The Study of Turbulent State in Quantum Fluid

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Contents

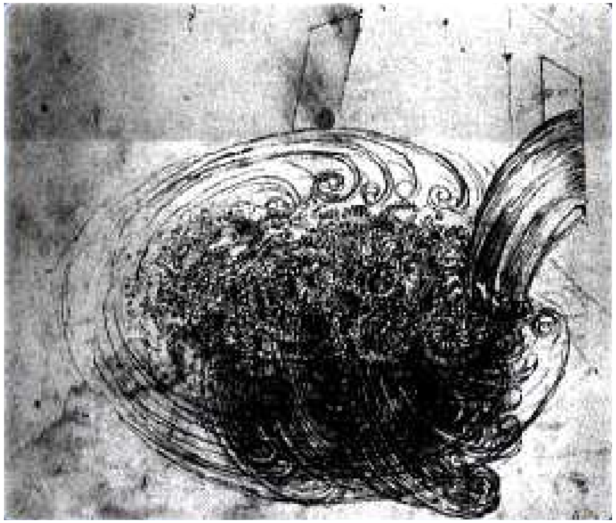
- I study the dynamics and statistics of turbulence in quantum fluid such as superfluid ^{4}He in which all rotational fluid is carried by quantized vortices.
- By numerically solving the nonlinear Schrödinger equation, I obtain the dynamics of quantized vortices in turbulence as tangled state and investigate the statistics like energy spectrum, fractal structure, *etc*.

Quantized Vortices and Quantum Turbulence

Ordinary fluid (air, water)

There is viscosity

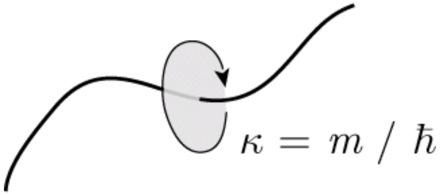
From large to small vortices exist in turbulence

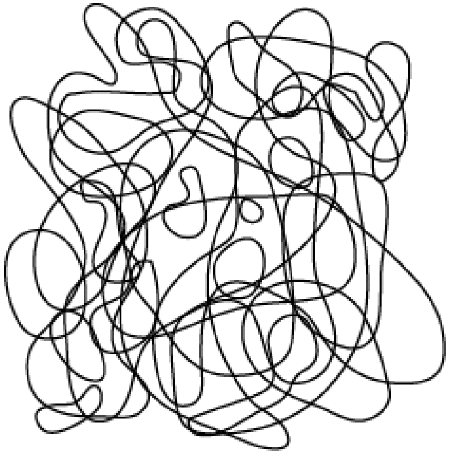


Classical fluid (He-II)

There is no viscosity

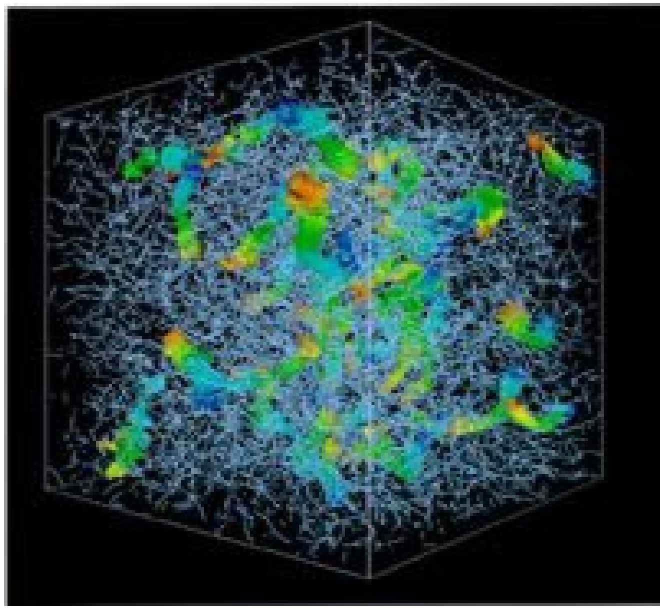
Circulations of vortices are quantized and turbulence is realized as vortex tangle





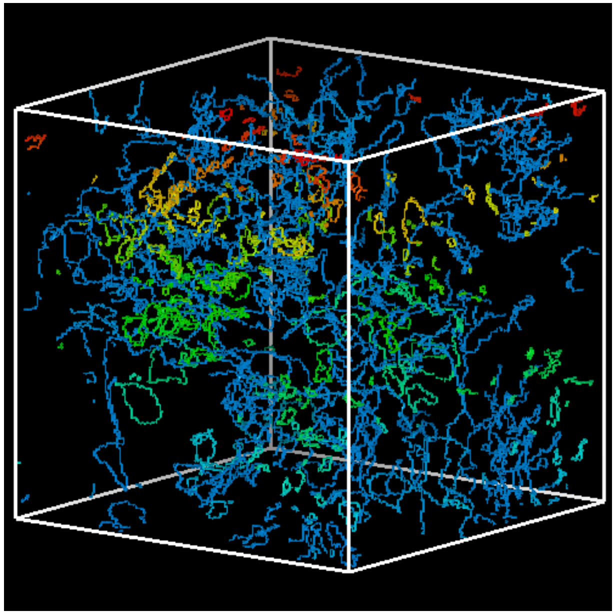
Quantum Turbulence As an Ideal System of Turbulence

Vortices in ordinary turbulence (Navier-Stokes simulation by S. Kida)



All circulations around vortices have arbitrary value and vortices are indefinite

Vortices in quantum fluid turbulence



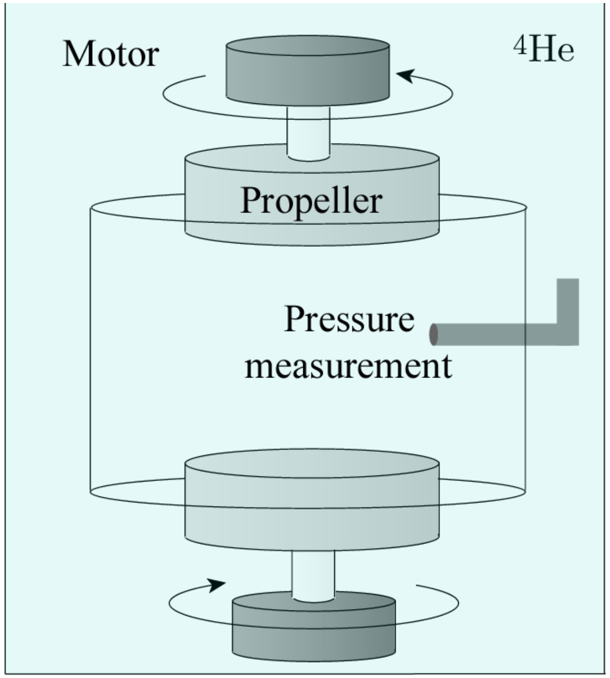
All circulations are quantized and vortices are definite

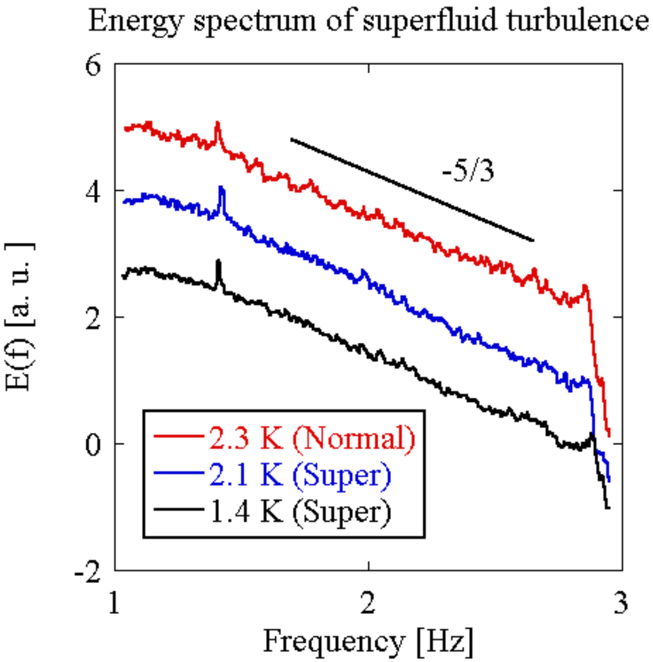
→ **Vortex skeletons in turbulence!**

Experiment of Superfluid ^{4}He-II

J. Maurer and P. Tabeling, Europhys. Lett. **43** (1), 29 (1998)

Two-counter rotating disks

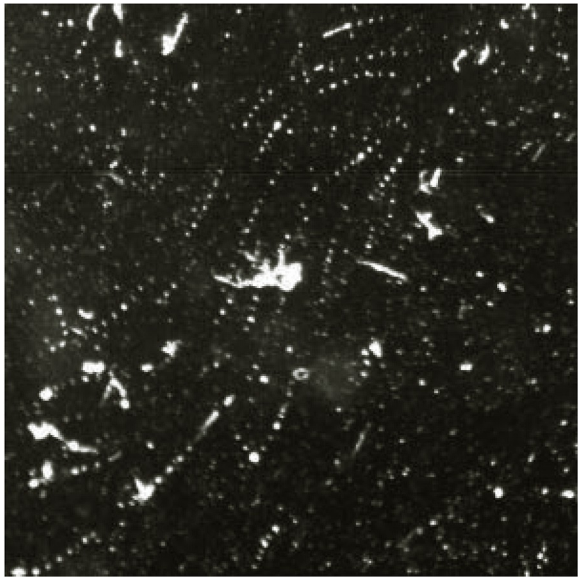




Quantum turbulence obeys the ordinary Kolmogorov law

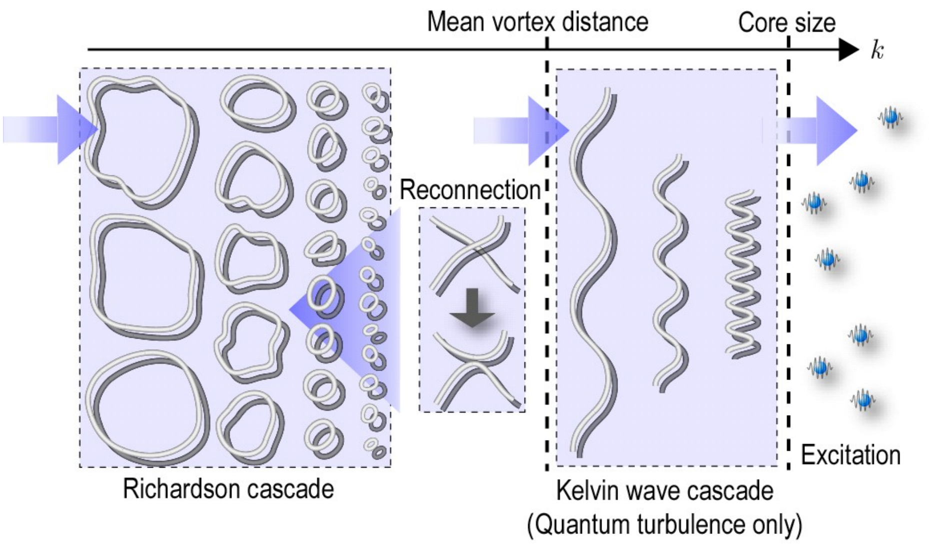
G. P. Bewley, et al, Nature **441**, 588(2006).

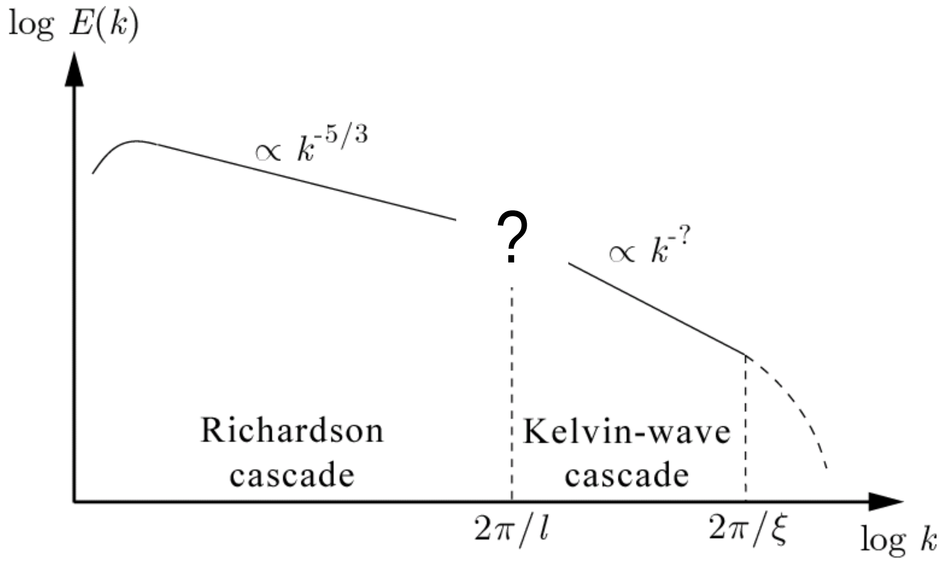
Visualization of quantized vortices in turbulence



Proposed Statistics of Vortices and the Energy Spectrum

3 regions : classical, quantum, and dissipative with elementary excitations





Structure of quantized vortices and the energy spectrum are closely related with each other.

Model of Quantum Fluid and Turbulence

Nonlinear Schrödinger equation

$$[\mathrm{i}-\gamma(\boldsymbol{k})]\frac{\partial\tilde{\Phi}(\boldsymbol{k})}{\partial t}$$

$$= \Big[(k^2-\mu)\tilde{\Phi}(\boldsymbol{k}) + \frac{g}{L^3}\sum_{\boldsymbol{k}_1}\tilde{V}(\boldsymbol{k}_1)\tilde{\Phi}(\boldsymbol{k}-\boldsymbol{k}_1)$$

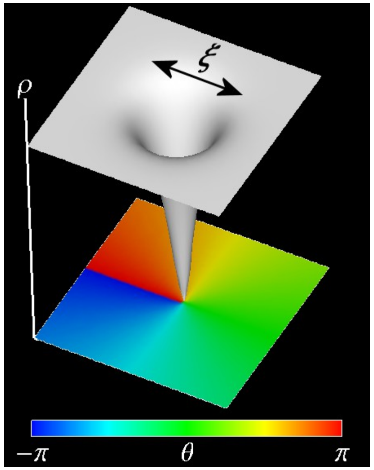
$$+ \frac{g}{L^6}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2}\tilde{\Phi}(\boldsymbol{k}_1)\tilde{\Phi}^*(\boldsymbol{k}_2)\tilde{\Phi}(\boldsymbol{k}-\boldsymbol{k}_1+\boldsymbol{k}_2)\Big]$$

$\Phi(\boldsymbol{x}) = |\Phi(\boldsymbol{x})|\exp[\mathrm{i}\theta(\boldsymbol{x})]$

$\rho(\boldsymbol{x}) = |\Phi(\boldsymbol{x})|^2$: Density

$\boldsymbol{v}(\boldsymbol{x}) = 2\nabla\theta(\boldsymbol{x})$: Velocity

$\xi = 1/\sqrt{g\bar{\rho}}$: Vortex core size



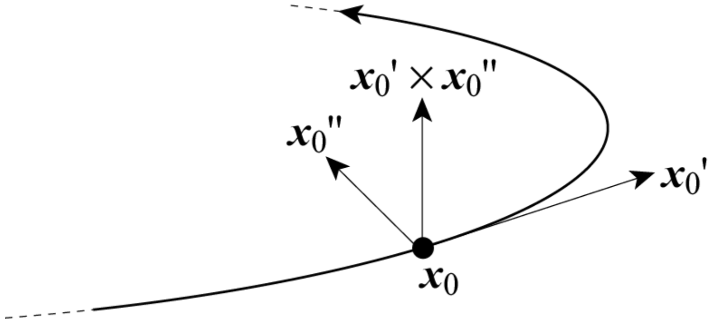
Quantized vortex

Vortex filament model

$$\frac{\partial \boldsymbol{x}_0(t)}{\partial t} = \boldsymbol{v}_{\mathrm{s}}(\boldsymbol{x}_0)$$

$$\boldsymbol{v}_{\mathrm{s}}(\boldsymbol{x}) = \boldsymbol{v}_{\mathrm{ind}}(\boldsymbol{x}) + \boldsymbol{v}_{\mathrm{sa}}(\boldsymbol{x})$$

$$\boldsymbol{v}_{\mathrm{ind}}(\boldsymbol{x}) = \frac{\kappa}{4\pi}\int\frac{[\boldsymbol{x}_0(t)-\boldsymbol{x}]\times \mathrm{d}\boldsymbol{x}_0(t)}{|\boldsymbol{x}_0(t)-\boldsymbol{x}|^3}$$



Numerical Simulation of Nonlinear Schrödinger Equation

Details of Simulation

$$\tilde{\gamma}(k) = \begin{cases} 0 \; (k < 2\pi/\xi) \\ \gamma_0 \; (k \geq 2\pi/\xi) \end{cases} \quad : \text{ Dissipation in scales smaller than } \xi$$

$$|\tilde{V}(k)| \begin{cases} V_0 \; (k_0 - \Delta k < k < k_0 + \Delta k) \\ 0 \; (\text{otherwise}) \end{cases} \quad : \text{ Energy injection}$$

$\xi = 1$: Healing length $\quad g = 1$: Coupling constant

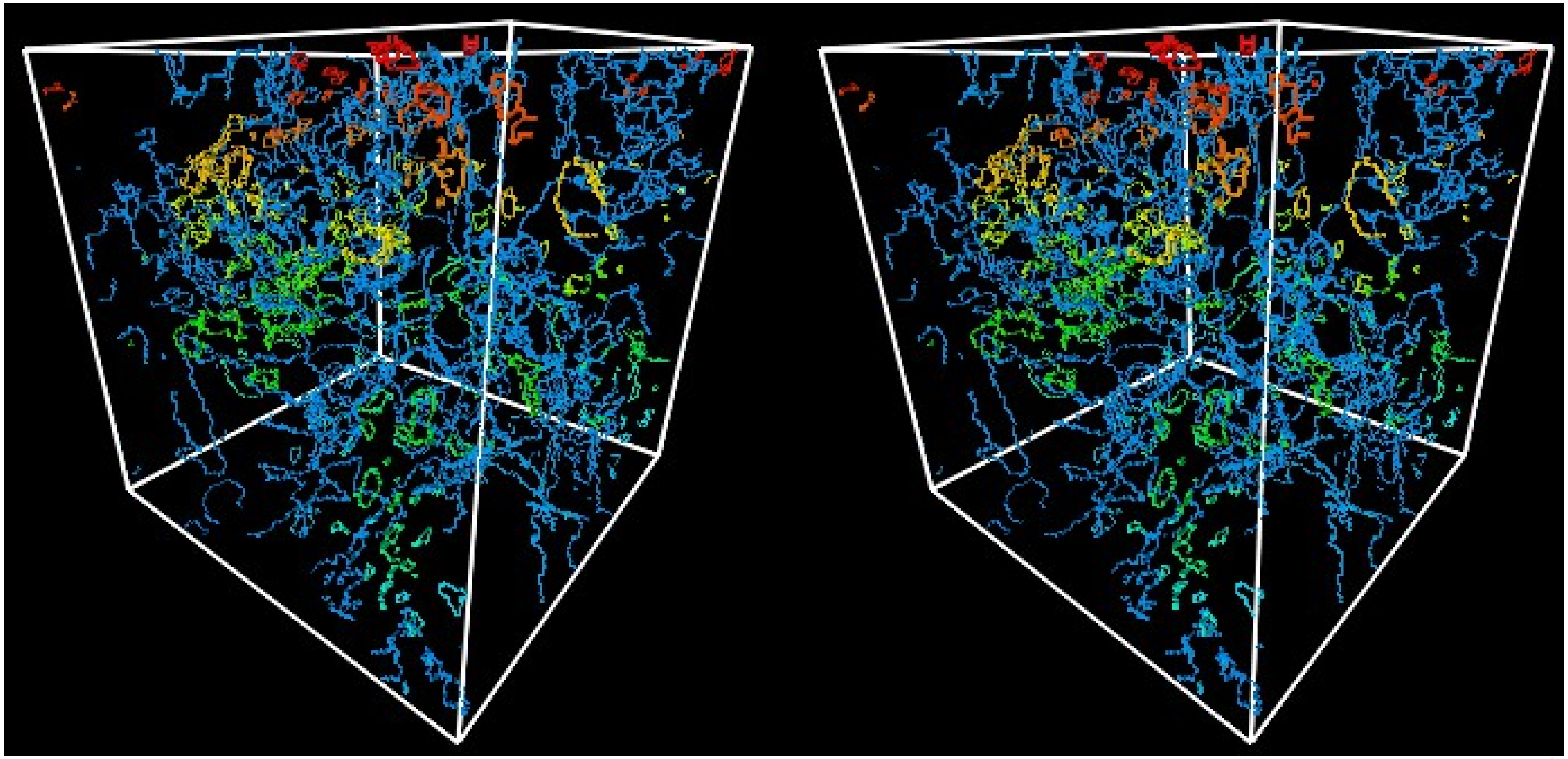
$\gamma_0 = 1 \quad V_0 = 50 \quad \Delta k = L/2\pi \quad k_0 = 2\Delta k$

Space : Fully dealiased pseudospectral-Galerkin method

Time : Runge-Kutta-Gill method

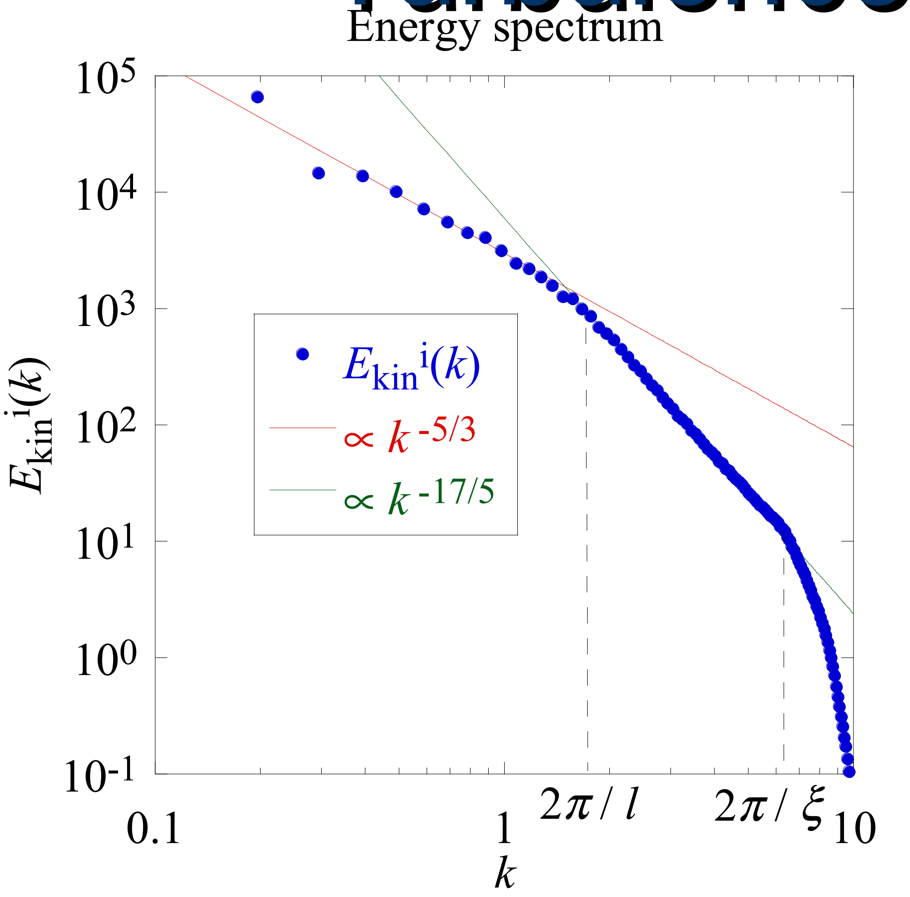
Turbulent State in the Simulation

Periodic box with 256^3 grids



(stereogram)

Energy Spectrum of Turbulence

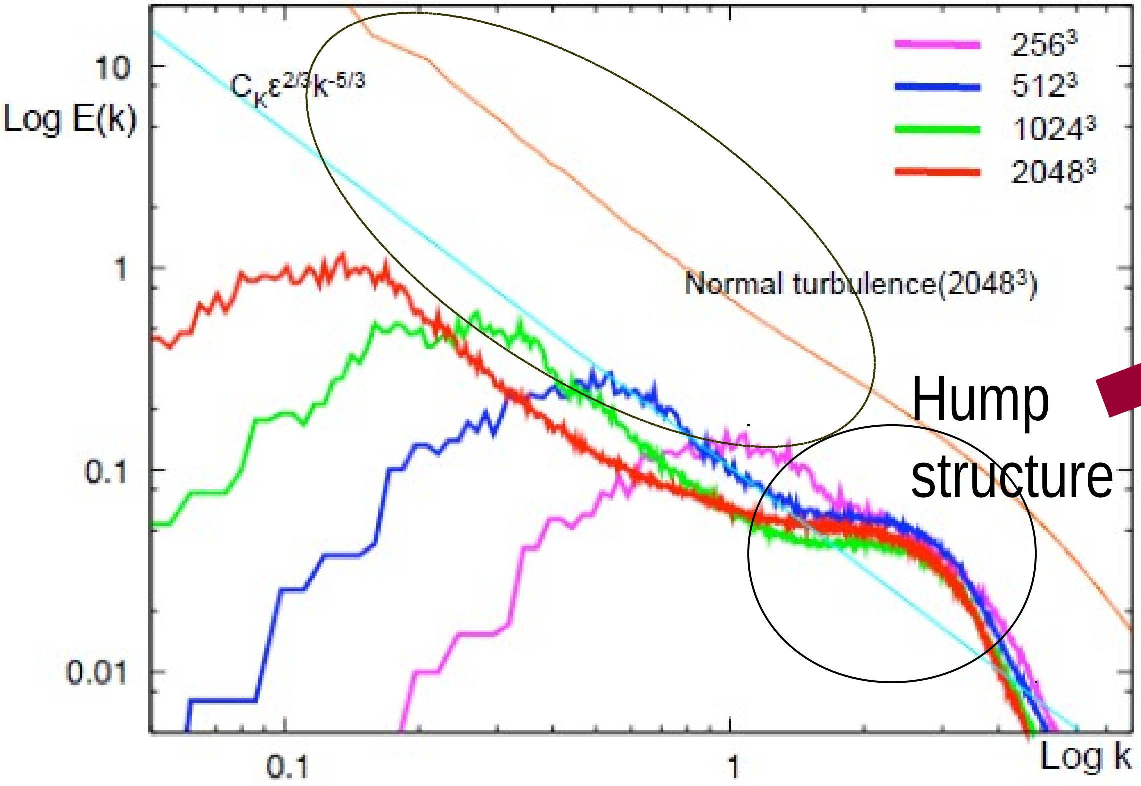


$k < 2\pi / l$: Quantum fluid turbulence shows the Kolmogorov law : there is a similarity between quantum and ordinary fluid.

$k > 2\pi / l$: There is Kelvin-wave turbulence characteristic in quantum fluid.

Huge Scale Simulations

In Japan Atomic Energy Agency

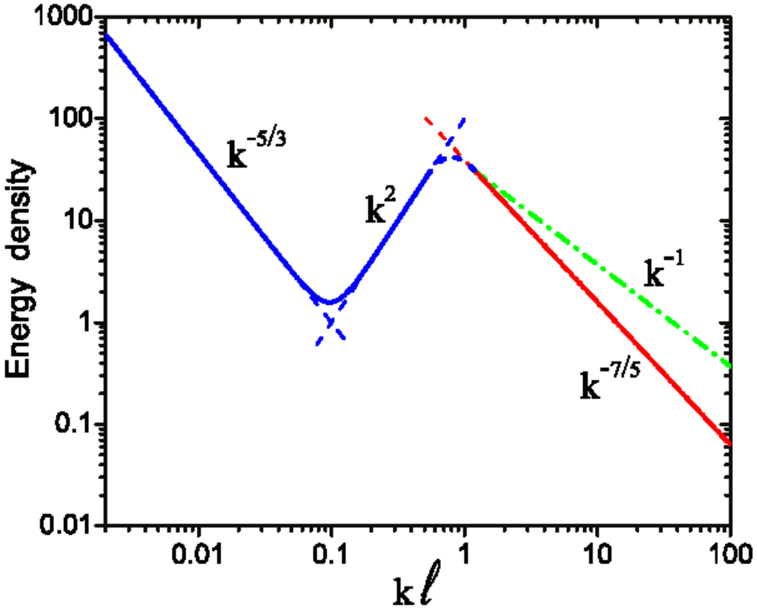


Some bottleneck effect between Richardson (Kolmogorov) and Kelvin-wave cascade?

Connection between Richardson and Kelvin-wave Cascade

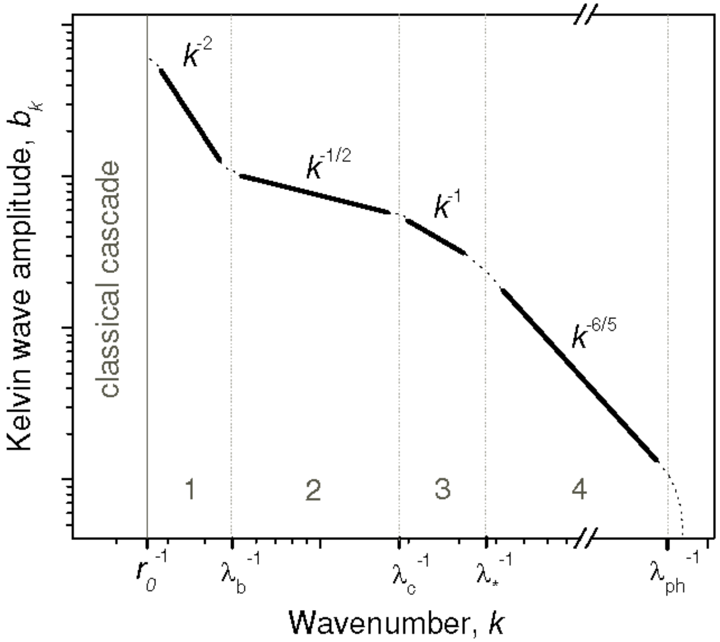
Two analytical proposals

V. S. L'vov et. al, PRB 76, 024520 (2007).



Bottleneck region as statistical equipartition.

E. Kozik and B. Svistunov, cond-mat/0703047



Complicated vortex bundle structure.

One of the big mystery in quantum fluid turbulence

Summary & Outlook of Quantum Fluid Turbulence

Quantum fluid turbulence consists of quantized vortices and shows the Kolmogorov law

⬇

Quantum fluid turbulence can become a ideal prototype to study turbulence from the view of elementary structure of vortices and the relation between dynamics of vortices and statistics like the Kolmogorov law.

Future Subject

- Details in the region of Kelvin-wave turbulence.
- Calculation of statistical and dynamical properties of vortices in real space, such as size-distribution of vortex loops, fractal dimension of vortex lines, vortex linking number etc.
- Investigation of relation between statistics and dynamics in real space and wave-number space.

MK and M. Tsubota, PRL **94**, 065302 (2005). MK and M. Tsubota, JPSJ **74**, 3248 (2005). MK and M. Tsubota, PRL **97**, 145301(2006). MK and M. Tsubota, PRA **76** 045603 (2007).