Analysis of Self-similar Reconnecting Dynamics of Quantized Vortices

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ULT25, Visualization workshop

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2. Analysis and numerical simulation.
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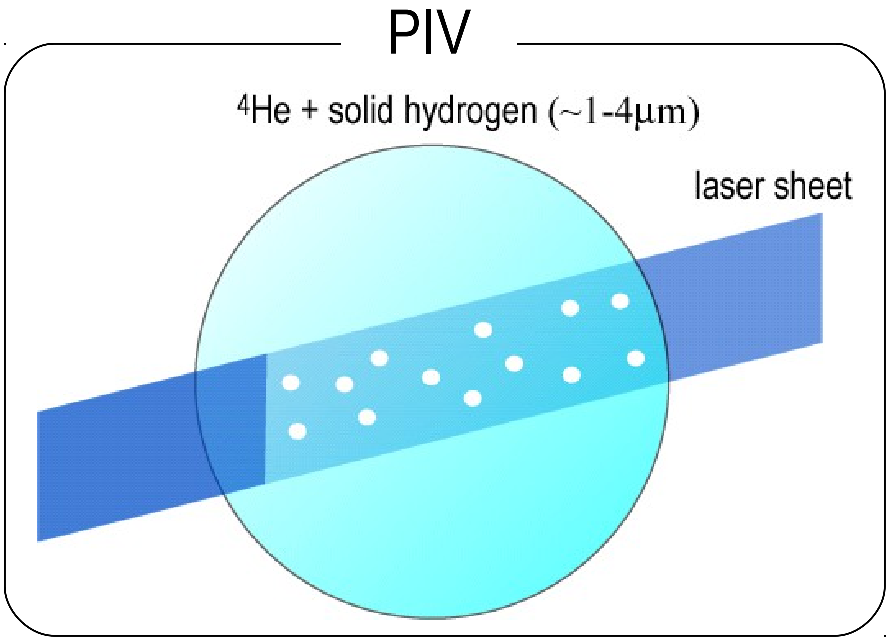
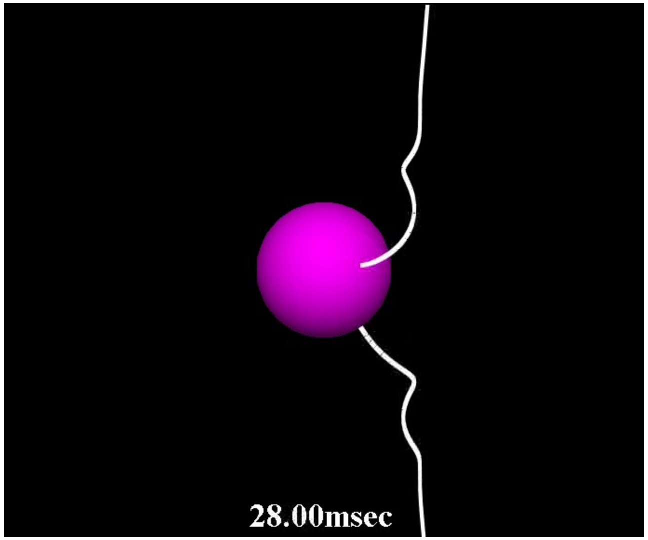
Observation of Quantized Vortices (Maryland University)

G. P. Bewley, D. P. Lathrop, K. R. Sreenivasan, Nature **441**, 588(2006).

PIV



Solid of hydrogen is captured by vortices rather than traces the velocity field



Force at hydrogen surface

$$\boldsymbol{F}_{\rm drag} = -6\pi\eta a \boldsymbol{v}_{\rm surface}$$

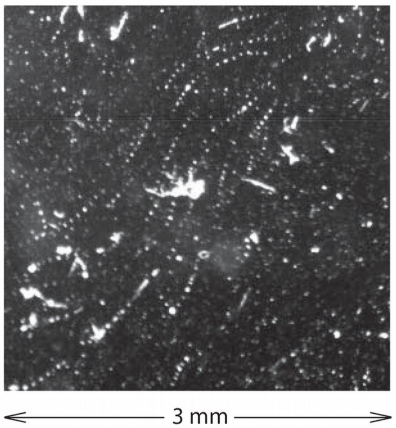
$$\boldsymbol{F}_{\rm trap} = \int_{\rm surface} \mathrm{d}S \, \nabla P$$

$$F_{\rm drag} \gg F_{\rm trap} \; : \; \text{viscous fluid}$$

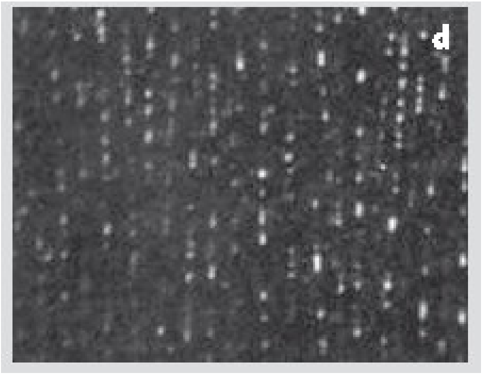
$$F_{\rm trap} \gg F_{\rm drag} \; : \; \text{superfluid}$$

Observation of Quantized Vortices

G. P. Bewley, D. P. Lathrop, K. R. Sreenivasan, Nature **441**, 588(2006).
D. Lathrop, talk at PSM2007 in Gifu.



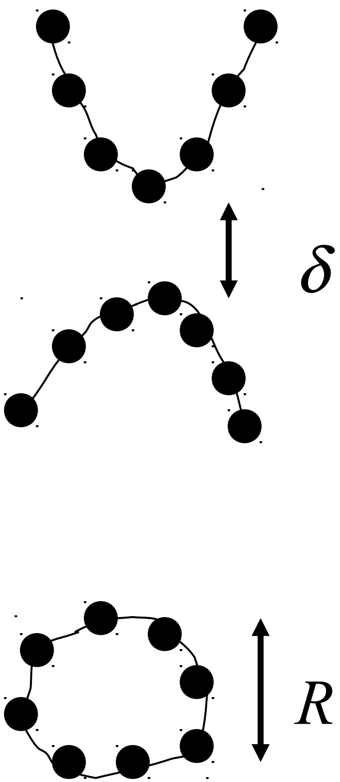
Vortex tangle in thermal counter flow



Vortex lattice under rotation

Disappearance of a vortex ring

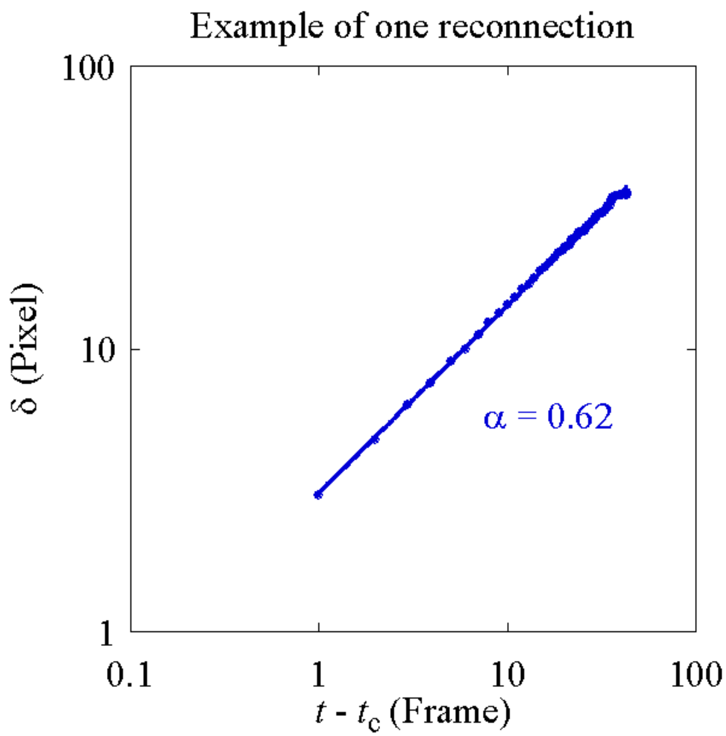
Analysis of Vortex Reconnections and Disappearance of a Vortex Ring

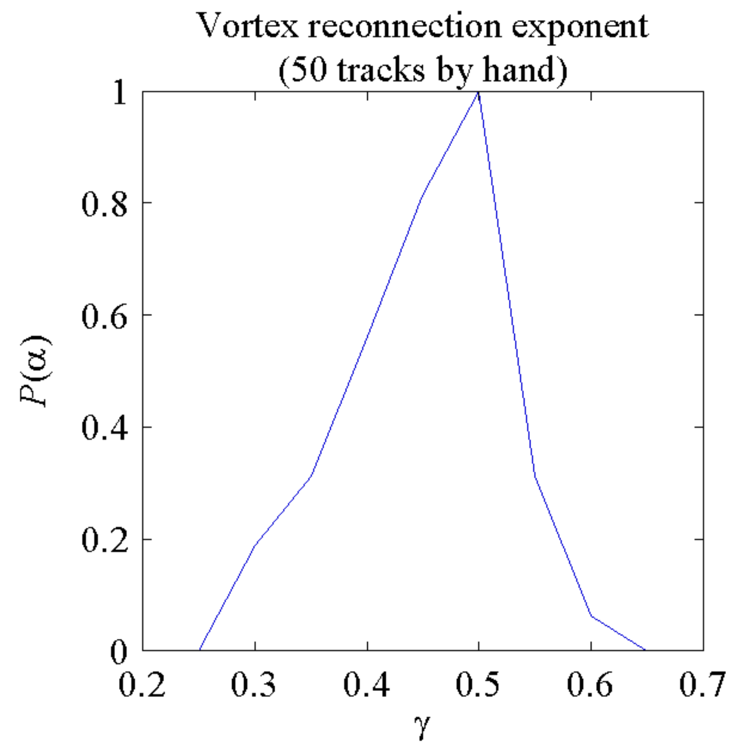


$$\delta \sim \kappa |t - t_c|^\alpha$$

$$R \sim \kappa |t - t_c|^\alpha$$

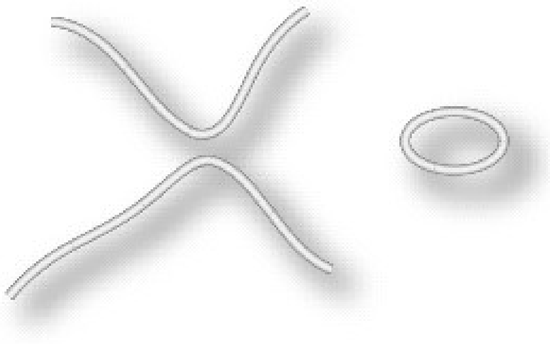
$\rightarrow$ $\alpha \sim 0.5$ (theory)





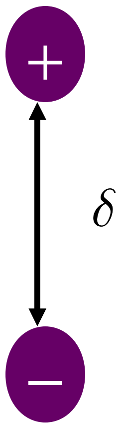
What is the origin of fluctuation from $\alpha = 0.5$?

Simple Analysis of Reconnection



→ Vortex elements becomes anti-parallel when reconnections occur

2 dimensional analysis



$$\dot{\delta}(t) = -\gamma \frac{\beta}{\delta(t)}$$

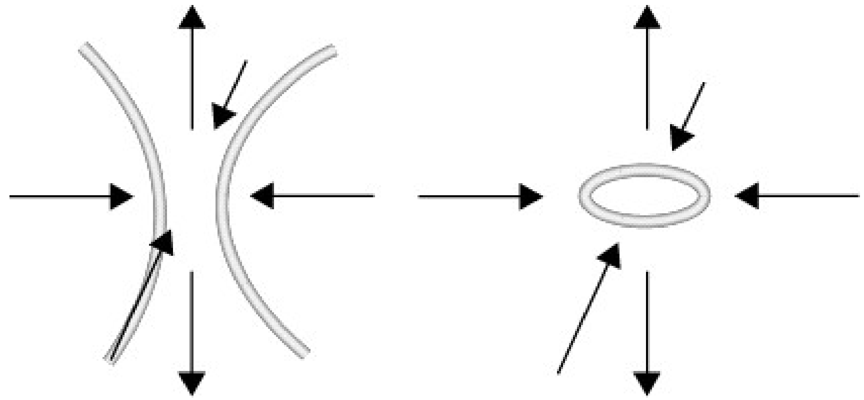
γ : parameter defined by the mutual friction etc

$$\beta = \frac{\kappa}{2\pi}$$

$$\delta(t) \simeq (\delta_0 - 2\gamma\beta t)^{0.5} \quad \Rightarrow \quad \alpha \sim 0.5$$

Main Origin of Fluctuation From $\alpha = 0.5$

The effect of strain field?



Strong strain field can be applied at the reconnection point under the effect of other vortex elements in turbulence (This is very popular in normal fluid turbulence)

$\rightarrow$Strongly deviate the exponents from $\gamma = 0.5$?

The Effect of Uniform Field (2D Analysis)

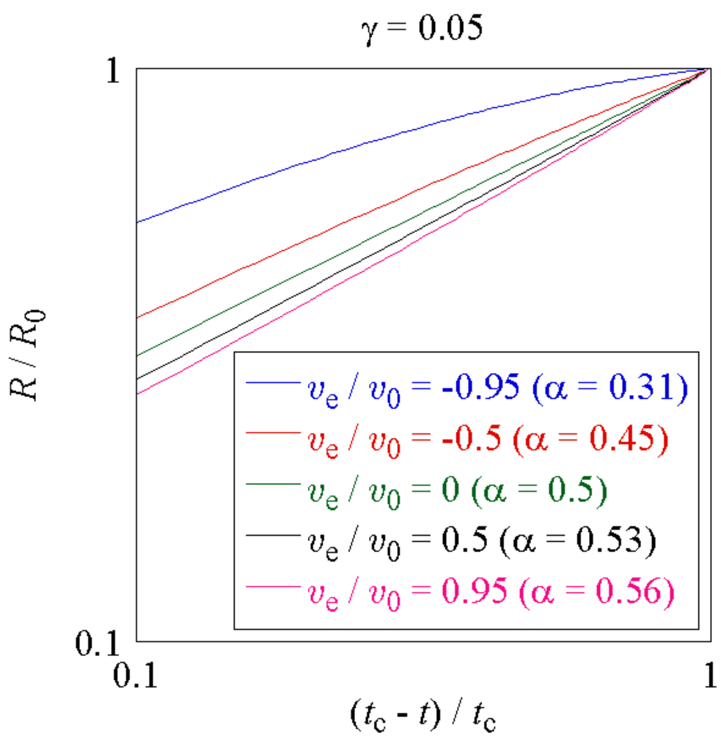
$$\dot{\delta}(t) = -\gamma\left[v_{\mathrm{e}} + \frac{\beta}{\delta(t)}\right]$$

v_{e} : external velocity field

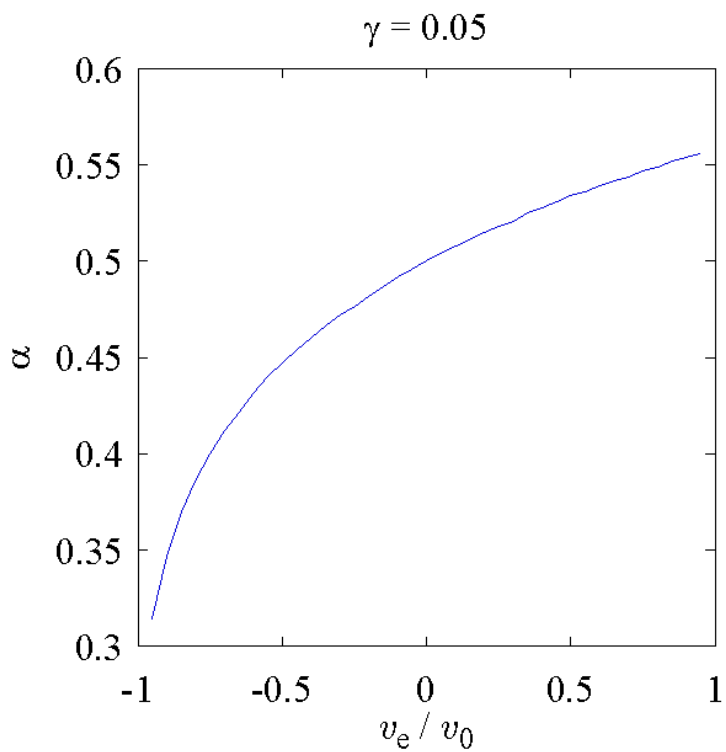
$$R(t) = \frac{-1 - W\left[(v_{\mathrm{s}}R_0 + 1)\mathrm{e}^{-\alpha v_{\mathrm{s}}^2 t + v_{\mathrm{s}}R_0 + 1}\right]}{v_{\mathrm{s}}}$$

$$t_{\mathrm{c}} = \frac{v_{\mathrm{s}}R_0 - \log(1 + v_{\mathrm{s}}R_0)}{\alpha v_{\mathrm{s}}^2}$$

$W(x)$: Lambert's W function : $x = W(x)\mathrm{e}^{W(x)}$



$$v_0 \equiv \frac{\beta}{R_0}$$



Motivation

1. We investigate the effect of the uniform and strain field on disappearance of vortex rings by numerically solving the Gross-Piteavskii equation.
2. We compare our results with experimental data.

Gross-Pitaevskii(GP) Equation

$$(\mathrm{i}-\gamma)\hbar\frac{\partial\Phi}{\partial t}=\left[-\frac{\hbar^2}{2m}\nabla^2-\mu+\frac{\hbar^2}{2m\xi^2\rho_0}|\Phi|^2-\mathrm{i}\boldsymbol{v}_\mathrm{e}\cdot\nabla\right]\Phi$$

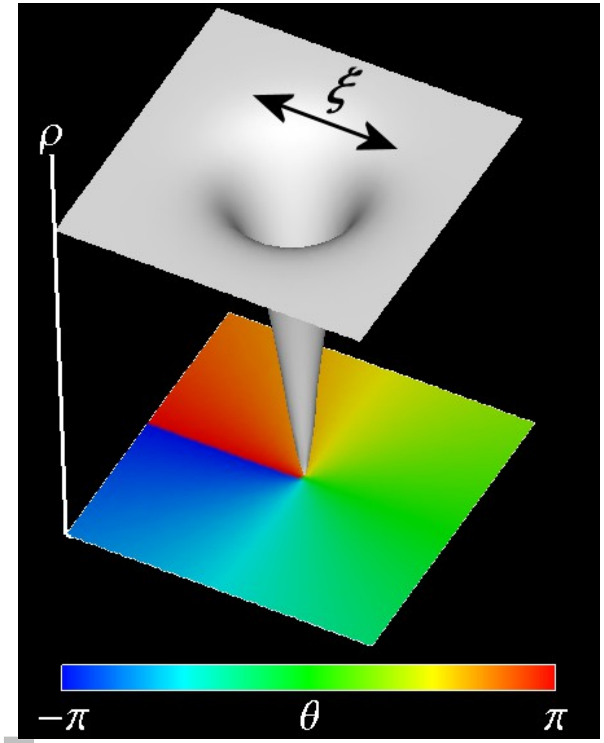
$\Phi=\sqrt{\rho}e^{\mathrm{i}\theta}$: Macroscopic wave function

ρ_0 : Mean density

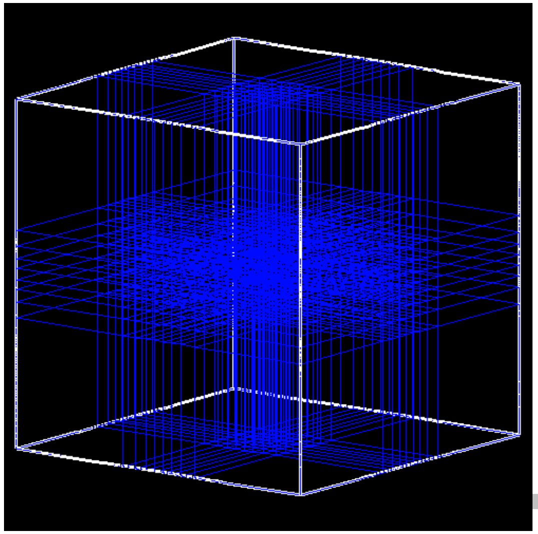
ξ : Healing length

γ : The mutual friction constant

Vortex in the GP equation

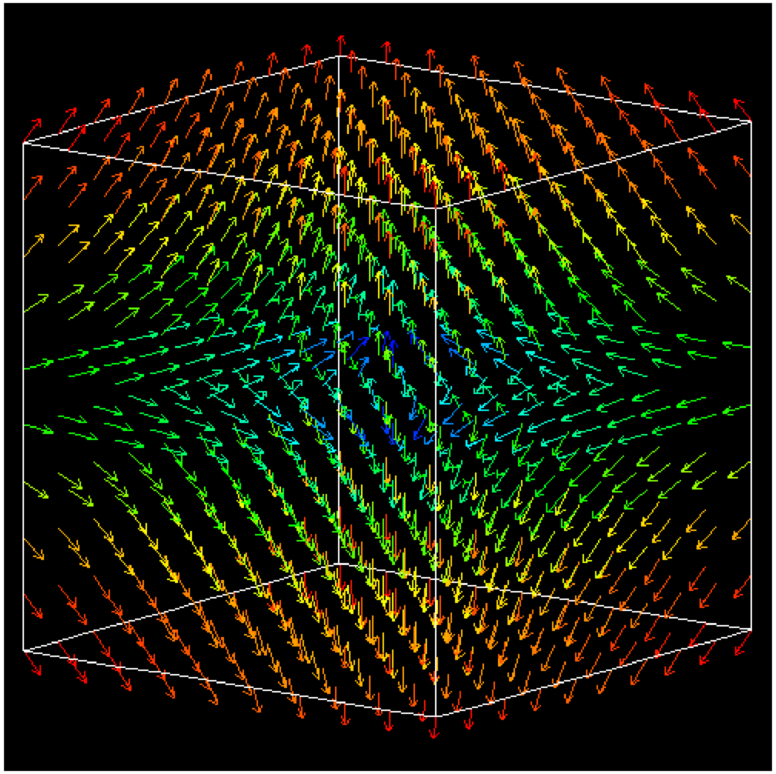


^{4}He : $\xi = 1$Å
Initial radius : $R_0 = 12.5$ μm
Box size : $L = 100$ μm
$\gamma = 0.05$ ($T = 1.4$ K)
Neumann boundary



Simulation grid

Strain Field



Strain field : $\boldsymbol{v}_{\mathrm{strain}} = \frac{2\pi v_{\mathrm{s}}}{L}[x, y, -2z]$

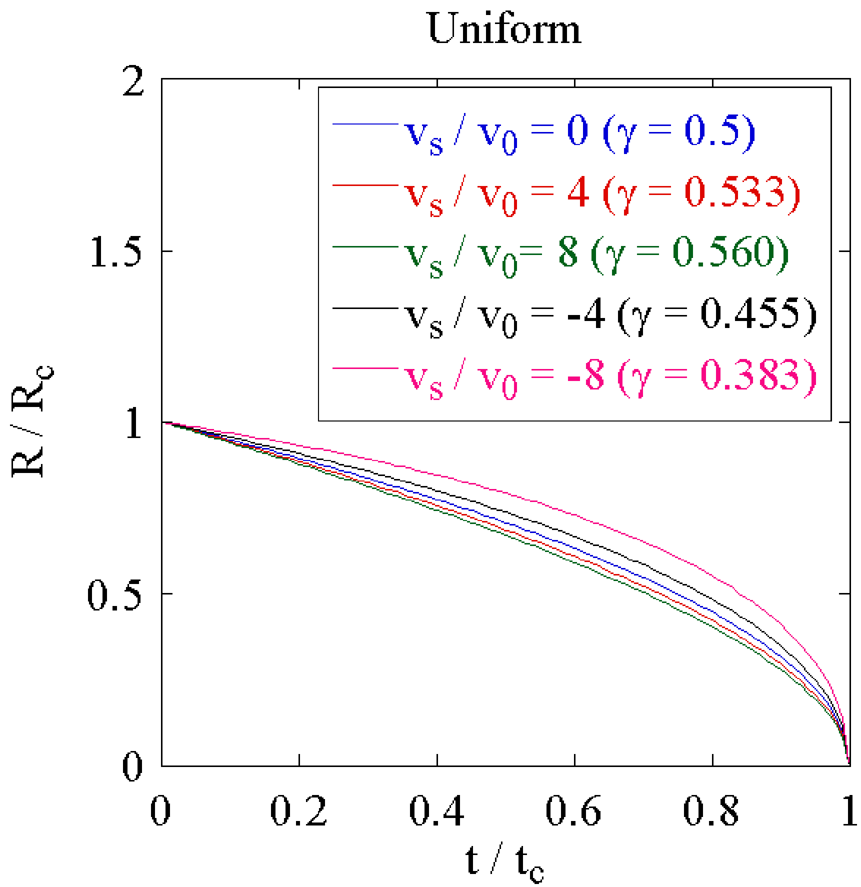
Modify

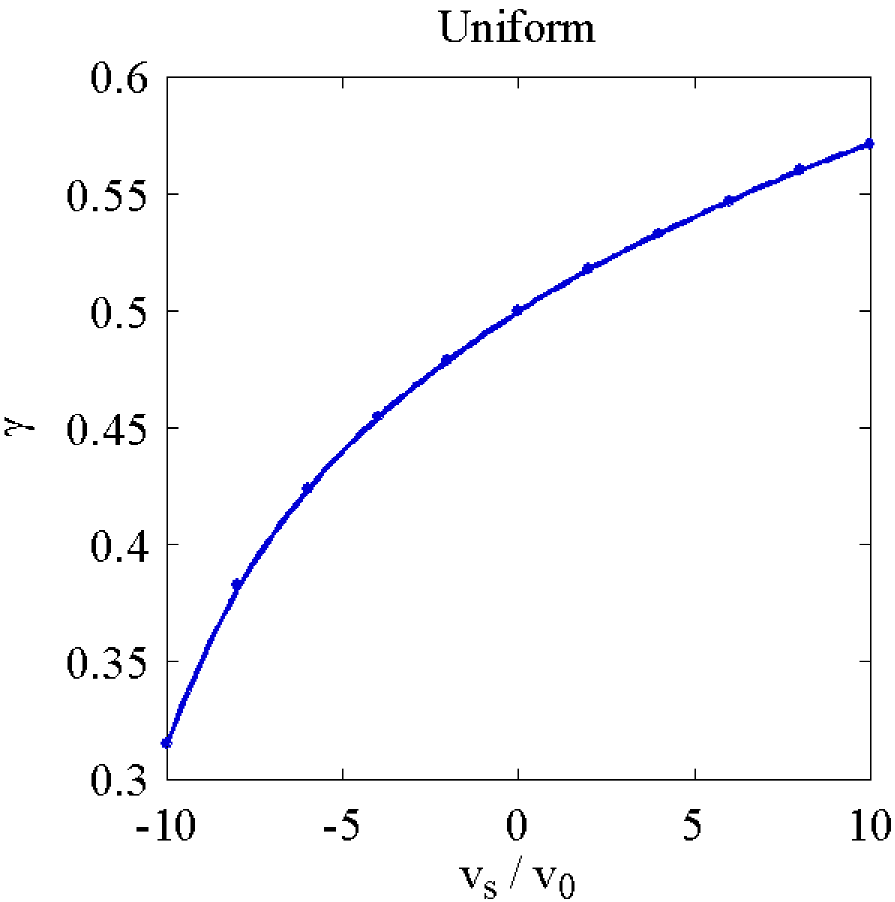
$$\boldsymbol{v}_{\mathrm{strain}} = v_{\mathrm{s}}[\sin(2\pi x/L), \sin(2\pi y/L), -2\sin(2\pi z/L)]$$

: satisfy the Neumann boundary

condi$x, y, z = \pm L/2$

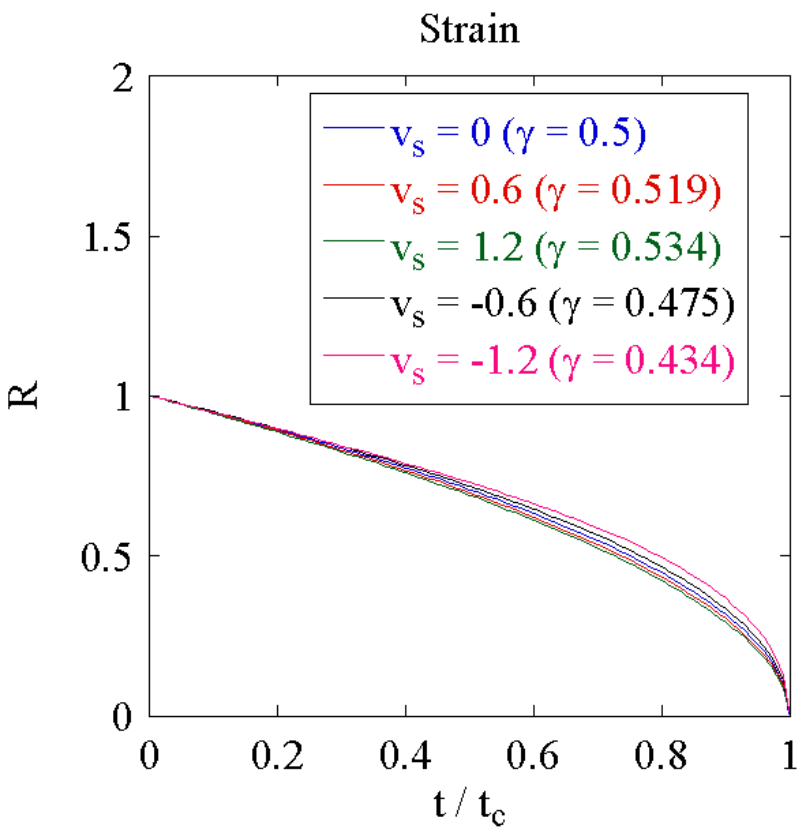
Uniform Field

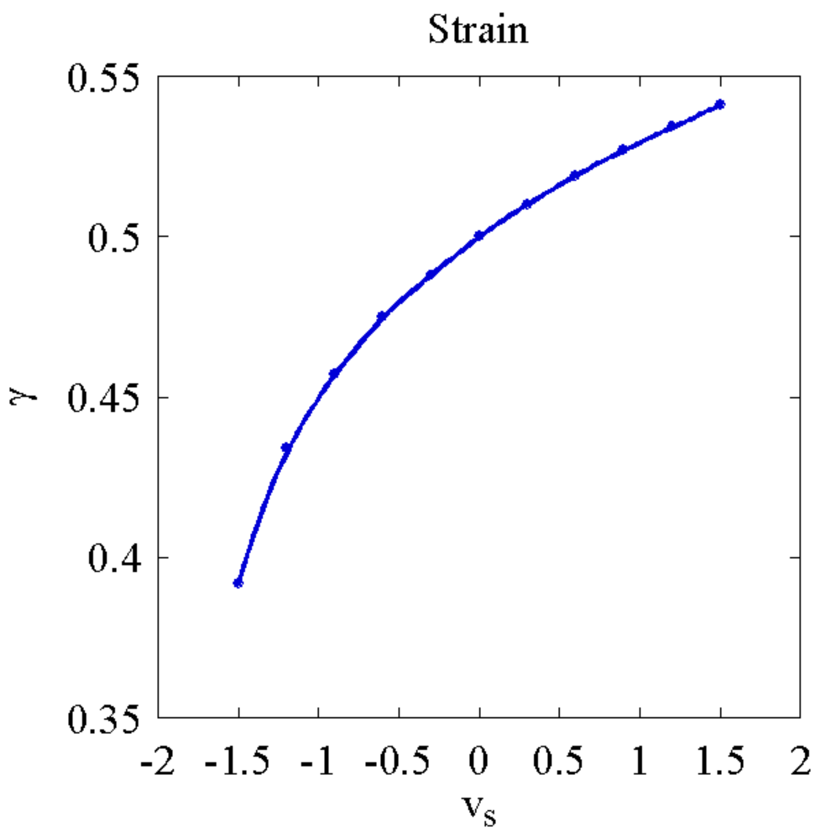




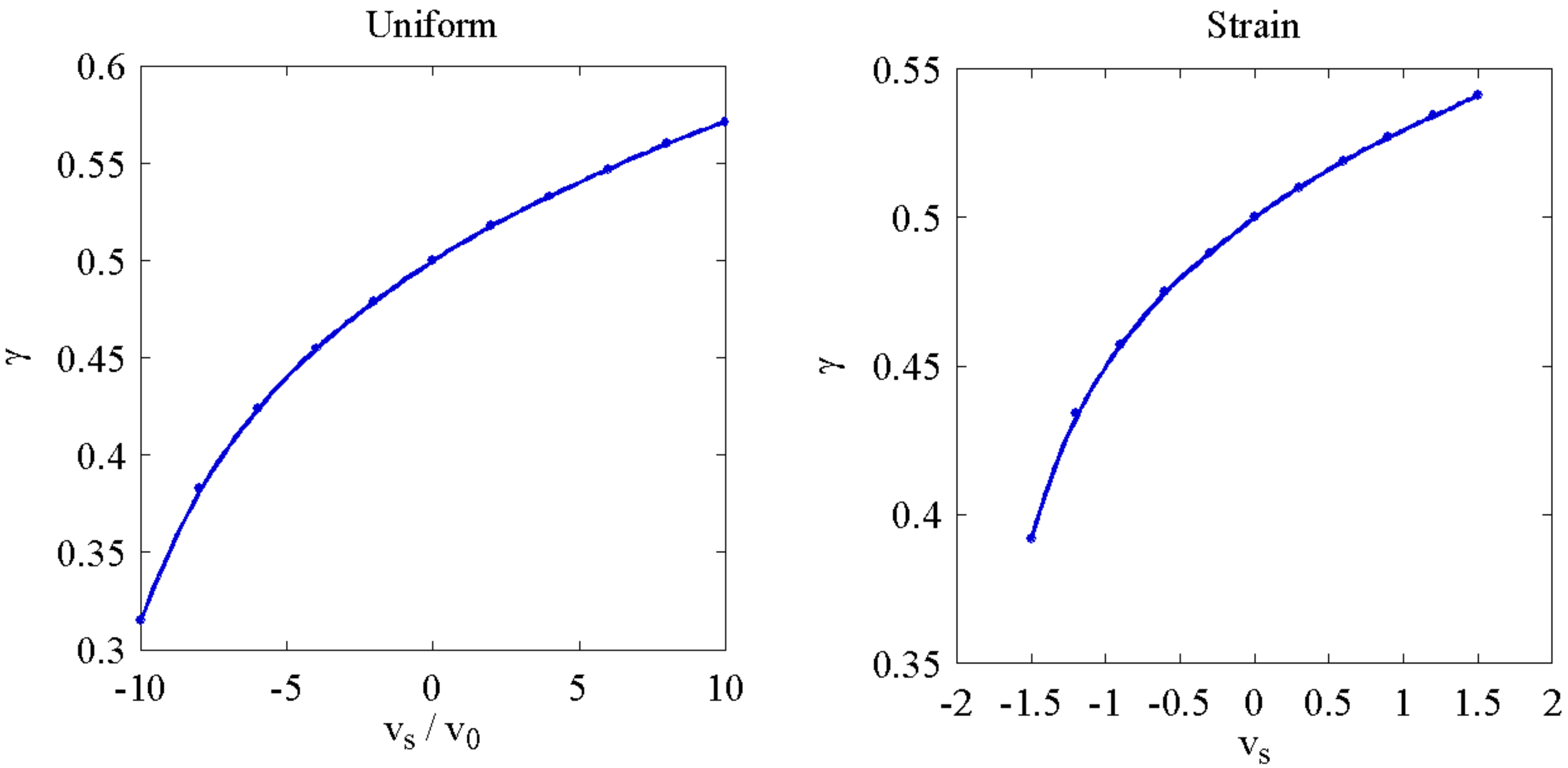
$$v_0 \equiv \frac{\kappa}{4\pi R_0} = 6.35 \times 10^{-4}$$

Strain Field





Comparison Between Uniform & Strain Field



Strain field can deviates the exponent more strongly.

Probability of the Exponent

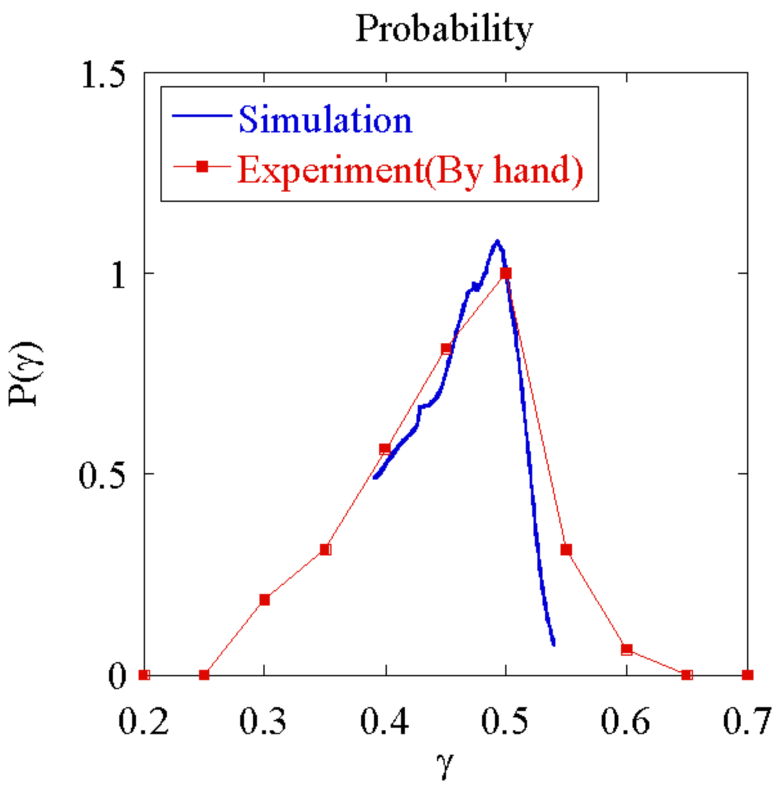
$$P(\gamma) \propto \frac{\mathrm{d}\gamma}{\mathrm{d}v_{\mathrm{s}}} P(v_{\mathrm{s}})$$

Example 1 : Assuming the Gaussian PDF

$$P(v_{\mathrm{s}}) \propto \exp[-(v_{\mathrm{s}}/v_{\mathrm{c}})^2]$$

$v_c \sim 2.0\ v_0$: critical strain between shrink and balloon of vortex rings

Comparison is good



Probability of the Exponent

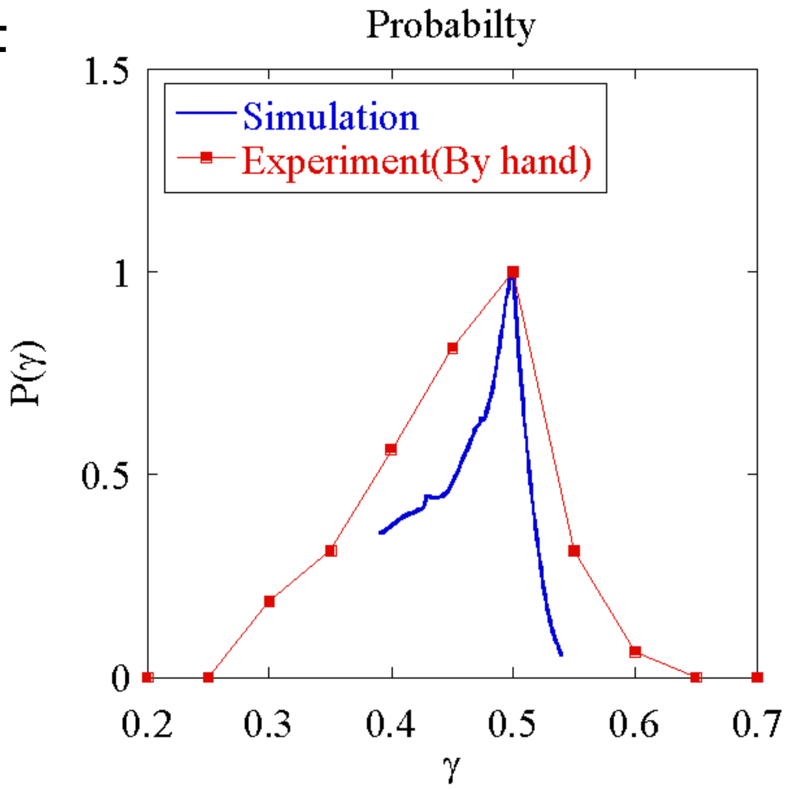
$$P(\gamma) \propto \frac{d\gamma}{dv_s} P(v_s)$$

Example 1 : Assuming the experimental PDF

$$P(v_s) \propto |v_s - v_c|^3$$

$v_c \sim 2.0\ v_0$: critical strain between shrink and balloon of vortex rings

Comparison becomes worse



Summary

- Strain field can strongly deviate the exponent of vortex reconnections.
- Probability of exponent in simulations is comparable to that in experiments (in particular, when assuming the Gaussian PDF of superfluid strain velocity).
- Future work : simulation of vortex reconnections in the strain field.