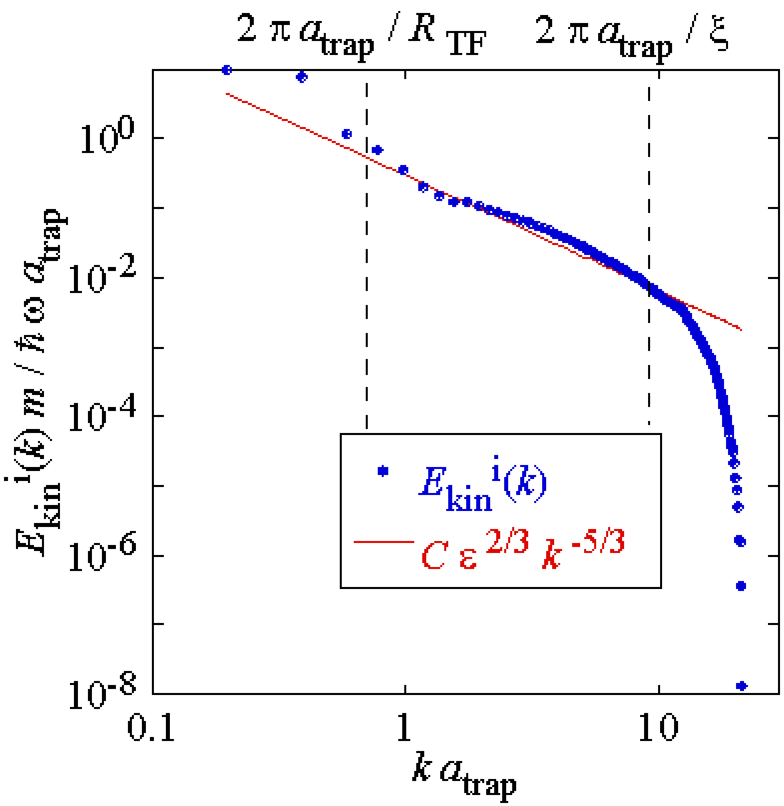
Vortices are not crystallized but tangled.

Quantum Turbulence Simulation

Energy spectrum

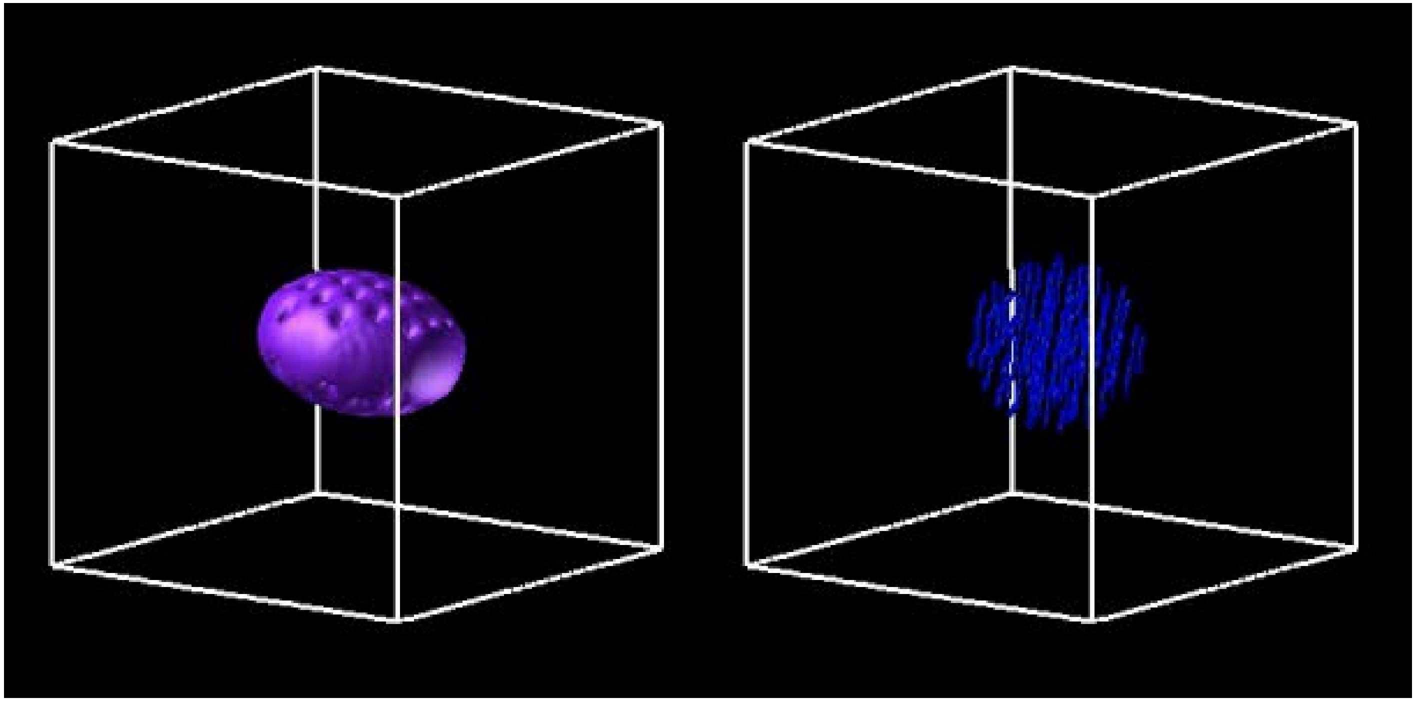


20 ensemble average

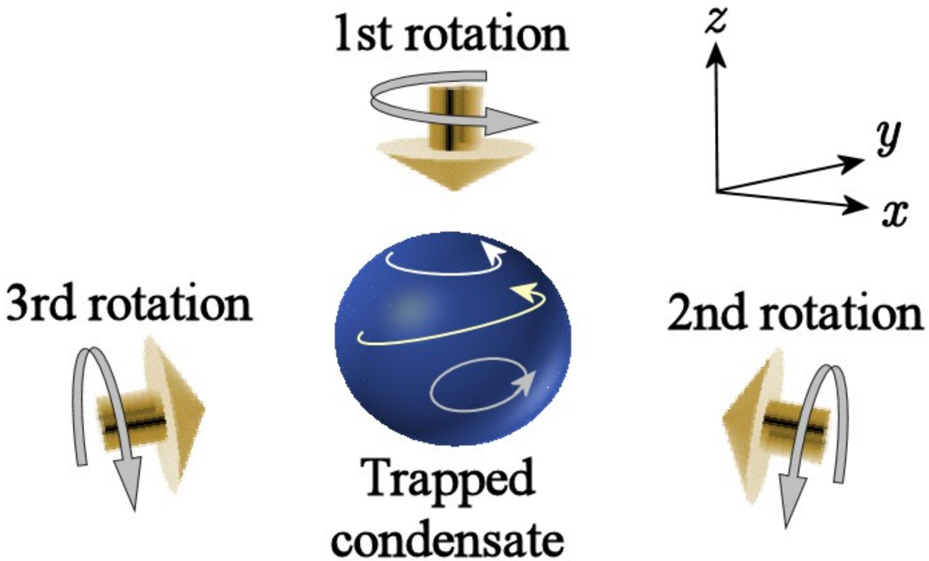


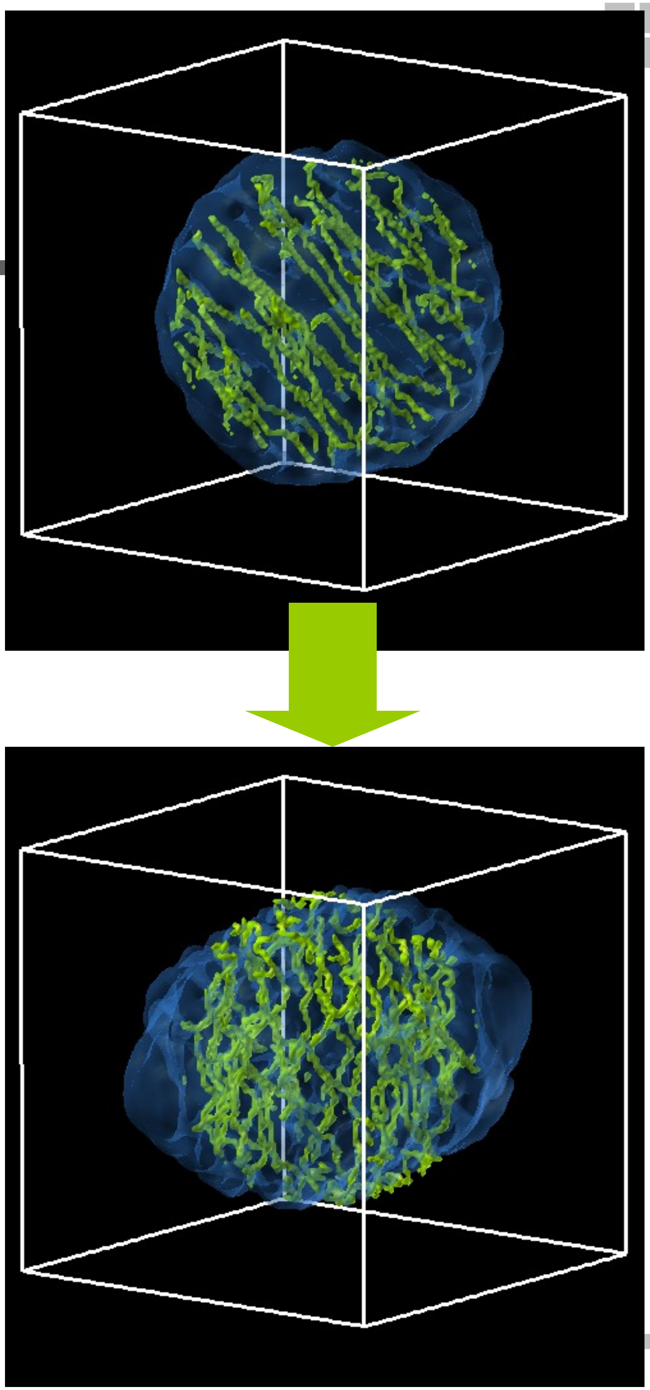
Quantum Turbulence Simulation

Starting from vortex lattice



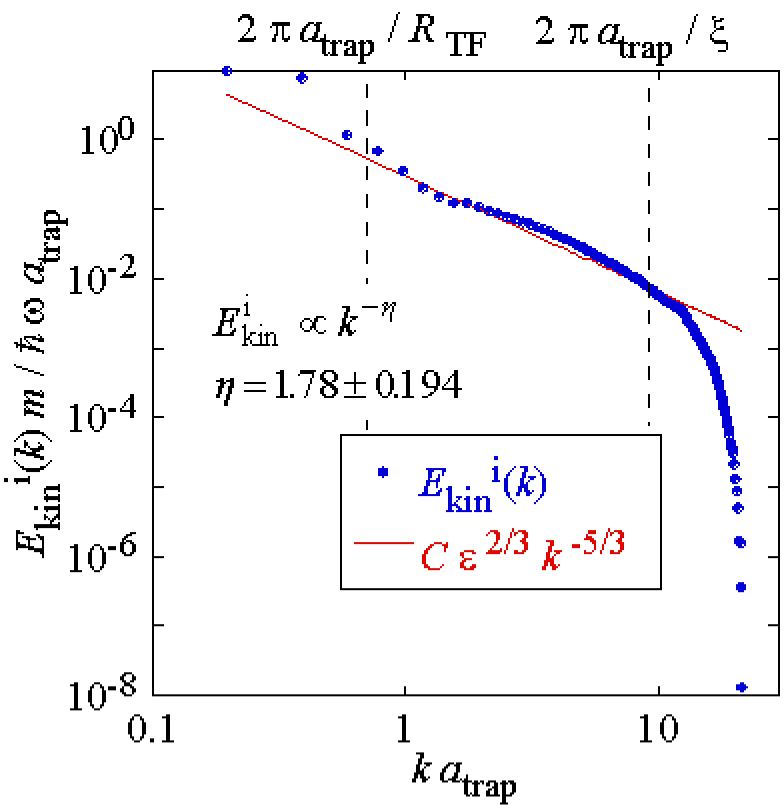
Three Axes Rotation

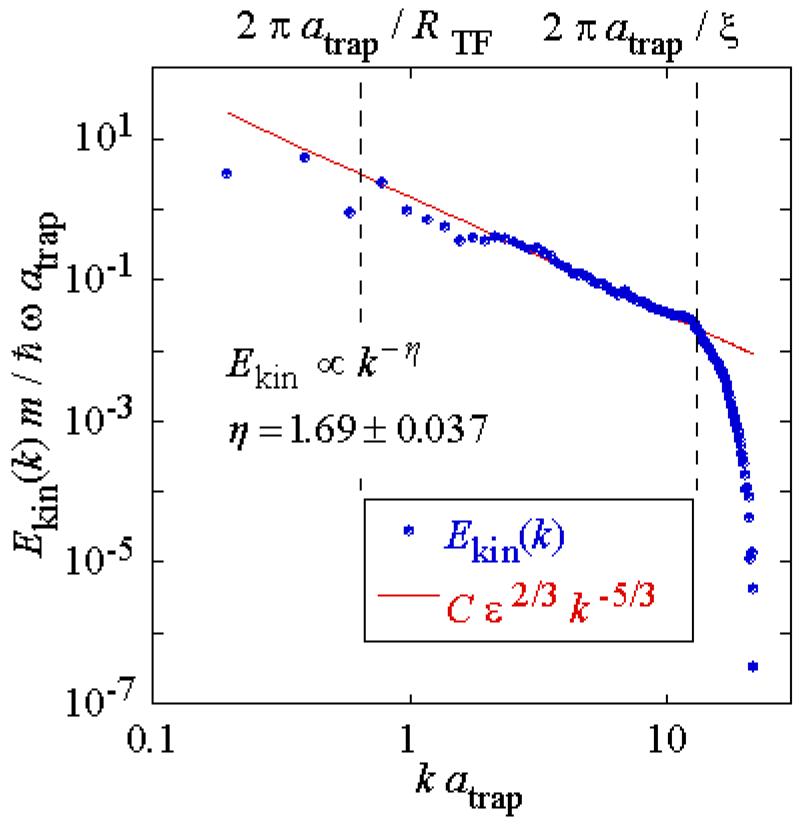




Vortex tangle becomes more isotropic

Three Axes Rotation



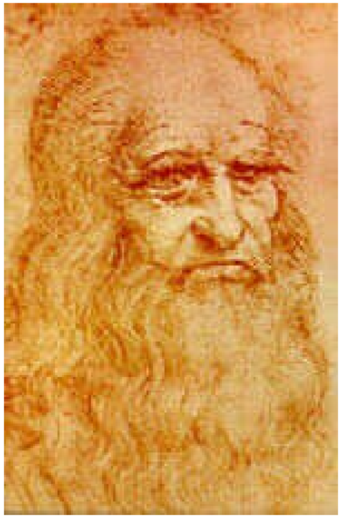


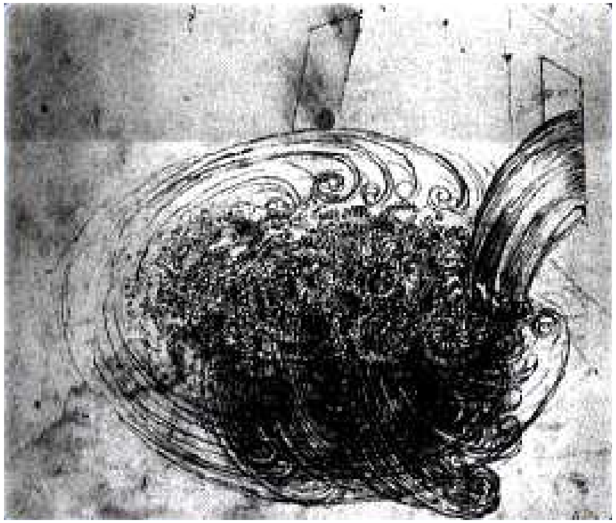
Agreement with Kolmogorov law becomes better

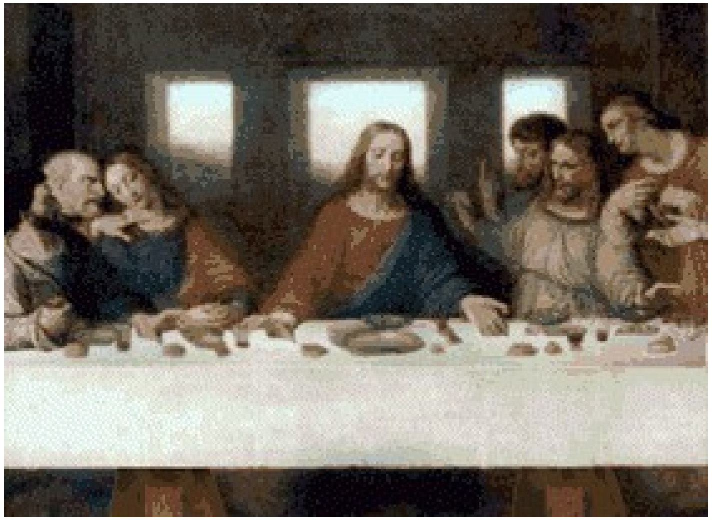
Summary

- Quantum turbulence is the good system to study turbulence because quantized vortices can be clearly identified (studying the Richardson cascade, the relation between cascade in wave number space and real space).
- Atomic Bose-Einstein condensation is the good experimental system to study quantum turbulence.

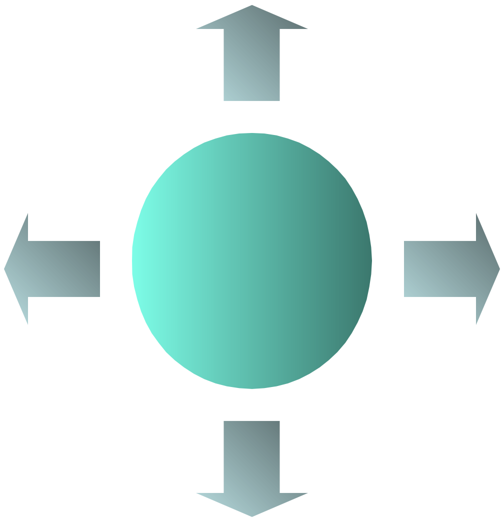
Thank You for Your Attention







Experimental Observation of the Kolmogorov Law



Expansion of BEC after switching off the magnetic trapping

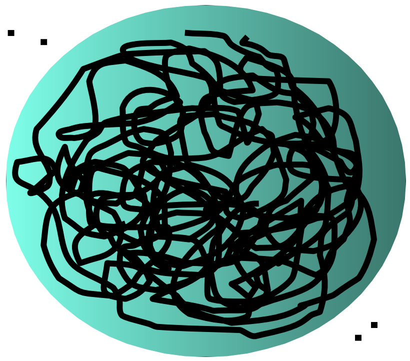
$$E \propto \int \mathrm{d}k\, k^{-5/3}$$

$$N \propto \int \mathrm{d}k\, k^{-11/3} \to N(k) \propto k^{-11/3}$$

$$v \sim k \to N(v) \propto v^{-11/6}$$

$$v \sim r(\mathrm{TOF}) \to N(r) \propto r^{-11/6}$$

Density distribution



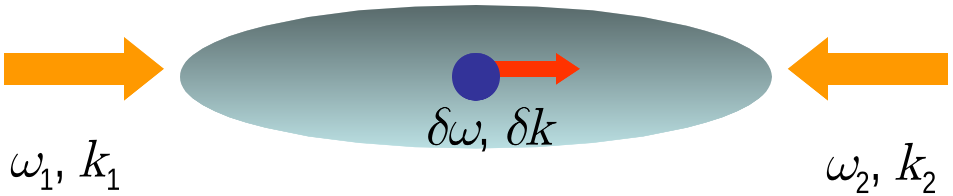
Two-dimensional projection of vortex configuration

$$L(k) \propto k^{1/3}$$

$$\to \int \mathrm{d}z\, L \cos\phi \propto \int \mathrm{d}k\, k^{-5/3}$$

Bragg Spectroscopy

Bragg spectroscopy with focused laser beam



Collective excitation of BEC

Spatial distribution of velocity field