

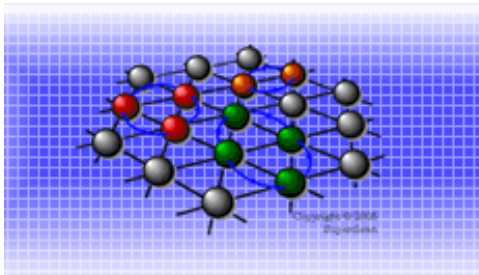


Quantum Turbulence Realized in Trapped Atomic Bose-Einstein Condensates

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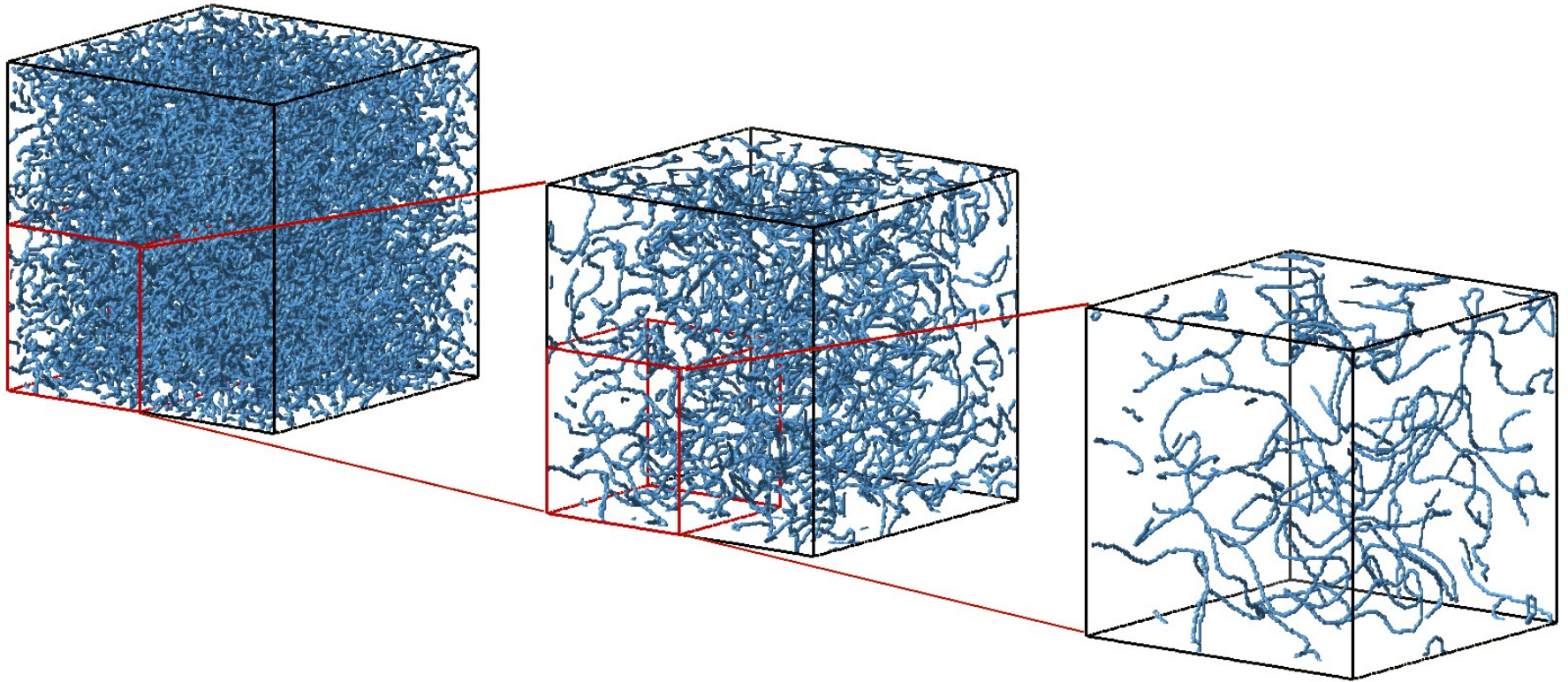
**ELEMENTARY EXCITATION
PHYSICS LABORATORY**
THEORY OF CONDENSED MATTER

“International Symposium on Physics of New Quantum Phases in Superclean Materials”, 29 Oct. – 1 Nov, 2007.



Quantum Turbulence

Quantum turbulence is realized as tangled state of quantized vortices

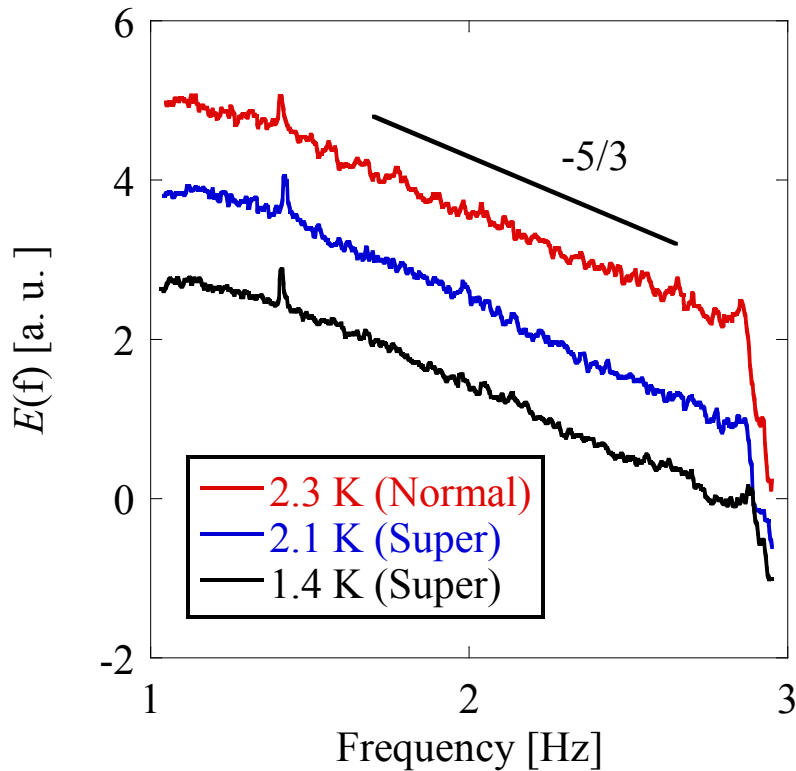


Energy Spectrum of Quantum Turbulence

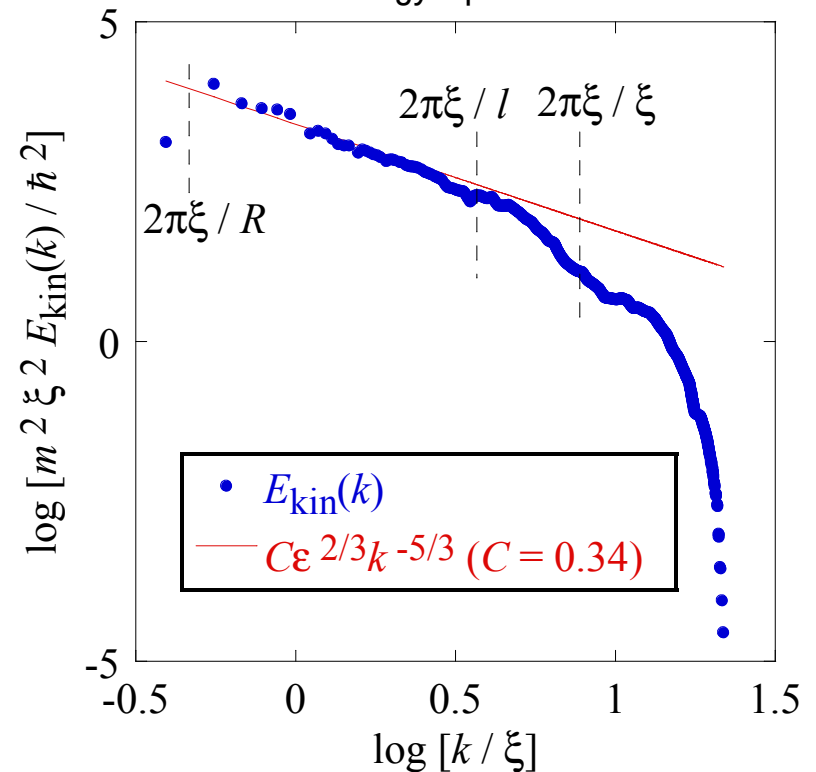
J. Maurer and P. Tabeling,
Europhys. Lett. **43** (1), 29 (1998)

MK & MT, J. Phys. Soc. Jpn
(English), **74**, 3248 (2005).

Energy spectrum of superfluid turbulence



Energy Spectrum



Quantized Vortices in Superfluid Helium

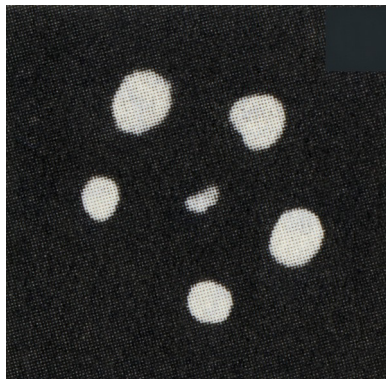
Observation of vortices in superfluid helium

- Second sound
- Vibrating wire
- NMR satellite peak



Only (total) vortex length (density)

It is very difficult to observe the spatial distribution of vortices in superfluid helium



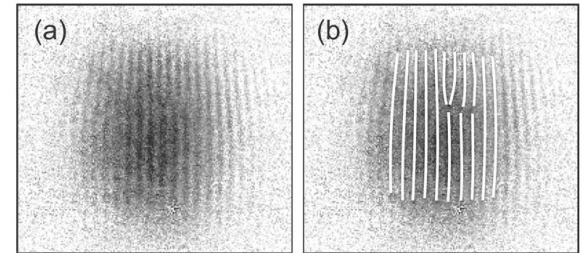
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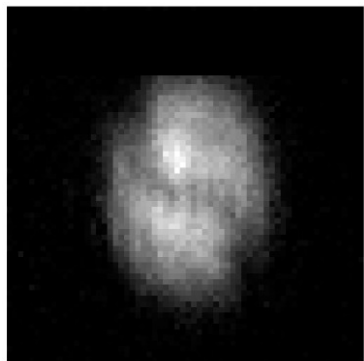
Quantized Vortices in Atomic BECs

Direct observation of quantized vortices is possible in atomic BECs

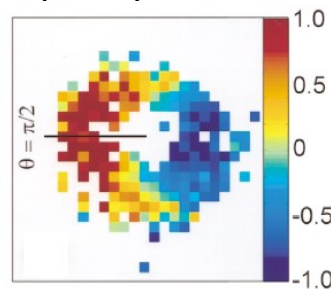
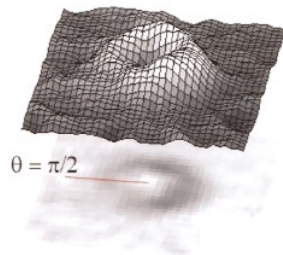


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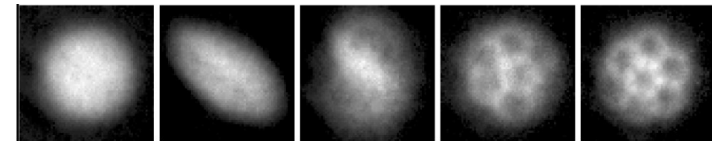
V. Bretin et al. PRL
90, 100403(2003)



M. R. Matthews et al.
PRL **83**, 2498(1999)

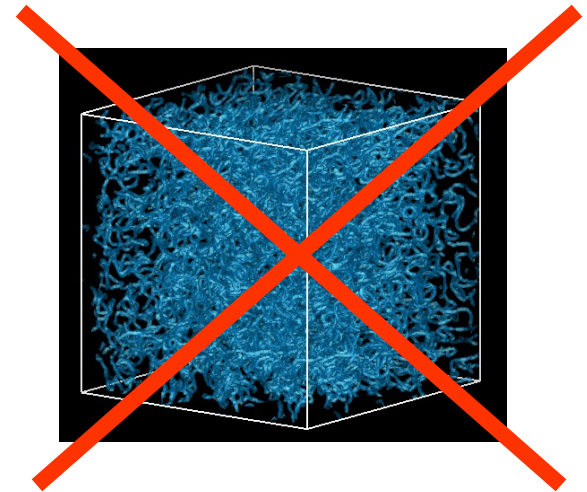
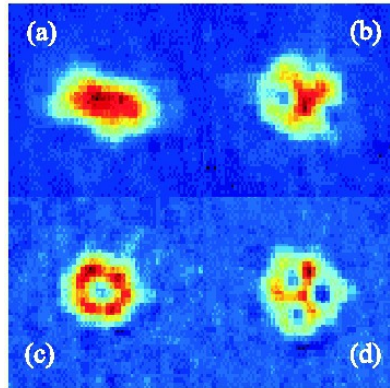
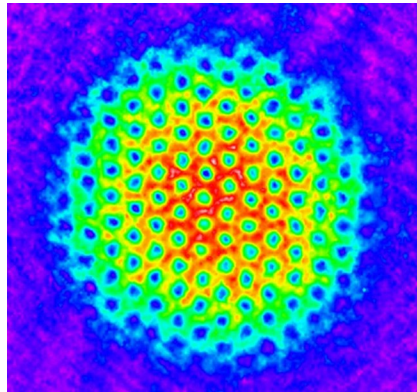


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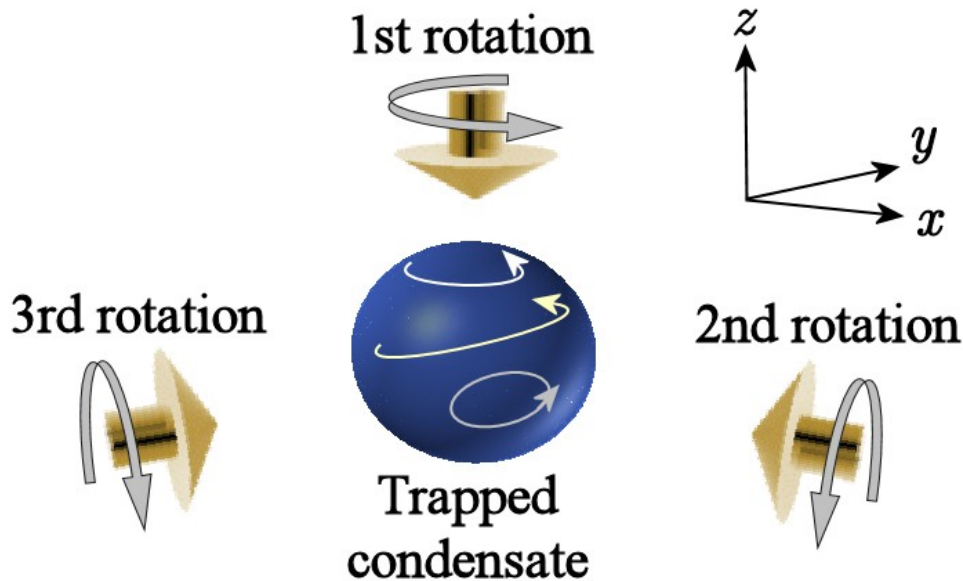
Turbulence in Atomic BECs

In Atomic BECs, only one vortex and vortex lattice state have been experimentally studied.



Even the concept of quantum turbulence has not existed

Atomic BECs Under Combined Rotation Around Two or Three-axis



- Without a other rotation, simple rotation is realized.
- Each rotation does not commute each other.
- 3rd and 2nd rotation are weak : rotating vortex lattice.
- 3rd rotation is weak : 2-D turbulence.



S. Goto et. al. Phys. Fluids **19**,
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Numerical Simulation of the Gross-Pitaevskii Equation

$$\hbar[i - \gamma(\mathbf{x})] \frac{\partial}{\partial t} \Phi(\mathbf{x}) = \left[-\frac{\hbar^2}{2m} \nabla^2 - \mu + U(\mathbf{x}) + \frac{4\pi\hbar^2 a}{m} |\Phi(\mathbf{x})|^2 - \boldsymbol{\Omega}(t) \cdot \mathbf{L}(\mathbf{x}) \right] \Phi(\mathbf{x})$$

$U(\mathbf{x})$: Magnetic trap potential a : s -wave scattering length

$\boldsymbol{\Omega}(t)$: Rotational frequency $\mathbf{L}(\mathbf{x})$: Angular momentum

$$U(\mathbf{x}) = \frac{m\omega^2}{2} [(1 - \varepsilon_1)(1 - \varepsilon_2)x^2 + (1 + \varepsilon_1)(1 - \varepsilon_2)y^2 + (1 + \varepsilon_2)z^2]: \text{Anisotropic trap}$$

$$\boldsymbol{\Omega}(t) = (\omega_x \cos \omega_y t + \omega_z \cos \omega_x t \sin \omega_y t, \omega_y + \omega_z \sin \omega_x t, \omega_x \sin \omega_y + \omega_z \cos \omega_x t \cos \omega_y t)$$

$\gamma(\mathbf{x})$: Dissipation obtained from BdG equation (MK & MT, PRL. **97**, 145301 (2006))

Numerical Simulation of the Gross-Pitaevskii Equation

Numerical parameters

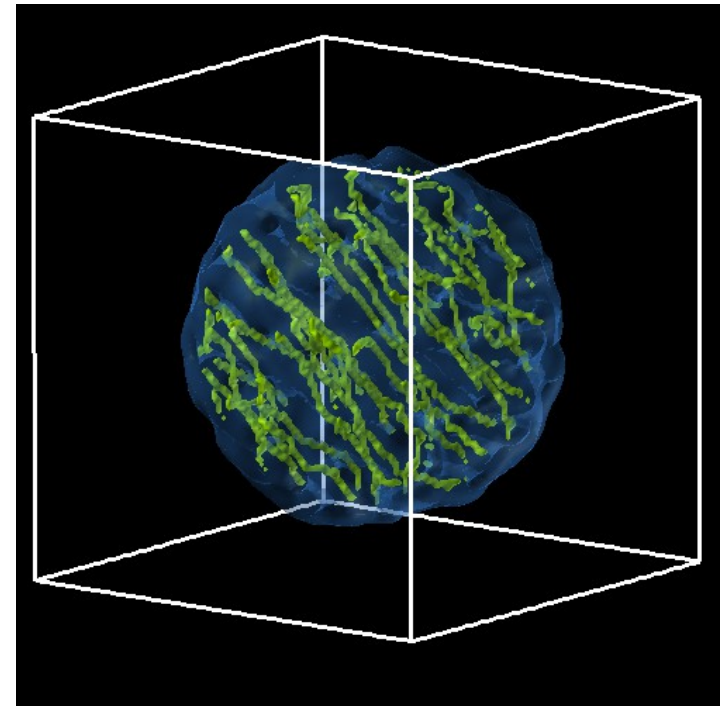
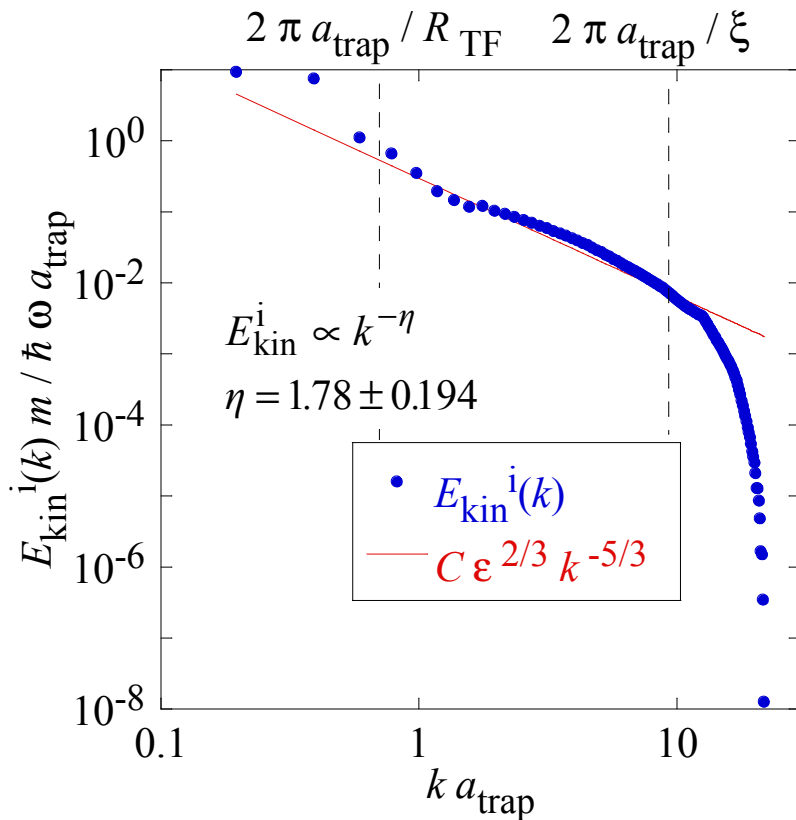
^{87}Rb atoms: $m = 1.46 \times 10^{-25}$ kg, $a = 5.61$ nm, $N = 250000$
 $\omega = 150 \times 2\pi$ Hz, $\boldsymbol{\Omega} = (0.2\omega, 0.05\omega, 0.9\omega)$, $\varepsilon_1 = \varepsilon_2 = 0.01$

Numerical method

Chebyshev-tau pseudo-spectral method with 512^3 grid points (volume = $100 \mu\text{m}^3$) Initial state : stationary state without rotation and anisotropy of potential

Numerical Results

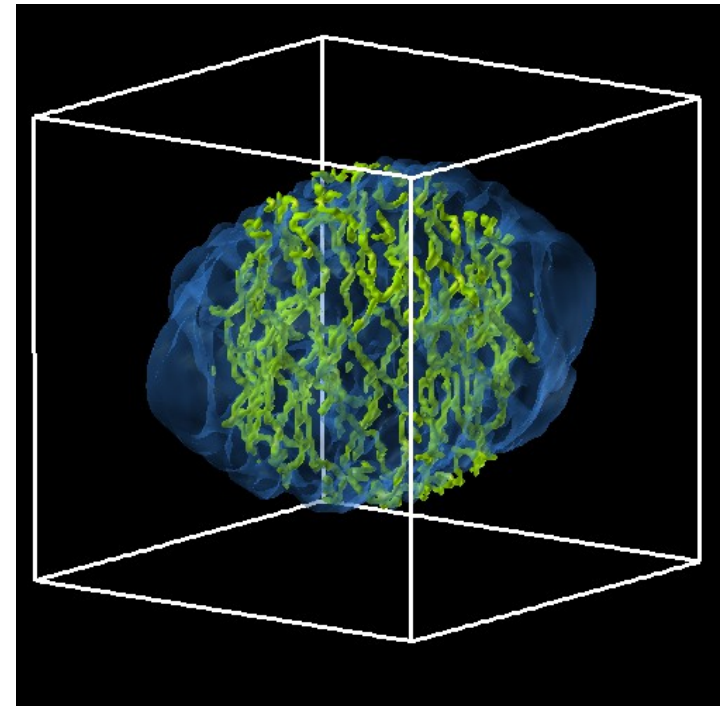
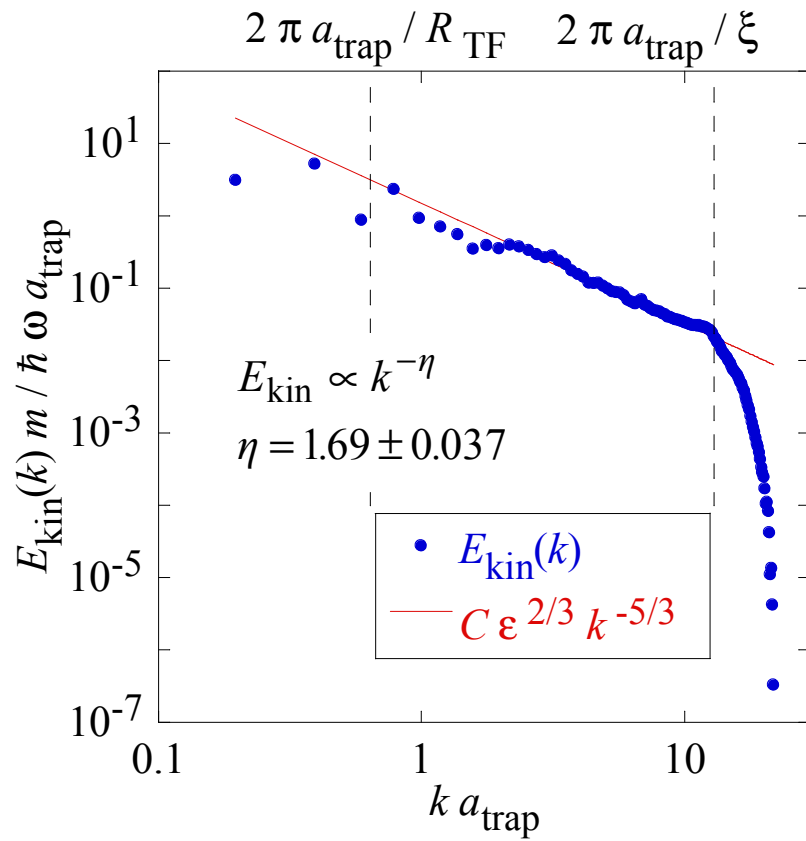
$\omega_y = 0$: biaxial rotation



$$(I_x, I_y, I_z) = \frac{1}{L} \left(\int d\xi [s \cdot \hat{x}]^2, \int d\xi [s \cdot \hat{y}]^2, \int d\xi [s \cdot \hat{z}]^2 \right) = (0.353, 0.276, 0.371)$$

Numerical Results

$\omega_y \neq 0$: triaxial rotation



$$(I_x, I_y, I_z) = \frac{1}{L} \left(\int d\xi [s \cdot \hat{x}]^2, \int d\xi [s \cdot \hat{y}]^2, \int d\xi [s \cdot \hat{z}]^2 \right) = (0.326, 0.327, 0.347)$$



Summary

- Atomic BECs can become a nice candidate to study quantum turbulence.
- Atomic BEC can become a system to study quantum turbulence with tangled quantized vortices by using precession rotation.