減衰量子乱流のエネルギースペクトルと渦糸長密度

大阪市立大理　小林未知数・坪田誠

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Osaka City Univ.
Elementary Excitation
Physics Lab.

発表概要

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3. Gross-Pitaevskii乱流を用いた減衰量子乱流のシミュレーション
4. 考察とまとめ

1, イントロダクション：量子乱流におけるKolmogorovスペクトル

M. Kobayashi and M. Tsubota, Phys. Rev. Lett. **94**, 065302 (2005).
M. Kobayashi and M. Tsubota, J. Phys. Soc. Jpn. **74**, 3248 (2005).

Gross-Pitaevskii方程式を用いた量子乱流

GP方程式

$$\mathrm{i}\frac{\partial}{\partial t}\Phi(\boldsymbol{x},t) = [-\nabla^2 - \mu + V(\boldsymbol{x},t) + g|\Phi(\boldsymbol{x},t)|^2]\Phi(\boldsymbol{x},t)$$

$\Phi(\boldsymbol{x},t)$:BECオーダーパラメーター

$V(\boldsymbol{x},t)$: エネルギー注入

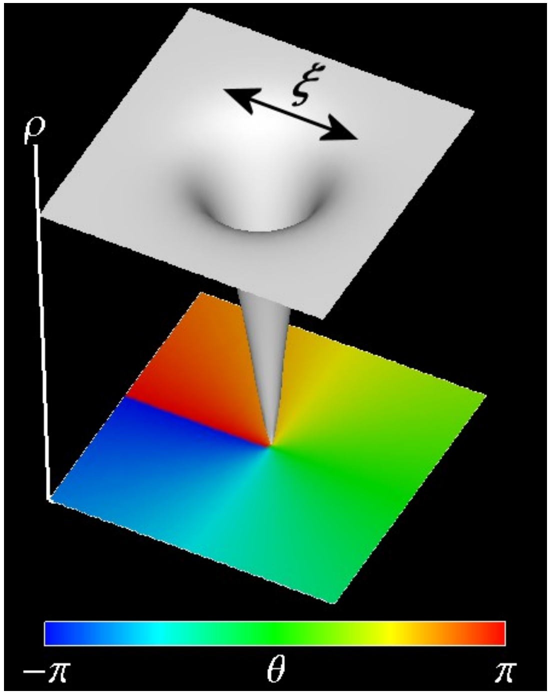
μ :化学ポテンシャル

g: 結合定数

量子渦

$$\mathrm{i}\frac{\partial}{\partial t}\Phi(\boldsymbol{x},t) = [-\nabla^2 - \mu + V(\boldsymbol{x},t) + g|\Phi(\boldsymbol{x},t)|^2]\Phi(\boldsymbol{x},t)$$

$$\boxed{\begin{aligned} \Phi(\boldsymbol{x}) &= |\Phi(\boldsymbol{x})|\exp[\mathrm{i}\theta(\boldsymbol{x})] \\ \rho(\boldsymbol{x}) &= |\Phi(\boldsymbol{x})|^2 : \text{量子渦} \\ \boldsymbol{v}(\boldsymbol{x}) &= 2\nabla\theta(\boldsymbol{x}) : \text{速度場} \\ \xi &= 1/\sqrt{g\bar{\rho}} : \text{回復長} \end{aligned}}$$



量子渦

小スケールのエネルギー散逸

フーリエ変換

$$[\mathrm{i}-\gamma(k)]\frac{\partial}{\partial t}\Phi(\boldsymbol{k},t)=(k^2-\mu)\Phi(\boldsymbol{k},t)+\sum_{\boldsymbol{k}_1}V(\boldsymbol{k}_1,t)\Phi(\boldsymbol{k}-\boldsymbol{k}_1,t)$$
$$+g\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2}\Phi(\boldsymbol{k}_1,t)\Phi^*(\boldsymbol{k}_2,t)\Phi(\boldsymbol{k}-\boldsymbol{k}_1+\boldsymbol{k}_2,t)$$

$$\gamma(k)=\gamma_0\theta(k-2\pi/\xi)$$

極低温では量子渦は直接散逸せず、渦芯よりも小さなスケールにのみ散逸が存在する

（量子および温度）揺らぎによる量子乱流の散逸機構

M. Kobayashi and M. Tsubota, cond-mat/0607434 (2006), Phys. Rev. Lett. in press.

GP & Bogoliubov-de-Gennes方程式による散逸構造の解析

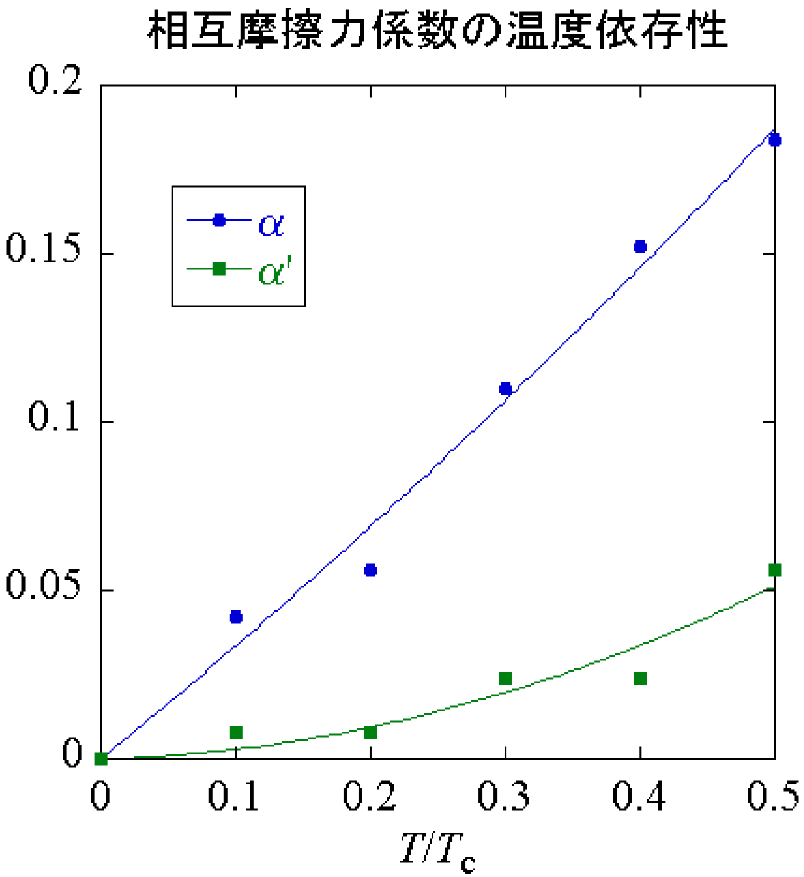
$$\mathrm{i}\frac{\partial \Phi}{\partial t} = [-\nabla^2 - \mu + g(|\Phi|^2 + 2\langle \hat{\chi}^\dagger \hat{\chi} \rangle)]\Phi + g\langle \hat{\chi}\hat{\chi} \rangle \Phi^* \ : \ \mathrm{GP}$$

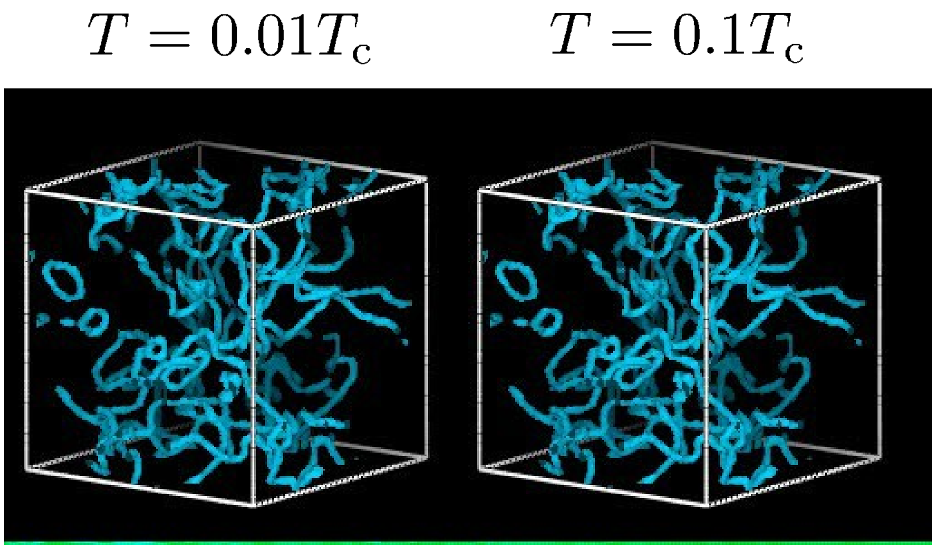
$$\mathrm{i}\frac{\partial \hat{\chi}}{\partial t} = [-\nabla^2 - \mu + 2g|\Phi|^2]\hat{\chi} + g\Phi^2 \chi^\dagger \ : \ \mathrm{BdG}$$

$\Phi(\boldsymbol{x}, t)$: BEC??????????

$\chi(\boldsymbol{x}, t)$: オーダーパラメータからのゆらぎ

量子乱流の散逸





極低温では量子渦は直接散逸せず、渦芯よりも小さなスケールにのみ散逸が存在する

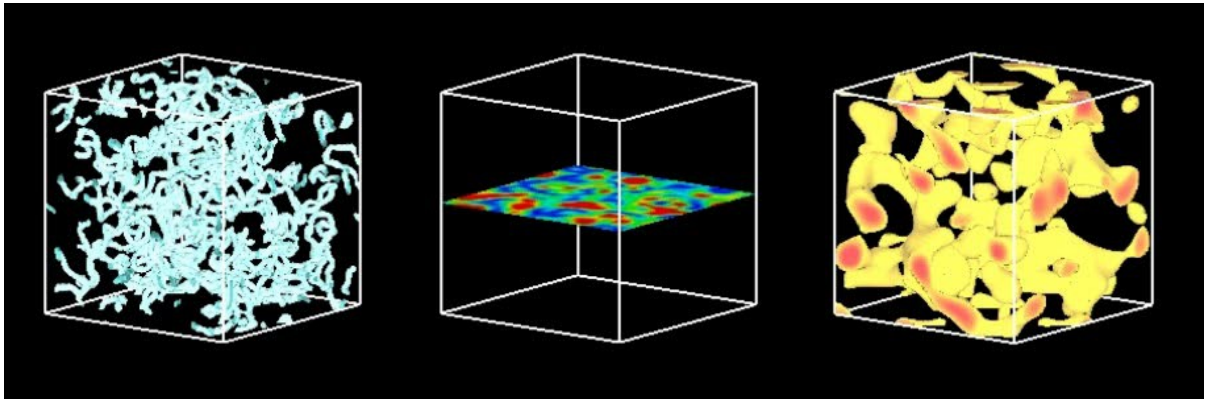
エネルギー注入による定常乱流の実現

$$[\mathrm{i}-\gamma(\boldsymbol{x},t)]\frac{\partial}{\partial t}\Phi(\boldsymbol{x},t)=[-\nabla^2-\mu+V(\boldsymbol{x},t)+g|\Phi(\boldsymbol{x},t)|^2]\Phi(\boldsymbol{x})$$

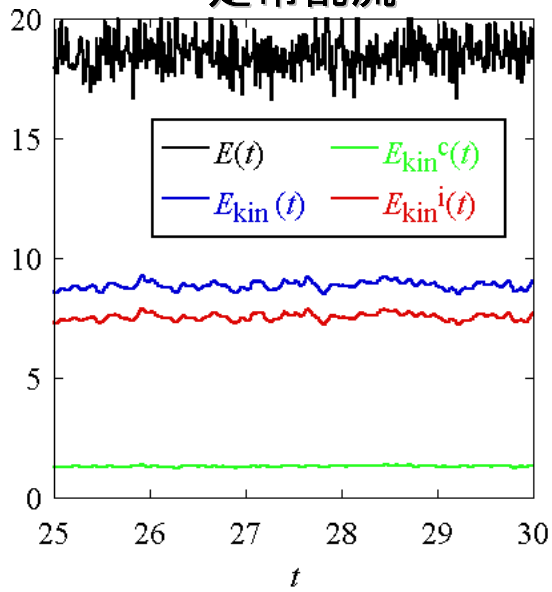
$$V(\boldsymbol{x}) : \text{Moving random potential}$$

$$\langle V(\boldsymbol{x},t)V(\boldsymbol{x}',t')\rangle = V_0^2\exp\left[-\frac{(\boldsymbol{x}-\boldsymbol{x}')^2}{2X_0^2}-\frac{(t-t')^2}{2T_0^2}\right]$$

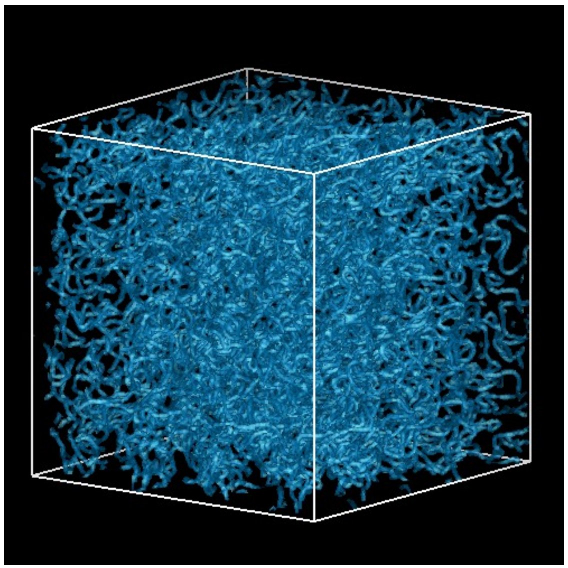
渦　　密度　　ポテンシャル

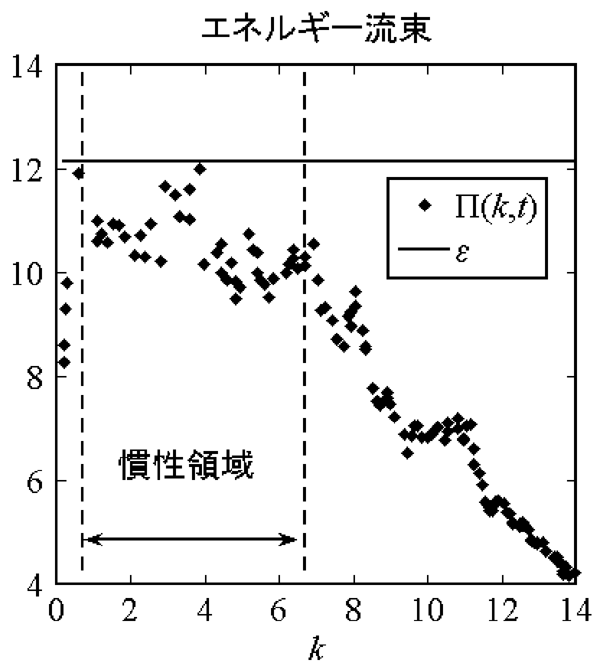


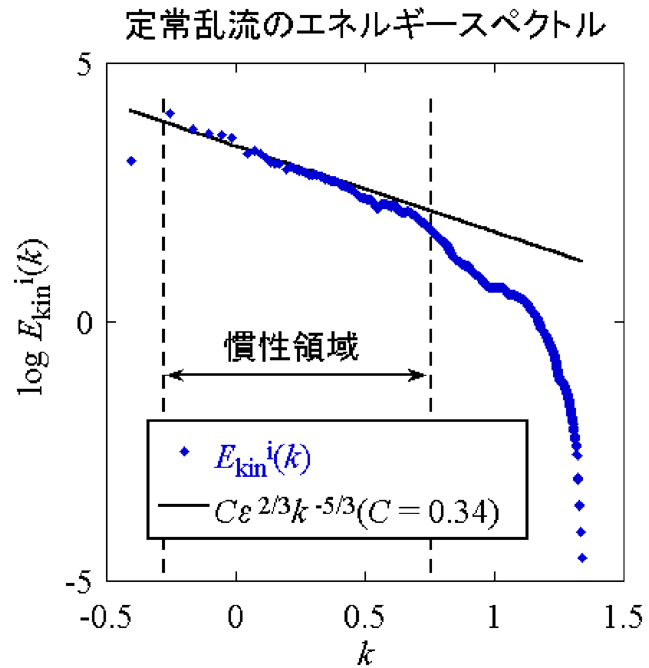
定常乱流



量子乱流のエネルギースペクトル





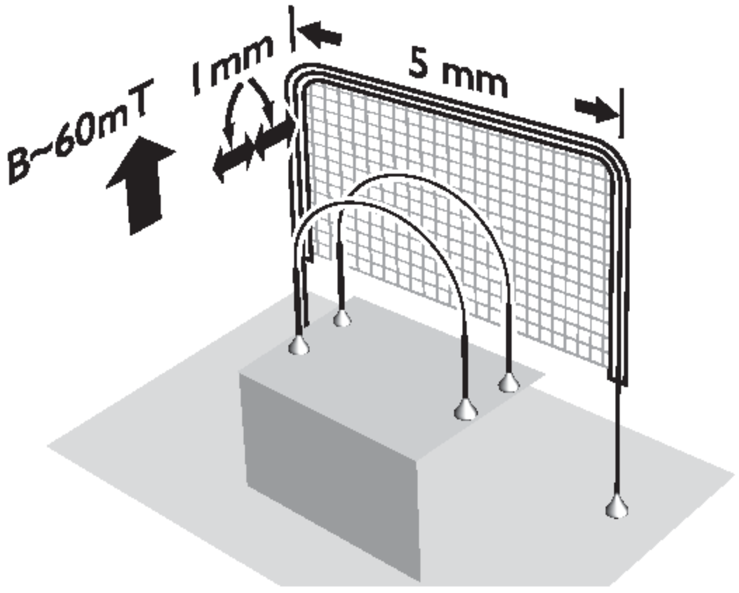


量子乱流は（古典）乱流の統計則である Kolmogorov則に従う

2, 超流動液体Heにおける量子乱流の減衰

D. I. Bradley, D. O. Clubb, S. N. Fisher, A. M. Guénault, R. P. Haley, C. J. Matthews, G. R. Pickett, V. Tsepelin and K. Zaki, Phys. Rev. Lett. **96**, 035301 (2006).

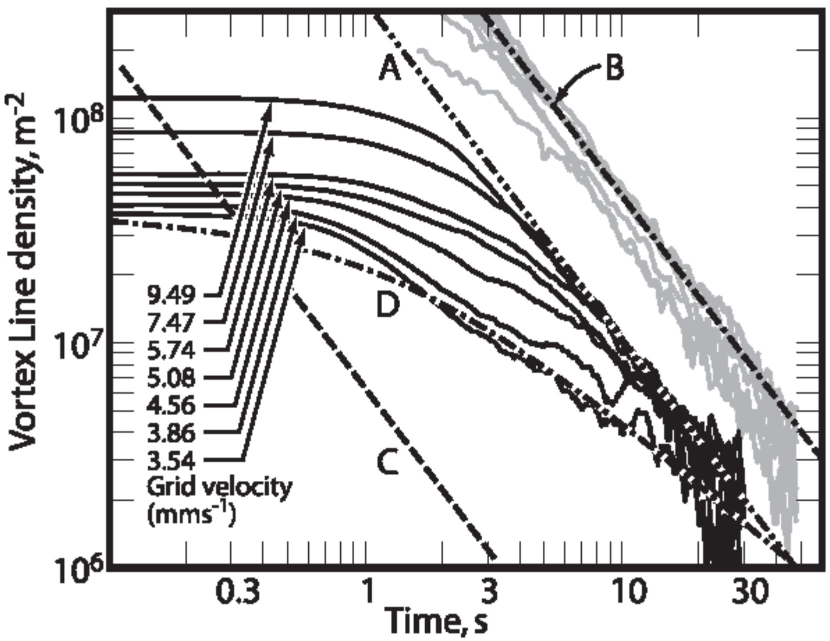
$T = 0.4$ [mK] における^{3}He-Bの乱流



振動格子により乱流状態を作り、ワイヤーによって量子渦の渦糸長密度を(アンドーレーエフ散乱を用いて)測定する

超流動液体Heにおける量子乱流の減衰

減衰乱流　$T = 0.4$ [mK] for ^{3}He-B



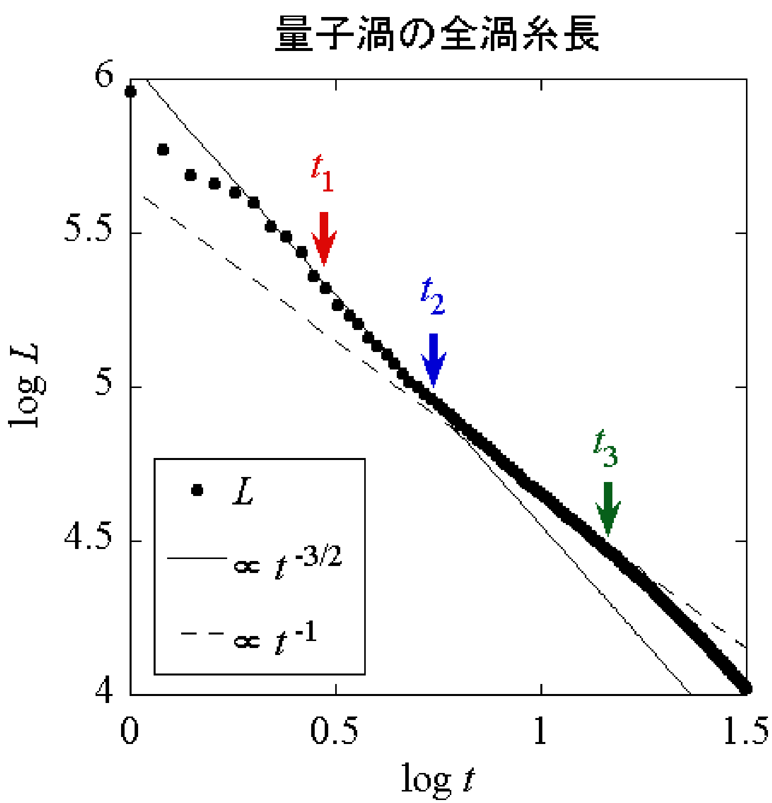
振動格子の速さが速いときに、渦糸長密度が$L \propto t^{-3/2}$ の振る舞いをする

→(間接的に)量子乱流がKolmogorov則に従っていることを支持する

GP乱流でも同様の結果が期待される

3, Gross-Pitaevskii乱流を用いた減衰量子乱流のシミュレーション

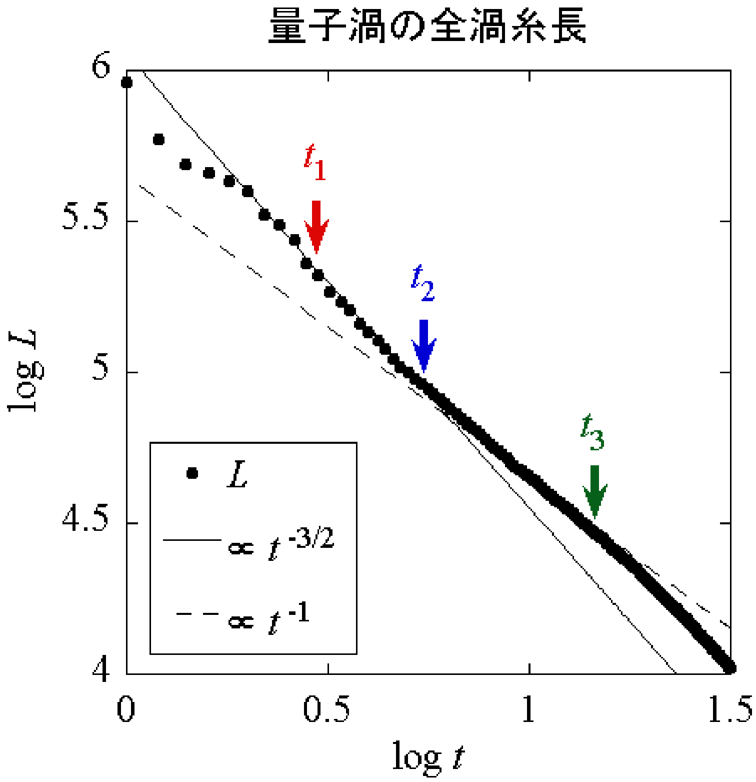
定常乱流の状態からポテンシャルを切って乱流を減衰させる

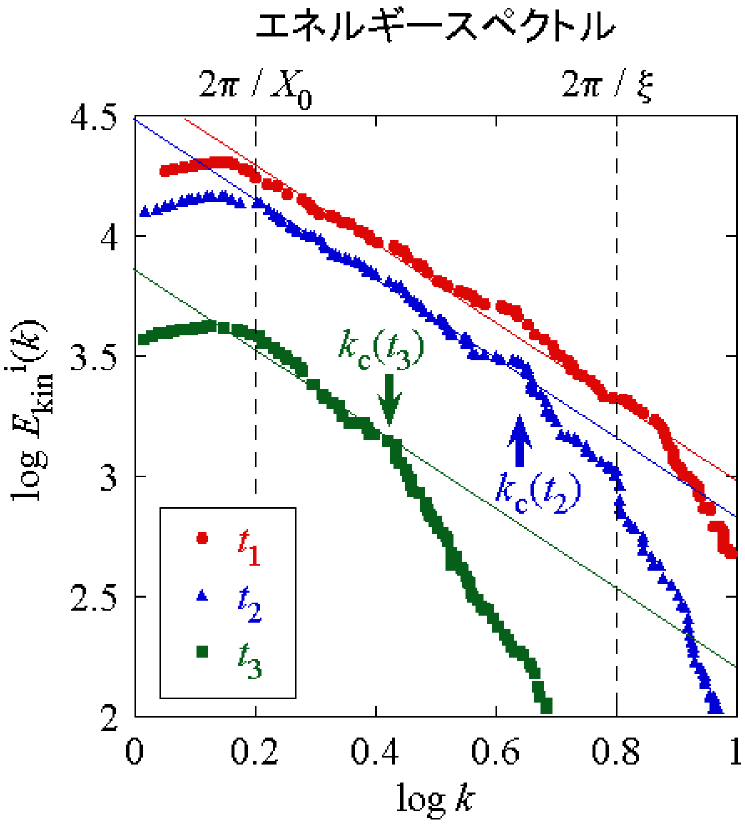


$$\boxed{\begin{array}{l} t_1 < t < t_2 : L \propto t^{-3/2} \\ t_2 < t < t_3 : L \propto t^{-1} \end{array}}$$

減衰の初期段階において実験と同様の結果が得られた

減衰とエネルギースペクトル





時間発展に伴い、エネルギースペクトルの高波数成分$k > k_c$がKolmogorov則からずれ始める

減衰とエネルギースペクトル

ずれはじ

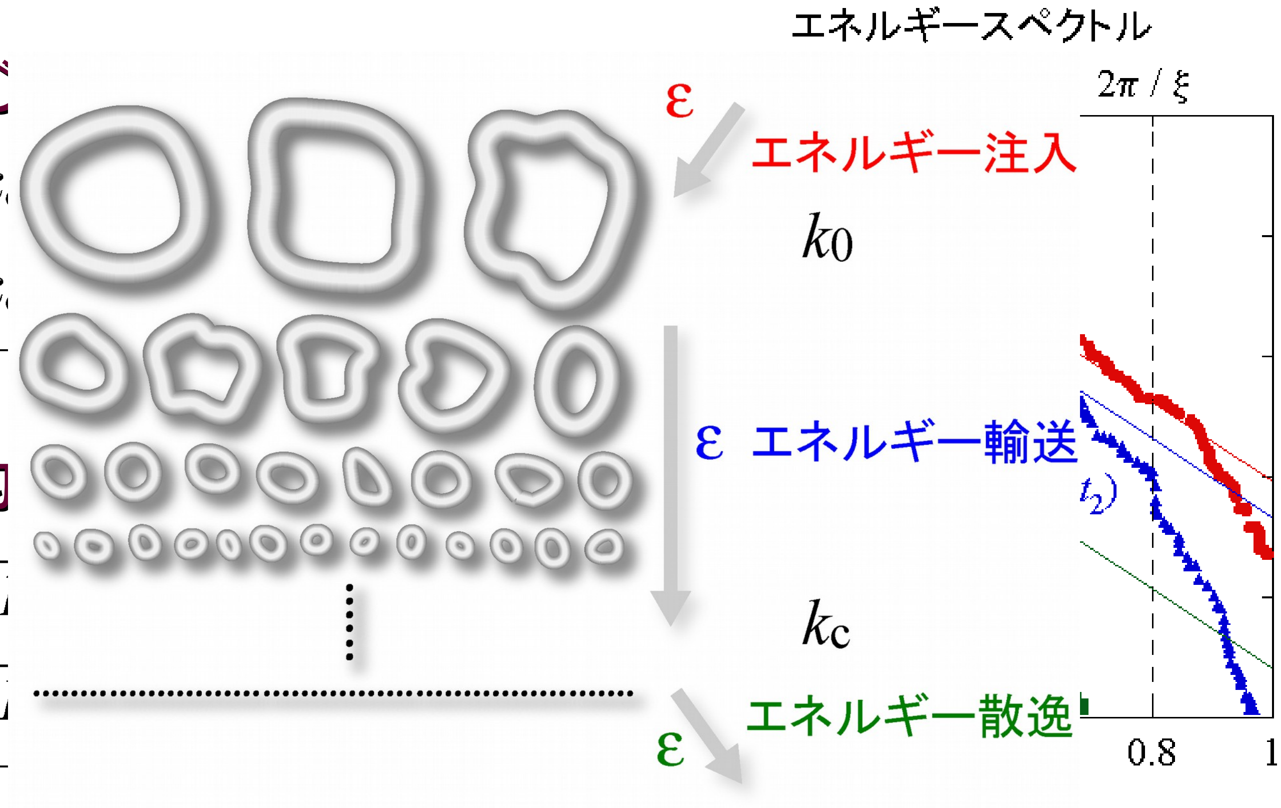
$2\pi/k$

$2\pi/k$

平均

$\sqrt{V/}$

$\sqrt{V/}$



Kolmogorov則は平均渦間距離よりも大きなスケールにおけるRichardsonカスケードによって支えられる

4, 考察とまとめ

減衰の初期段階に対する全渦糸長の解析

- 減衰の初期段階では、慣性領域が$2\pi/X_0 < k < 2\pi/\xi$である
- 乱流の全エネルギーに対して、慣性領域におけるエネルギースペクトルからの寄与がほとんどである

$$E_{\mathrm{kin}}^{\mathrm{i}} \simeq \int_{2\pi/X_0}^{2\pi/\xi} \mathrm{d}k\, E_{\mathrm{kin}}^{\mathrm{i}}(k) = C\varepsilon^{2/3} \int_{2\pi/X_0}^{2\pi/\xi} \mathrm{d}k\, k^{-5/3} = \frac{3C\varepsilon^{2/3}}{2(2\pi)^{2/3}}(X_0^{2/3} - \xi^{2/3})$$

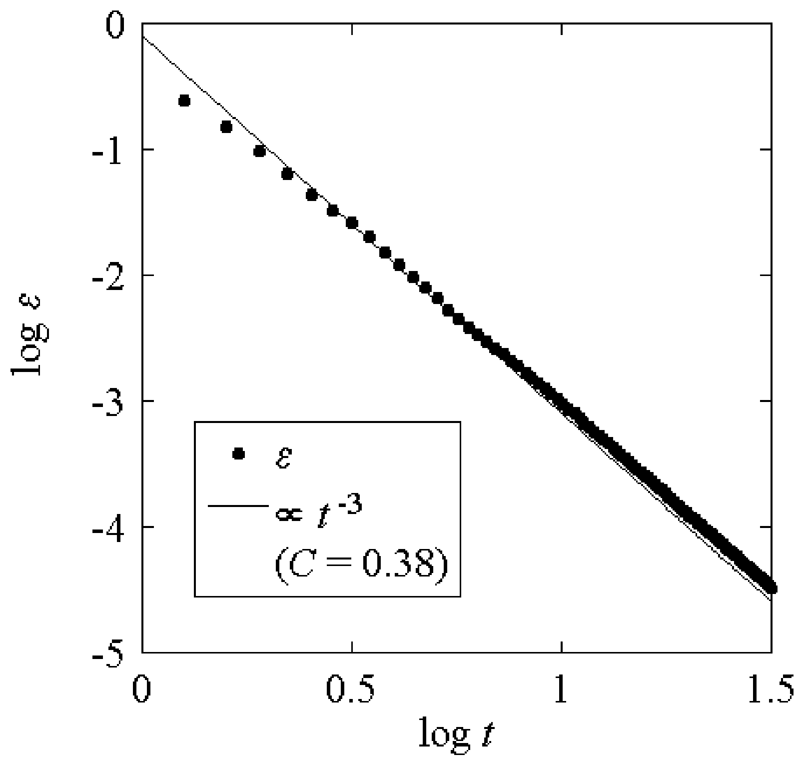
$$\varepsilon = -\frac{\mathrm{d}E_{\mathrm{kin}}^{\mathrm{i}}}{\mathrm{d}t} = -C\frac{\varepsilon^{-1/3}}{(2\pi)^{2/3}}(X_0^{2/3} - \xi^{2/3})\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}$$

全渦糸長に対する解析

微分方程式の解

$$\varepsilon = \frac{27C^3}{(2\pi)^2}\left(\frac{X_0^{2/3} - \xi^{2/3}}{t + t_0}\right)^3$$

エネルギー散逸率

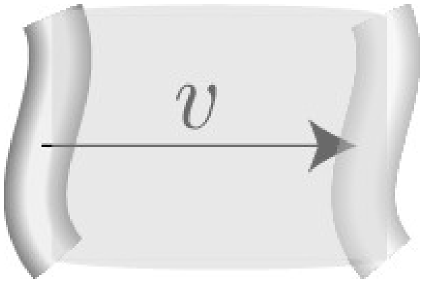


解析とシミュレーション両方において$\varepsilon \propto t^{-3}$の振る舞いが確認された

全渦糸長に対する解析

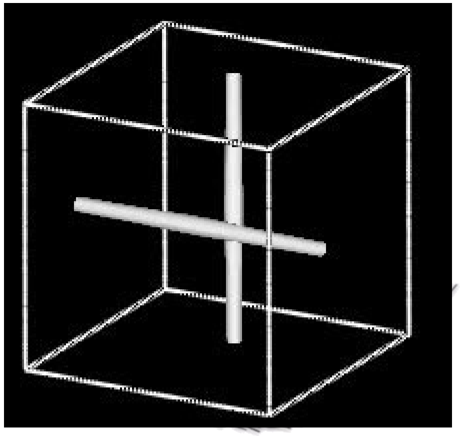
$\xi \bar{v} = \dfrac{\kappa}{4\pi}$:単位時間に単位長の渦糸が掃く平均体積

$\dfrac{L\xi^2}{V}$:全体積に対する量子渦の割合



$\dfrac{\kappa L \xi^2}{4\pi V}$:単位時間に量子渦が衝突する全衝突断面積

$\dfrac{\kappa L}{4\pi V}$:単位時間における量子渦の全衝突回数



量子渦の衝突(再結合)

全渦糸長に対する解析

乱流中において量子渦は主に再結合を通して減衰すると仮定する

$\dfrac{\kappa L \xi^2}{4\pi V}$:単位時間に量子渦が衝突する全衝突断面積

$\dfrac{\rho\kappa^2}{4\pi\xi^2}$:単位長さあたりの渦のエネルギー

$$\Rightarrow \quad \varepsilon = \frac{\alpha\rho\kappa^3 L^2}{16\pi^2 V}$$

$$L = \frac{6\sqrt{3C^3V}}{\kappa^{3/2}\sqrt{\alpha\rho}}\left(\frac{X_0^{2/3} - \xi^{2/3}}{t+t_0}\right)^{3/2}$$

減衰の初期段階と一致する

減衰後半における$L \propto t^{-1}$の振る舞いはよく分かっていない（ランダムな量子渦タングルの減衰に関するVinen方程式に関連？）

まとめ

1. 定常乱流から出発した減衰乱流のシミュレーションを行い、量子乱流におけるKolmogorov則のシグナル($L \propto t^{-3/2}$)を得た。
2. 量子渦の全渦糸長とエネルギースペクトルを比較することにより、Kolmogorov則が平均渦間距離よりも大きなスケールで起こり得るRichardsonカスケードによって引き起こされることを明らかにした。

3, Dissipation of Quantum Turbulence Coupled With Thermal Excitations

What is the microscopic mechanism of dissipation ?
What is the realistic nature of introduced dissipation ?
How does dissipation change at finite temperatures ?

Is the form of dissipation

$$\gamma(k) = \gamma_0 \theta(k - 2\pi/\xi) \text{ correct?}$$

Mutual Friction in Superfluid Helium

For the case of superfluid helium

Quantized vortices dissipate through mutual friction between vortices and viscous normal fluid

$$\dot{\boldsymbol{x}}_0 = \alpha \boldsymbol{x}_0' \times (\boldsymbol{v}_{\mathrm{n}} - \boldsymbol{v}_{\mathrm{s}}) - \alpha' \boldsymbol{x}_0' \times [\boldsymbol{x}_0' \times (\boldsymbol{v}_{\mathrm{n}} - \boldsymbol{v}_{\mathrm{s}})]$$

How is the dissipation for the case of GP turbulence?

Dissipation of GP Turbulence : Coupled System of GP and BdG Equations

Dissipation of GP equation can be discussed by considering the Bogoliubov-de Gennes equation of excitations

$$\hat{H} = \int \mathrm{d}\boldsymbol{x} \hat{\Psi}^\dagger(\boldsymbol{x},t)[-\nabla^2 - \mu + \frac{g}{2}|\hat{\Psi}(\boldsymbol{x},\boldsymbol{t})|^2]|\hat{\Psi}(\boldsymbol{x},t)$$

$$\mathrm{i}\frac{\partial}{\partial t}\hat{\Psi}(\boldsymbol{x},t) = [-\nabla^2 - \mu + g\hat{\Psi}^\dagger(\boldsymbol{x},t)\hat{\Psi}(\boldsymbol{x},t)]\hat{\Psi}(\boldsymbol{x},t)$$

$$\boxed{\hat{\Psi}(\boldsymbol{x},t) \ : \ \text{Field operator of bosons}}$$

Dissipation of GP Turbulence : Coupled System of GP and BdG Equations

$$\hat{\Psi}(\boldsymbol{x},t) = \Phi(\boldsymbol{x},t) + \hat{\chi}(\boldsymbol{x},t) + \hat{\zeta}(\boldsymbol{x},t)$$

$$: \ \text{Bogoliubov approximation}$$

$$\Phi(\boldsymbol{x},t) = O(\sqrt{N_0/V})$$

$$\hat{\chi}(\boldsymbol{x},t) = O(1/\sqrt{V})$$

$$\hat{\zeta}(\boldsymbol{x},t) = O(1/\sqrt{N_0 V}) \ : \ \text{Neglect}$$

$$\mathrm{i}\frac{\partial \Phi}{\partial t} = [-\nabla^2 - \mu + g(|\Phi|^2 + 2\langle \hat{\chi}^\dagger \hat{\chi} \rangle)]\Phi + g\langle \hat{\chi}\hat{\chi} \rangle \Phi^* \ : \ \text{GP}$$

$$\mathrm{i}\frac{\partial \hat{\chi}}{\partial t} = [-\nabla^2 - \mu + 2g|\Phi|^2]\hat{\chi} + g\Phi^2 \chi^\dagger \ : \ \text{BdG}$$

Effective Dissipation of GP Equation

$$\boxed{\mathrm{i}\frac{\partial \Phi}{\partial t} = [-\nabla^2 - \mu + g(|\Phi|^2 + 2\langle \hat{\chi}^\dagger \hat{\chi} \rangle)]\Phi + g\langle \hat{\chi}\hat{\chi} \rangle \Phi^*}$$

GP equation has the imaginary part in Hamiltonian

$$\hat{H}_{\mathrm{GP}} = -\gamma(\boldsymbol{x},t) = \mathrm{Im}\left[g\frac{\langle \hat{\chi}(\boldsymbol{x},t)\hat{\chi}(\boldsymbol{x},t)\rangle \Phi^*(\boldsymbol{x},t)}{\Phi(\boldsymbol{x},t)} \right]$$

Dissipation can be obtained naturally!

Calculation of Excitations

Bogoliubov transformation

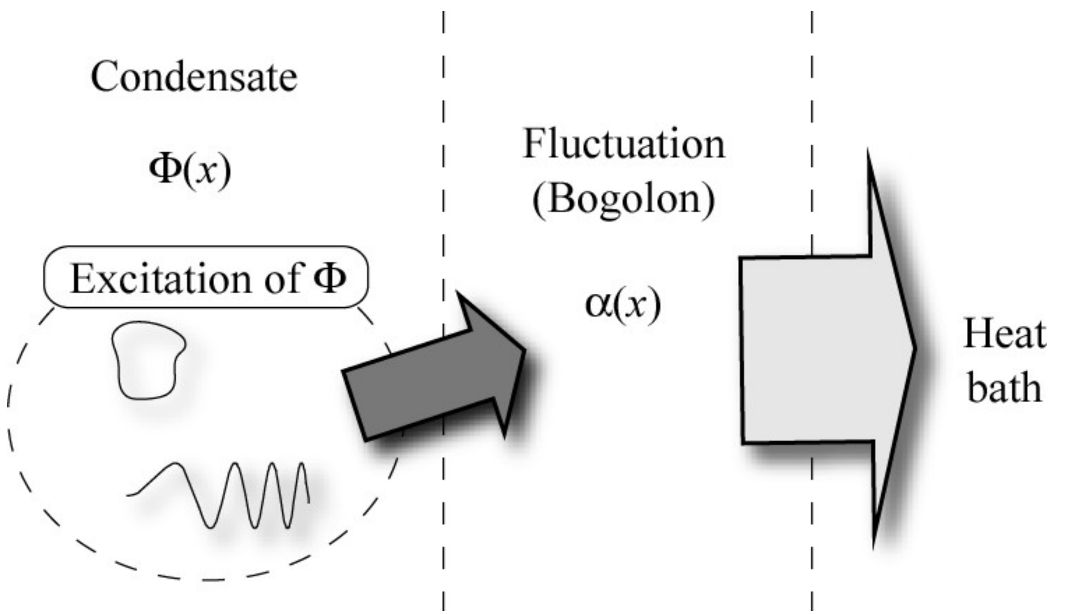
$$\hat{\chi}(\boldsymbol{x},t) = \frac{1}{\sqrt{V}}\sum_j \phi_j(\boldsymbol{x},t)\hat{a}_j = \frac{1}{\sqrt{V}}\sum_j [u_j(\boldsymbol{x},t)\hat{\alpha}_j + v_j^*(\boldsymbol{x},t)\hat{\alpha}_j^\dagger]$$

$\hat{\alpha}_j\ (\hat{\alpha}_j^\dagger)$: annihilation (creation) operator of excitation bogolon

Calculation of Excitations

$$\langle \alpha_j^\dagger \alpha_j \rangle = N_j = \frac{1}{\exp[E_j/T] - 1} \ : \ \text{Local equilibrium approximation}$$

: Bogolons are coupled with the heat bath



Final Form of Equations

$$\mathrm{i}\frac{\partial \Phi}{\partial t} = [-\nabla^2 - \mu + g(|\Phi|^2 + 2n_\mathrm{e})]\Phi + g m_\mathrm{e}\Phi^*$$

$$\mathrm{i}\frac{\partial u_j}{\partial t} = [-\nabla^2 - \mu + 2g|\Phi|^2]u_j - g\Phi^2 v_j = A_j$$

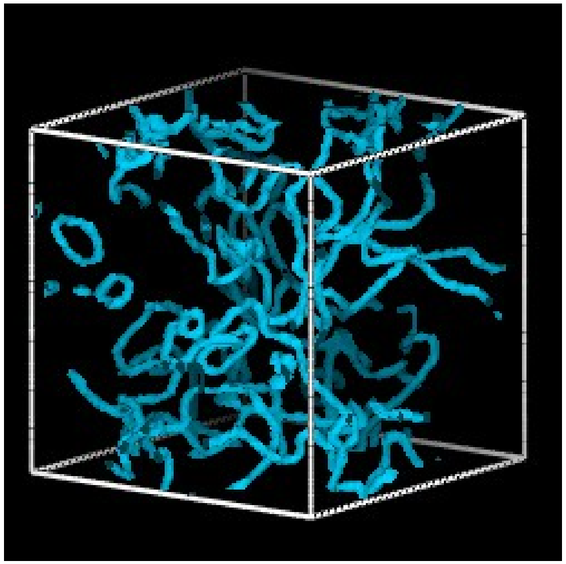
$$\mathrm{i}\frac{\partial v_j}{\partial t} = -[-\nabla^2 - \mu + 2g|\Phi|^2]v_j + g\Phi^{*2} u_j = B_j$$

$$n_\mathrm{e} = \sum_j [|u_j|^2 N_j + |v_j|^2 (N_j + 1)] : \text{ Noncondensate density}$$

$$m_\mathrm{e} = -\sum_j [u_j v_j^* (2N_j + 1)]$$

$$E_j = \int \mathrm{d}\boldsymbol{x}\ \mathrm{Re}[u_j^* A_j + v_j^* B_j] : \text{ Excitation spectrum}$$

Initial State for Numerical Simulation



Condensate : randomly placed some vortices

$$u_j(\boldsymbol{x}, t=0) = \mathrm{e}^{\mathrm{i}\boldsymbol{k}_j\cdot\boldsymbol{x}}\sqrt{\frac{1}{2V}\frac{k_j^2 + g|\Phi|^2}{E_j} + 1}$$

$$v_j(\boldsymbol{x}, t=0) = \mathrm{e}^{-\mathrm{i}\boldsymbol{k}_j\cdot\boldsymbol{x}}\sqrt{\frac{1}{2V}\frac{k_j^2 + g|\Phi|^2}{E_j} - 1}$$

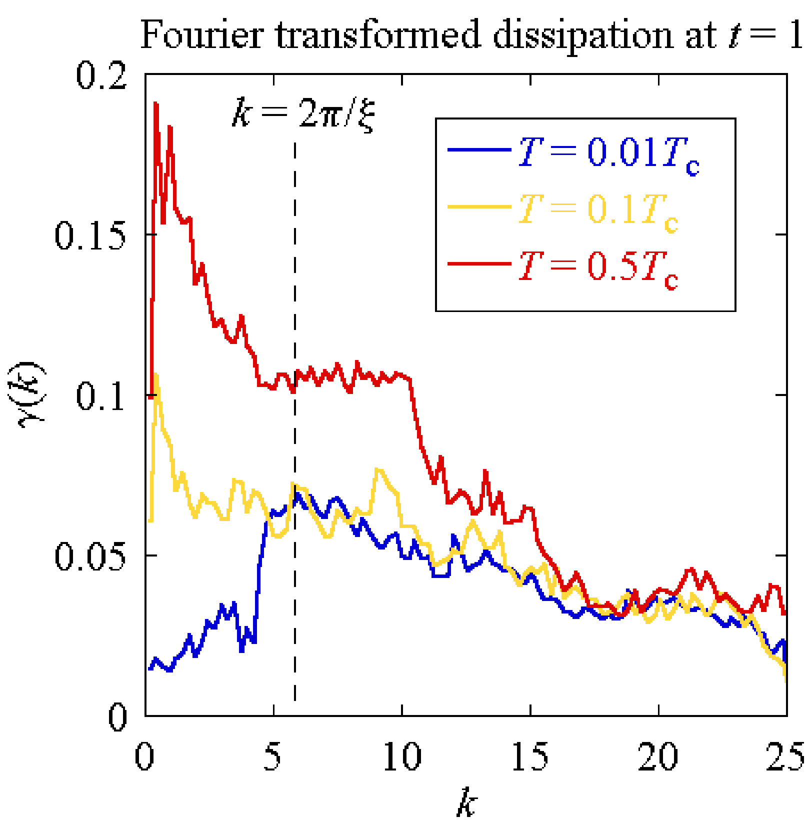
Excitation : uniform solution

Simulation Parameters

$N = 32^3$ grids :

$g = 1 \qquad \Delta x = 0.125 \qquad V = 4^3 \qquad \Delta t = 5 \times 10^{-4}$

Numerical Result : Dissipation Term



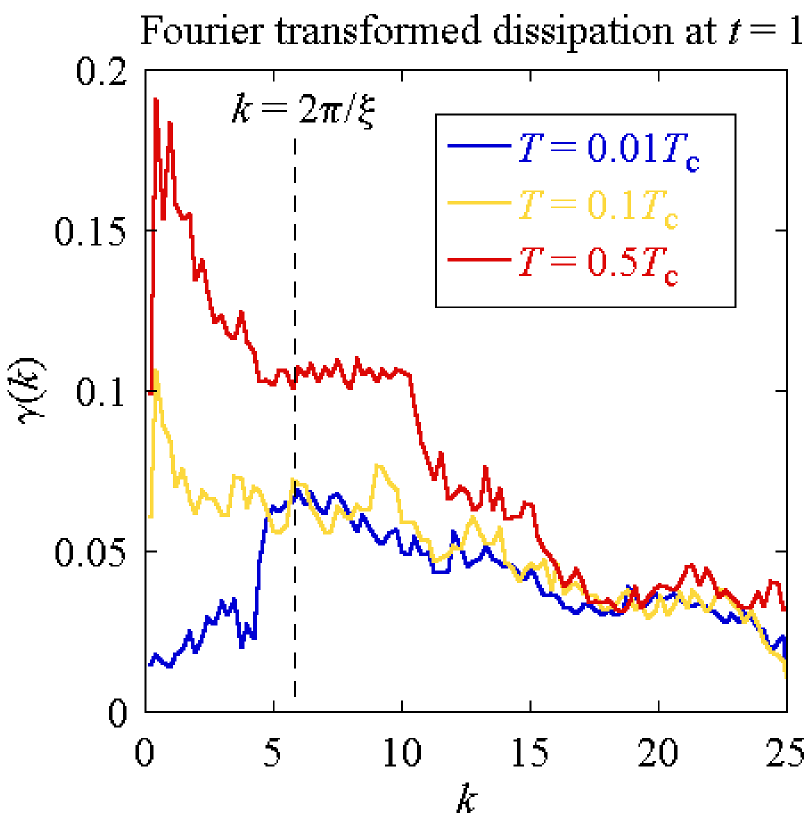
$T_c = 4\pi/\{\zeta(3/2)\}^{2/3}$: Critical temperature of the BEC of free bosons

At low temperature :

Dissipation works at scales smaller than the healing length and consistent with the dissipation introduced in our previous work

→Only short wavelength excitations are dissipated

Numerical Result : Dissipation Term



$T_c = 4\pi/\{\zeta(3/2)\}^{2/3}$: Critical temperature of the BEC of free bosons

At high temperature :

Dissipation works at large scales as well.

→Vortices are dissipated and vortex dynamics is affected by the dissipation

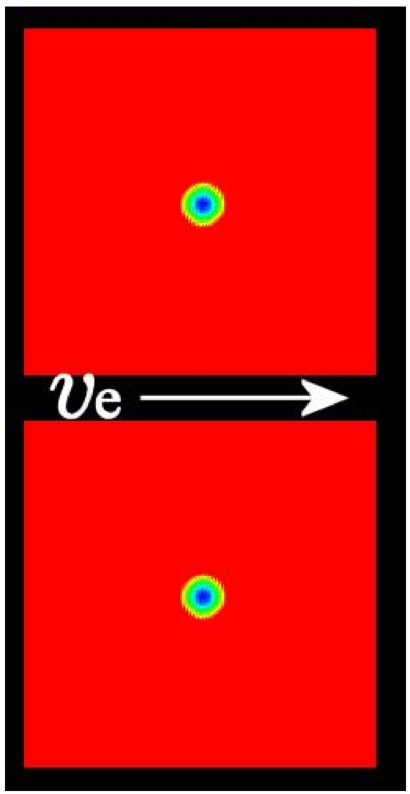
→Similar to mutual friction

Comparison With Mutual Friction

Dynamics of 1 straight vortex (2D simulation) under the velocity field

$T = 0.01T_{\mathrm{c}}$

$T = 0.1T_{\mathrm{c}}$



$$\dot{\boldsymbol{x}}_0 = \alpha \boldsymbol{x}_0' \times (\boldsymbol{v}_{\mathrm{n}} - \boldsymbol{v}_{\mathrm{s}}) - \alpha' \boldsymbol{x}_0' \times [\boldsymbol{x}_0' \times (\boldsymbol{v}_{\mathrm{n}} - \boldsymbol{v}_{\mathrm{s}})]$$

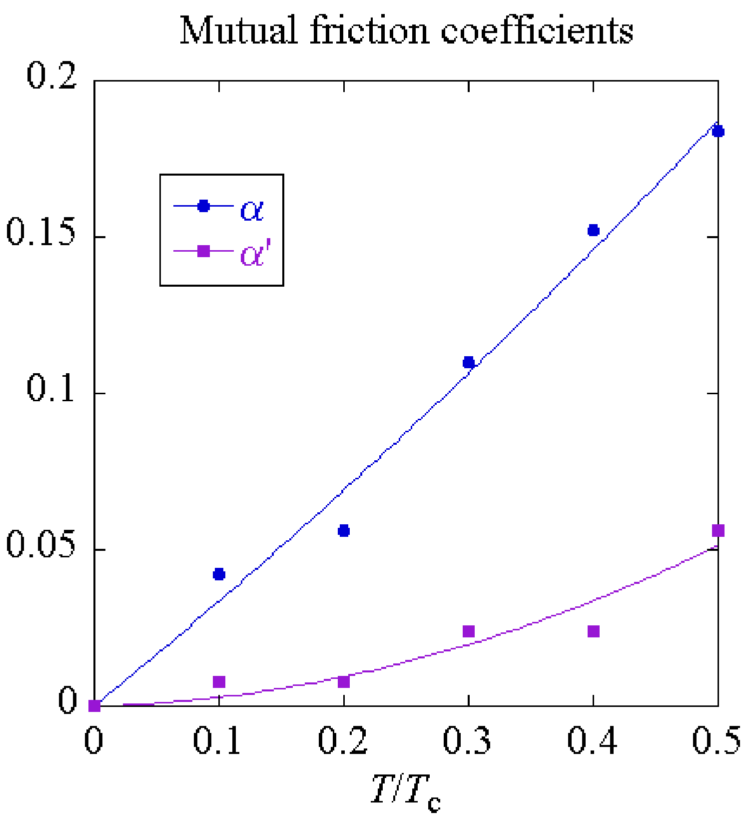
Drag force in vertical direction $\rightarrow \alpha$

Drag force in horizontal direction $\rightarrow \alpha'$

$\boldsymbol{v}_{\mathrm{e}}$: Velocity field

Moving flame with $\boldsymbol{v}_{\mathrm{e}}$

Comparison With Mutual Friction



We successfully calculate the mutual friction coefficients for the case of GP turbulence

→ need to be experimentally observed in dilute BECs

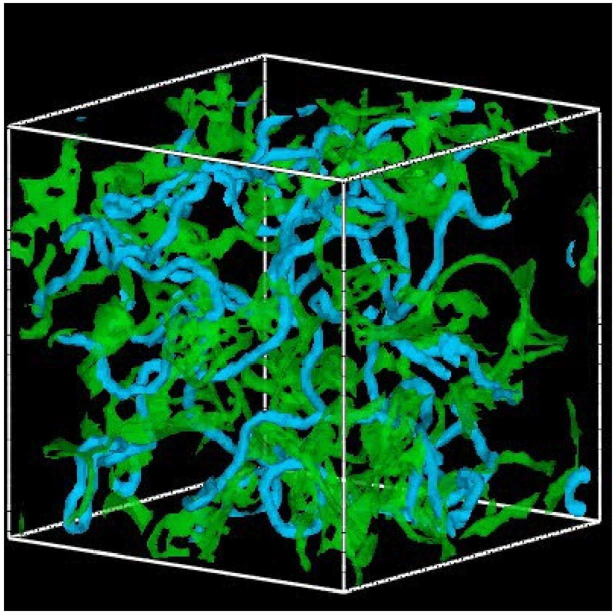
Dynamics of Excitations

W. F. Vinen, Phys. Rev. B **61**, 1410 (2000).

In superfluid helium, superfluid and normal fluid are likely coupled together at large scales due to mutual friction and behave similar to the turbulence in a one-component fluid

We can expect a similar coupled turbulence in which the dynamics of thermal excitations is strongly coupled with that of the condensation and both the dynamics become comparable at large scales.

Coupled Turbulence



$T = 0.1T_{\rm c}$

Blue : Quantized vortices

Green : Region of high vorticity of noncondensate

$$|\boldsymbol{\omega}_{\rm e}(\boldsymbol{x})| > 0.95 \, \langle |\boldsymbol{\omega}_{\rm e}| \rangle$$

We can see highly tangled turbulence made of quantized vortices and noncondensate eddies

Correlation Between Quantized Vortices and Noncondensate Eddies

Correlation function : $$C(t) \equiv \frac{L_{\mathrm{e}}/L}{\int \mathrm{d}\boldsymbol{x}\, p_{\mathrm{e}}(\boldsymbol{x})/V}$$

L : Total line length of quantized vortices

L_{e} : Total line length of quantized vortices in the region of $|\boldsymbol{\omega}_{\mathrm{e}}(\boldsymbol{x})| > 0.95\,\langle|\boldsymbol{\omega}_{\mathrm{e}}|\rangle$

V : Total volume

$p_{\mathrm{e}}(\boldsymbol{x}) = 1$ if $|\boldsymbol{\omega}_{\mathrm{e}}(\boldsymbol{x})| > 0.95\,\langle|\boldsymbol{\omega}_{\mathrm{e}}|\rangle$

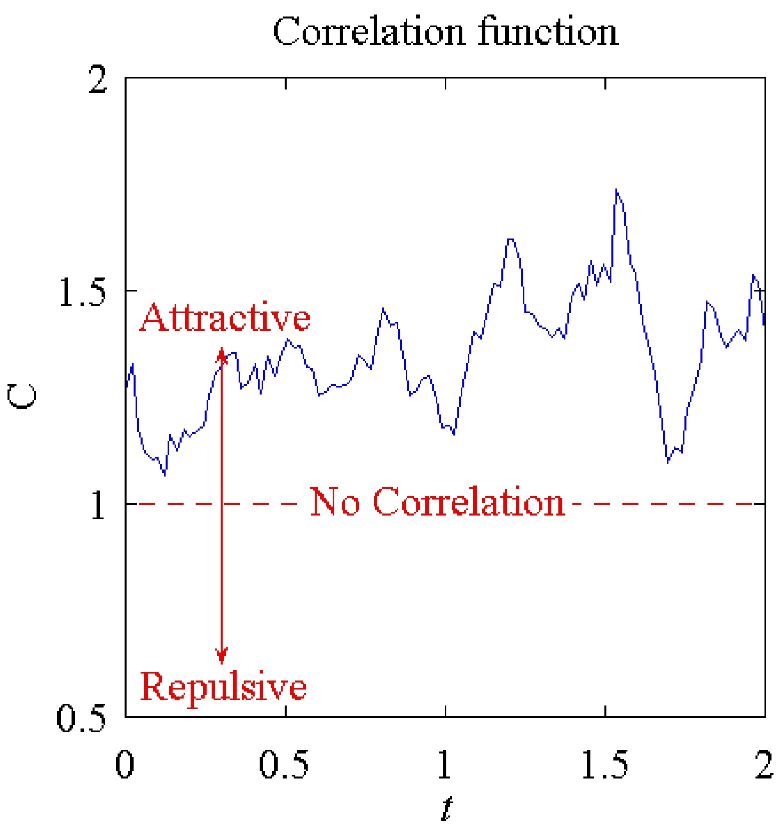
$p_{\mathrm{e}}(\boldsymbol{x}) = 0$ if $|\boldsymbol{\omega}_{\mathrm{e}}(\boldsymbol{x})| > 0.95\,\langle|\boldsymbol{\omega}_{\mathrm{e}}|\rangle$

$C(t) < 1$: Vortices and eddies are repulsive

$C(t) = 1$: No correlation between vortices and eddies

$C(t) > 1$: Vortices and eddies are attractive

Correlation Between Quantized Vortices and Noncondensate Eddies



- Correlation function is always larger than 1 : Quantized vortices and noncondensate eddies are attractive.

We confirm the signal of coupled turbulence!

4, Summary : Dissipation Mechanism of Quantum Turbulence

1. We calculate the coupled system of GP and BdG equations and investigate the microscopic mechanism of the dissipation in quantum turbulence.
2. At low temperatures, dissipation works only at scales smaller than the vortex core size, which is consistent with the dissipation introduced in our previous work.
3. At high temperatures, dissipation works at large scales as well and directly affect the vortex dynamics.

4, Summary : Dissipation Mechanism of Quantum Turbulence

1. We successfully relate the dissipation at high temperature with mutual friction in superfluid helium by calculating the mutual friction coefficients as functions of temperature.
2. We confirm the signal of coupled tangle and resulting coupled turbulence of quantized vortices and noncondensate eddies by calculating the correlation between them.