Non-Abelian Vortices in Spinor Bose-Einstein Condensates

Michikazu Kobayashi[a]

Collaborators: Yuki Kawaguchi[a], Muneto Nitta[b], and Masahito Ueda[a]

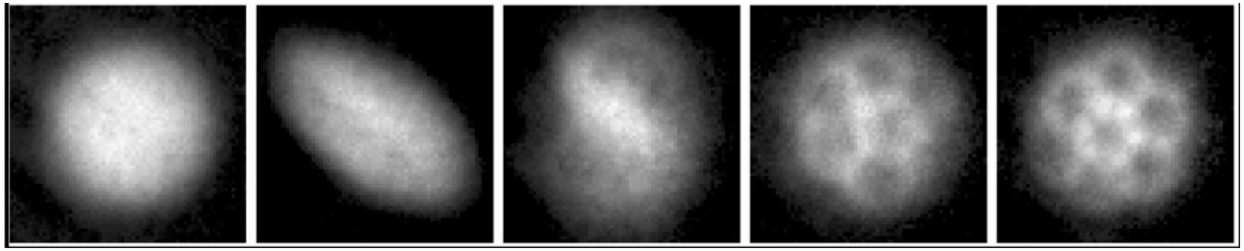
University of Tokyo[a] and Keio University[b]

Apr. 21, 2009, Workshop of A03-A04 groups for Physics of New Quantum Phases in Superclean Materials(O22)

Contents

1. Vortices in Bose-Einstein condensates
2. Spin-2 Bose-Einstein condensates
3. Vortices in spin-2 Bose-Einstein condensates
4. Collision dynamics of vortices
5. Summary

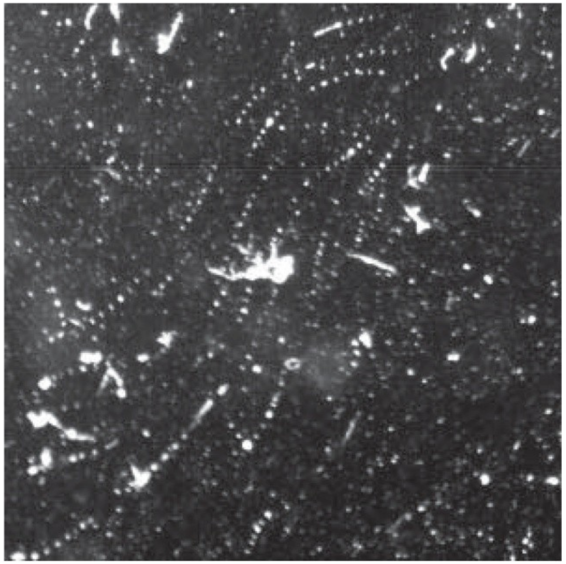
Vortices in Bose-Einstein Condensates



vortex in ^{87}Rb BEC

K. W. Madison et al.
PRL **86**, 4443 (2001)

vortex in ^{4}He



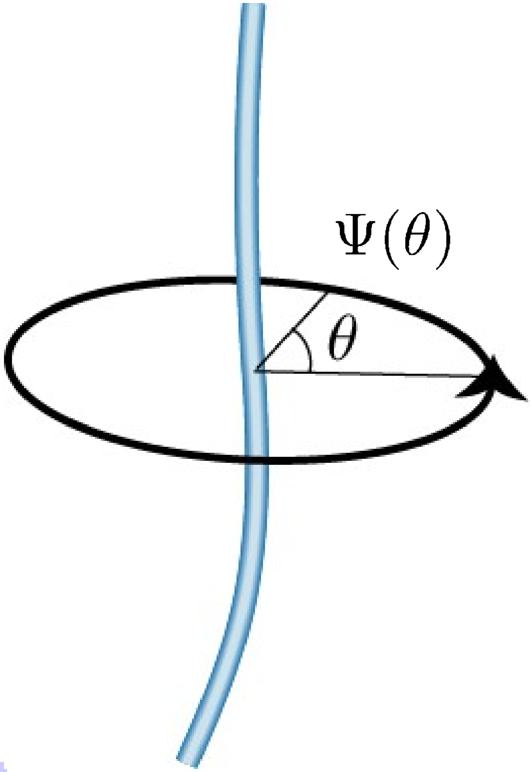
G. P. Bewley et al.
Nature **441**, 588 (2006)

Vortices appears as line defects when symmetry breaking happens

- Vortices are Abelian for single-component BEC

Quantized Vortex and Topological Charge

Topological charge of a vortex can be considered how order parameter changes around the vortex core



Single component BEC : $\Psi(\theta) \propto \exp[in\theta]$

Topological charge can be expressed by integer n

Quantized Vortex and Topological Charge

Topological charge of a vortex can be considered how order parameter changes around the vortex core

Topological charge can be expressed by the first homotopy group

single component BEC
$\pi_1(G/H) = Z$

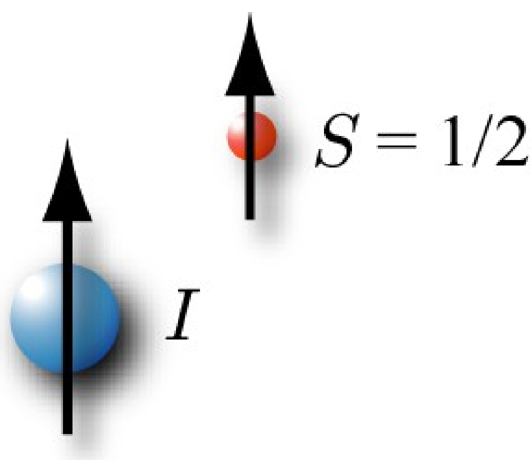
$G\ (= U(1))$: Symmetry of the system
$H\ (= 1)$: Symmetry of the order-parameter

When topological charge can be expressed by non-commutative algebra (: first homotopy group π_1 is non-Abelian), we define such vortices as "**non-Abelian vortices**"

Spin-2 BEC

Bose-Einstein condensate in optical trap
(spin degrees of freedom is alive)

Hyperfine coupling
(F $= I +$ S)



^{87}Rb ($I = 3/2$)

$$F = 2 \begin{cases} m_F = 2 \\ m_F = 1 \\ m_F = 0 \\ m_F = -1 \\ m_F = -2 \end{cases} \qquad F = 1 \begin{cases} m_F = 1 \\ m_F = 0 \\ m_F = -1 \end{cases}$$

BEC characterized by m_F

Introduction of spinor BEC

Hamiltonian of spinor boson system (without trapping and magnetic field)

$$H = -\int d\boldsymbol{x}\, \frac{\hbar^2}{2M} \nabla \Psi^\dagger_m(\boldsymbol{x}) \nabla \Psi_m(\boldsymbol{x})$$
$$+ \frac{1}{2} \int d\boldsymbol{x}_1 \int d\boldsymbol{x}_2 \Psi^\dagger_{m_1}(\boldsymbol{x}_1) \Psi^\dagger_{m_2}(\boldsymbol{x}_2) V_{m_1 m_2 m'_1 m'_x}(\boldsymbol{x}_1 - \boldsymbol{x}_2) \Psi_{m'_2}(\boldsymbol{x}_2) \Psi_{m'_1}(\boldsymbol{x}_1)$$

Contact interaction ($l = 0$)

$$V_{m_1 m_2 m'_1 m'_x}(\boldsymbol{x}_1 - \boldsymbol{x}_2) = \delta(\boldsymbol{x}_1 - \boldsymbol{x}_2) \sum_{F=even} g_F P_F$$

$$P_F = \sum_{m_1, m_2, m'_1, m'_2, M} O^{F,M}_{m_1 m_2} \left(O^{F,M}_{m'_1 m'_2} \right)^* |F, m'_1\rangle \otimes |F, m'_2\rangle \langle F, m_2| \otimes \langle F, m_1|$$

Mean Field Approximation for BEC at $T = 0$

Case of Spin-2

$$H \simeq \int d\boldsymbol{x} \left[-\Psi_m^* \frac{\hbar^2}{2M} \nabla^2 \Psi_m + \frac{c_0}{2} n_{\rm tot}^2 + \frac{c_1}{2} \boldsymbol{F}^2 + \frac{c_2}{2} |A_{00}|^2 \right]$$

$$c_0 = \frac{4g_2 + 3g_4}{7}, \quad c_1 = \frac{g_4 - g_2}{7}, \quad c_2 = \frac{7g_0 - 10g_2 + 3g_4}{7}$$

$$n_{\rm tot}(\boldsymbol{x}) = \Psi_m^*(\boldsymbol{x})\Psi_m(\boldsymbol{x}), \quad \boldsymbol{F}(\boldsymbol{x}) = \Psi_m^*(\boldsymbol{x})\hat{\boldsymbol{F}}_{mm'}(\boldsymbol{x})\Psi_{m'}(\boldsymbol{x})$$

$$A_{00}(\boldsymbol{x}) = \frac{1}{\sqrt{5}}[2\Psi_2(\boldsymbol{x})\Psi_{-2}(\boldsymbol{x}) - 2\Psi_1(\boldsymbol{x})\Psi_{-1}(\boldsymbol{x}) + \Psi_0(\boldsymbol{x})^2]$$

$n_{\rm tot}$: total density

$\boldsymbol{F}$: magnetization

A_{00} : singlet pair amplitude

Spin-2 BEC

$$H \simeq \int d\boldsymbol{x}\left[-\Psi_m^* \frac{\hbar^2}{2M}\nabla^2\Psi_m + \frac{c_0}{2}n_{\rm tot}^2 + \frac{c_1}{2}\boldsymbol{F}^2 + \frac{c_2}{2}|A_{00}|^2\right]$$

1. $c_1 < 0 \rightarrow$ ferromagnetic phase : $\boldsymbol{F} \neq 0$
2. $c_1 > 0,\ c_2 < 0 \rightarrow$ polar phase : $\boldsymbol{F} = 0,\ A_{00} \neq 0$
3. $c_1 > 0,\ c_2 > 0 \rightarrow$ cyclic phase : $\boldsymbol{F} = A_{00} = 0$

ferromagnetic

$$e^{i\phi}e^{-i\boldsymbol{e}\cdot\hat{\boldsymbol{F}}\alpha}\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$$

polar

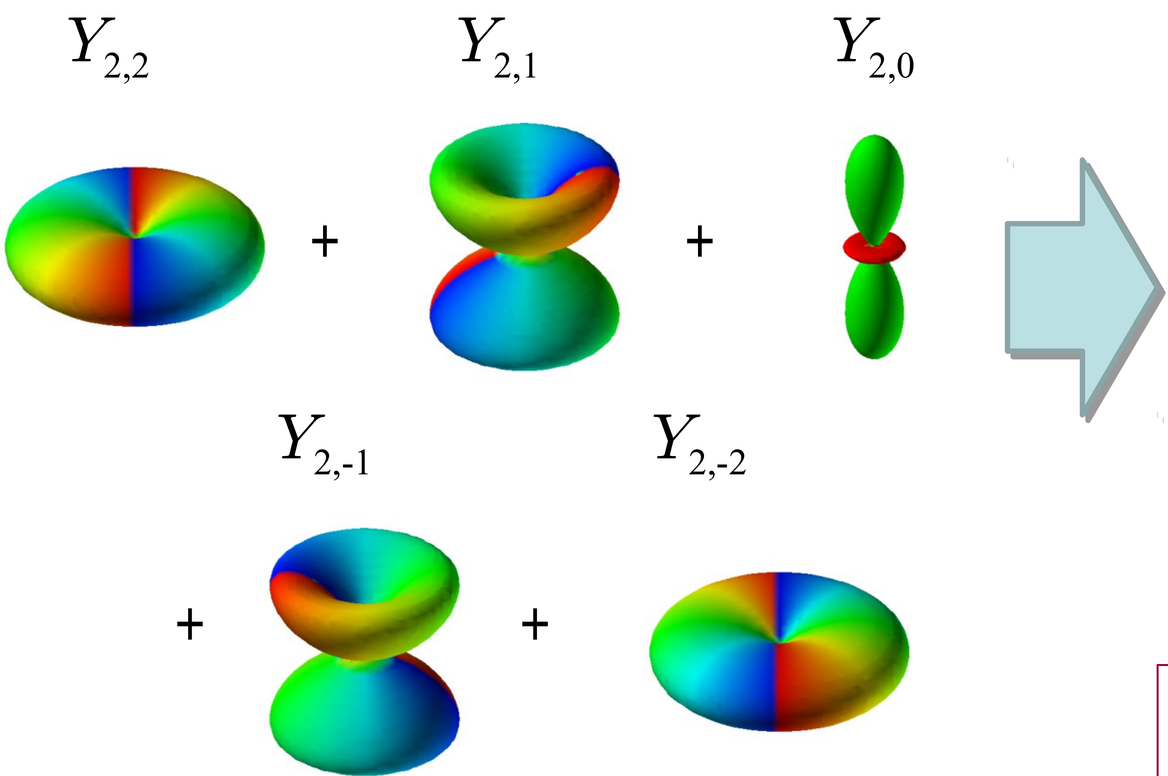
$$e^{i\phi}e^{-i\boldsymbol{e}\cdot\hat{\boldsymbol{F}}\alpha}\begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix} \text{ or } \begin{pmatrix}1/\sqrt{2}\\0\\0\\0\\1/\sqrt{2}\end{pmatrix}$$

cyclic

$$e^{i\phi}e^{-i\boldsymbol{e}\cdot\hat{\boldsymbol{F}}\alpha}\begin{pmatrix}i/2\\0\\1/\sqrt{2}\\0\\i/2\end{pmatrix}$$

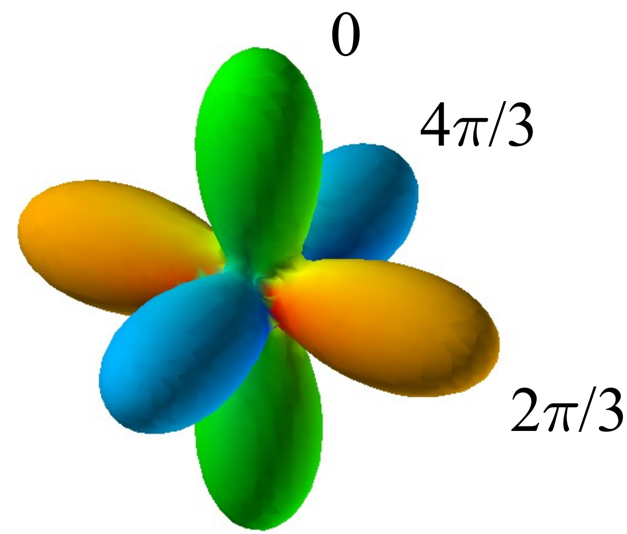
Spin-2 BEC

$$\sum_{m=-2}^{2} \Psi_m Y_{2,m}$$



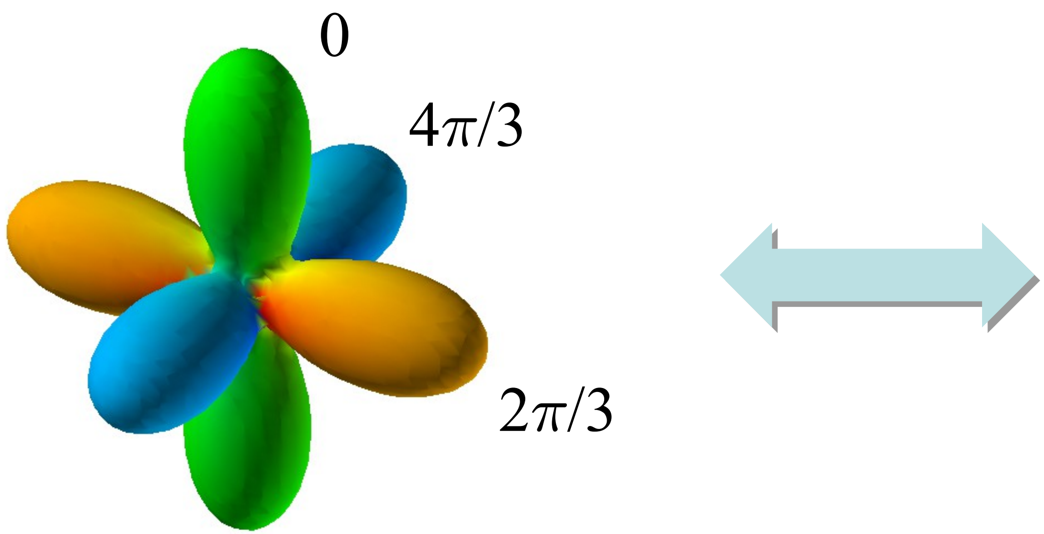
Cyclic phase

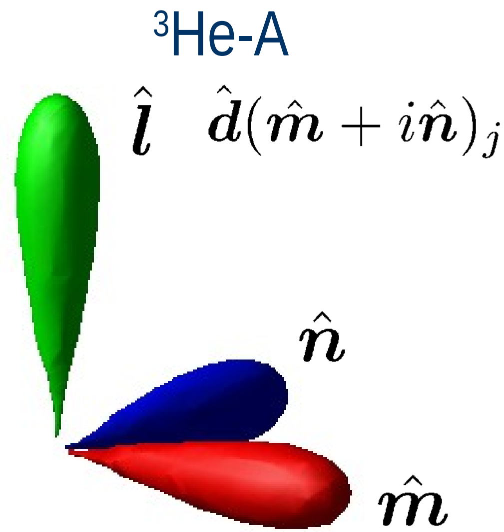
$$e^{i\phi}(3\cos^2\theta + \sqrt{3}i\sin^2\theta\cos 2\varphi - 1)$$

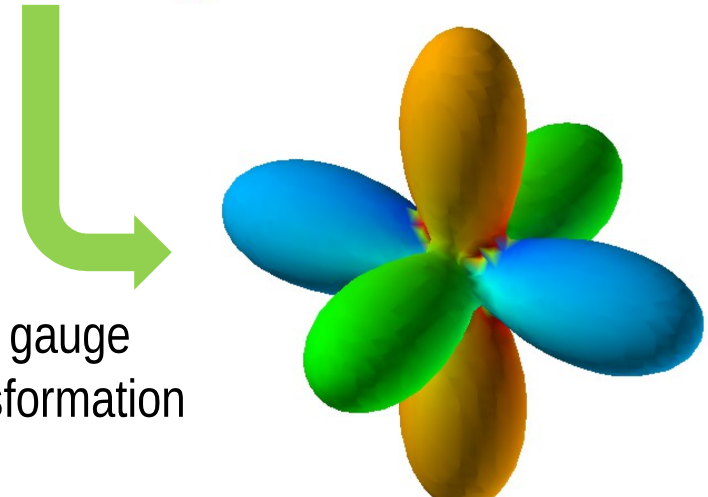


headless triad

Triad of ^{3}He-A and cyclic phase







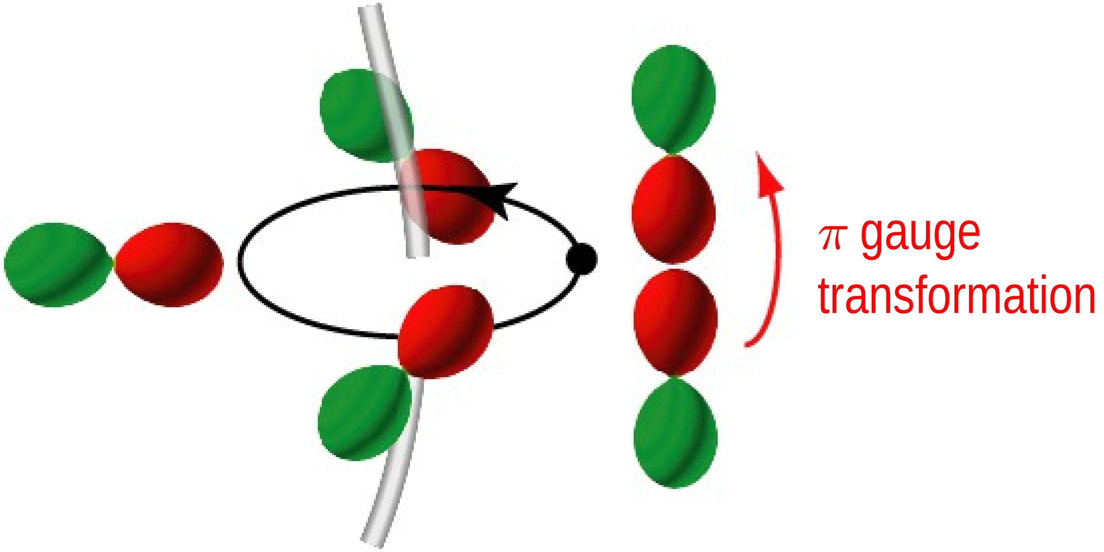
$2\pi/3$ gauge transformation

1. Having a π–rotational symmetry
2. Three axes can interchange each other by $2\pi/3$ gauge transformation

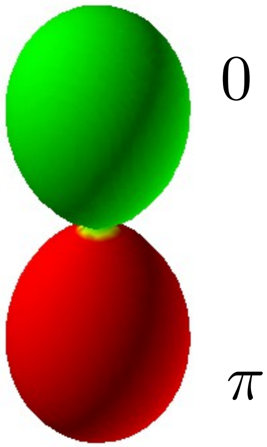
Vortices in Spinor BEC

$S = 1$ Polar phase

$$e^{i\phi} e^{-i\boldsymbol{e}\cdot\hat{\boldsymbol{F}}\alpha} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$



headless vector



Half quantized vortex : spin & gauge rotate by π around vortex core

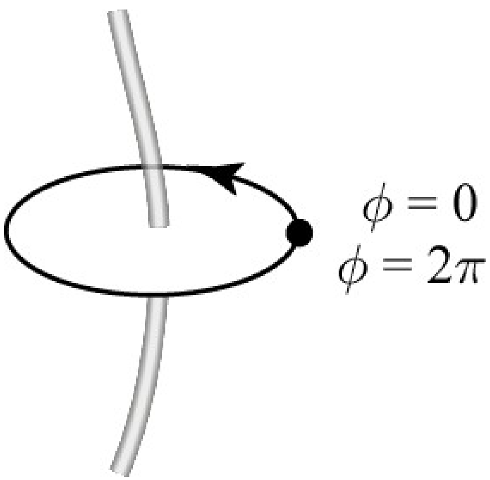
Topological charge can be expressed by integer and half integer (Abelian vortex)

$$\pi_1(G/H) = Z_2 \ltimes Z$$

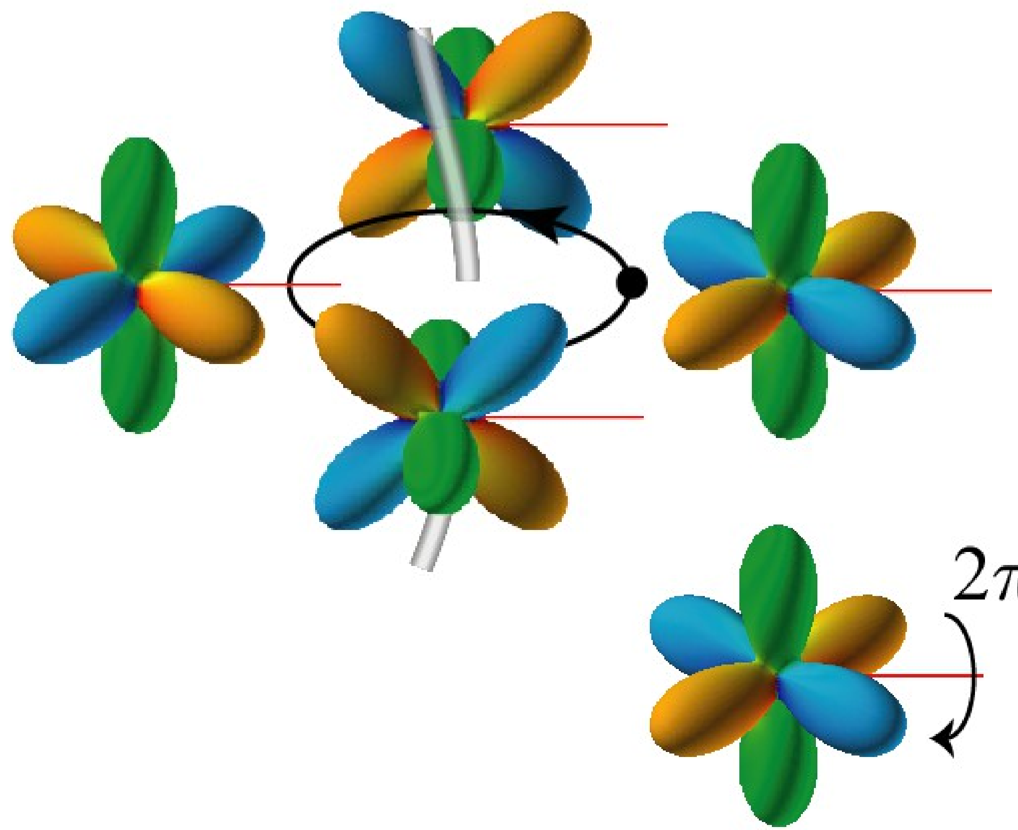
Vortices in Spin-2 BEC

There are 5 types of vortices in the cyclic phase

gauge vortex

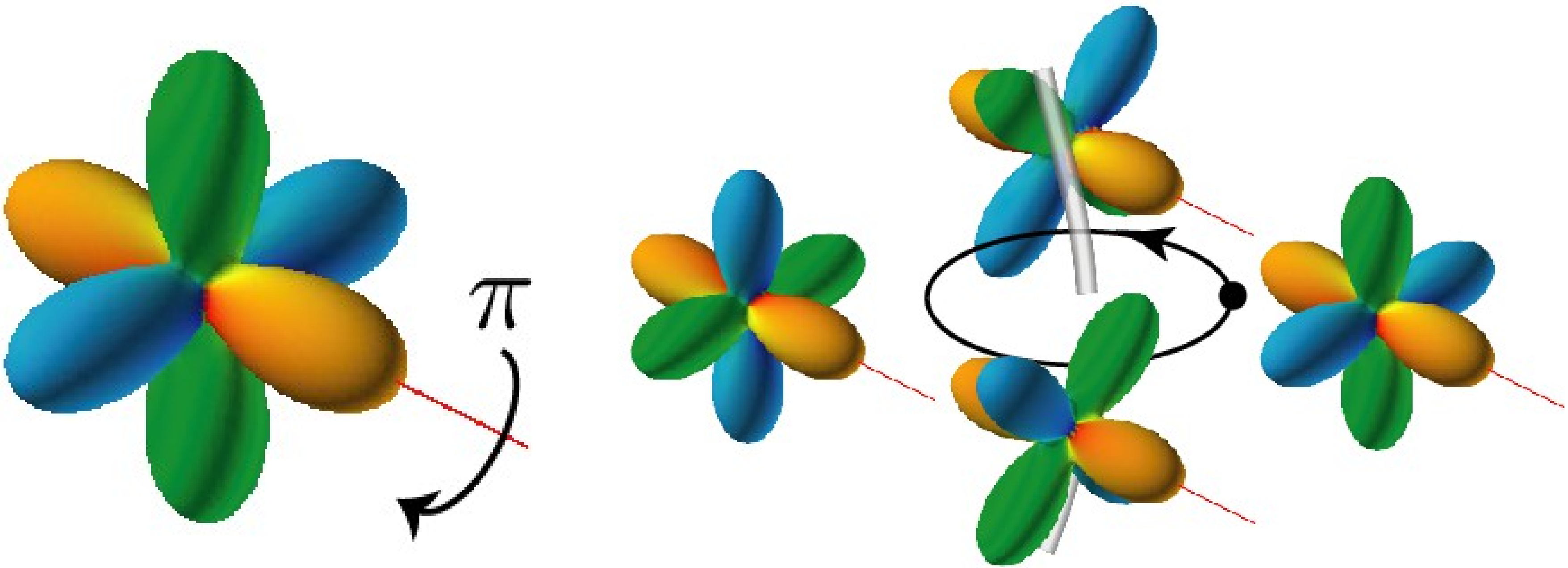


integer spin vortex



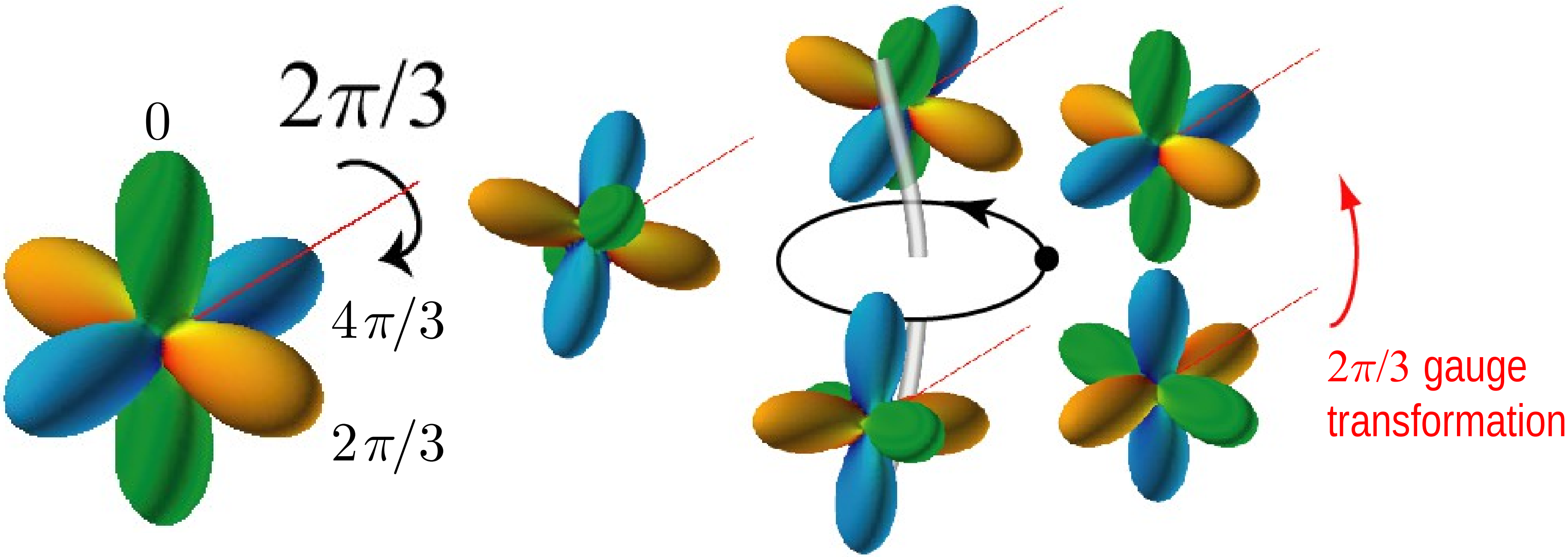
Vortices in Spin-2 BEC

1/2-spin vortex : triad rotate by π around three axis $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$



Vortices in Spin-2 BEC

1/3 vortex : triad rotate by $2\pi/3$ around four axis $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$, $\boldsymbol{e}_4$ and $2\pi/3$ gauge transformation

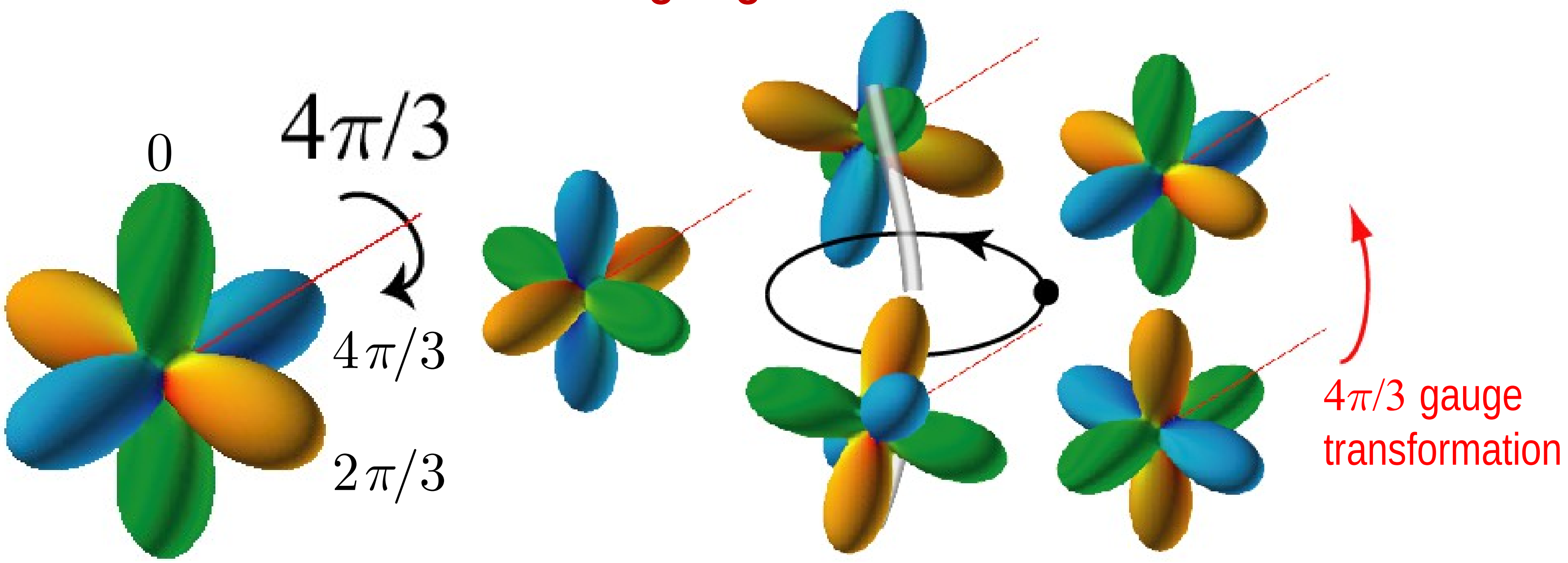


$$\boldsymbol{e}_1 = (1,1,1)/\sqrt{3}, \quad \boldsymbol{e}_2 = (1,-1,-1)/\sqrt{3}$$

$$\boldsymbol{e}_3 = (-1,1,-1)/\sqrt{3}, \quad \boldsymbol{e}_4 = (-1,-1,1)/\sqrt{3}$$

Vortices in Spin-2 BEC

4, 2/3 vortex : triad rotate by $4\pi/3$ around four axis $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$, $\boldsymbol{e}_4$ and $4\pi/3$ gauge transformation



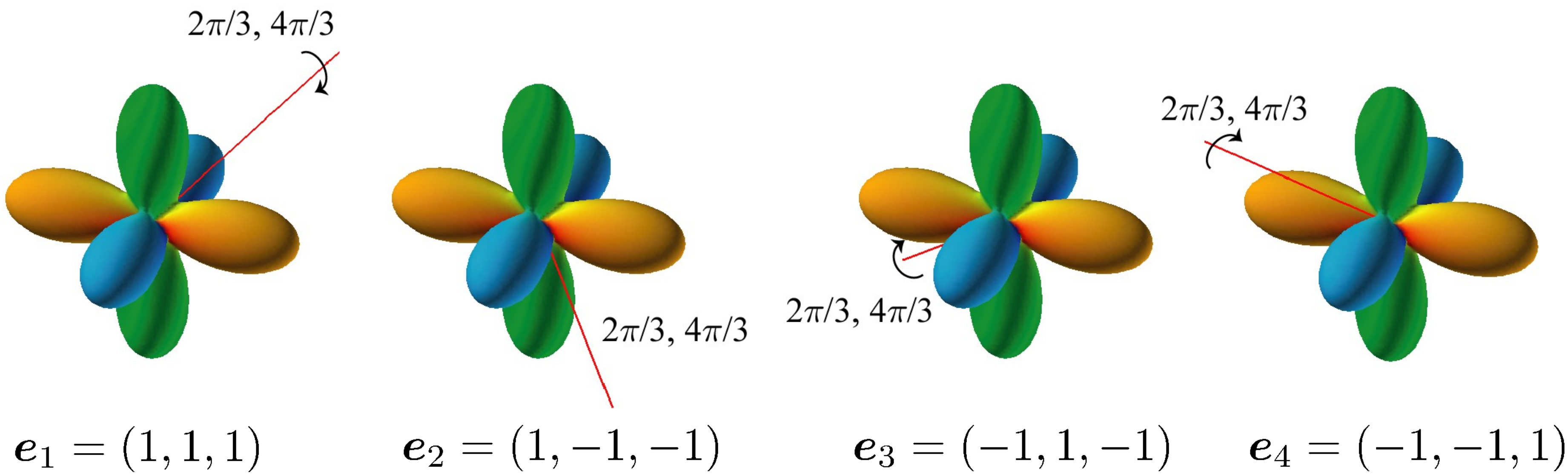
$$\boldsymbol{e}_1 = (1,1,1)/\sqrt{3}, \quad \boldsymbol{e}_2 = (1,-1,-1)/\sqrt{3}$$

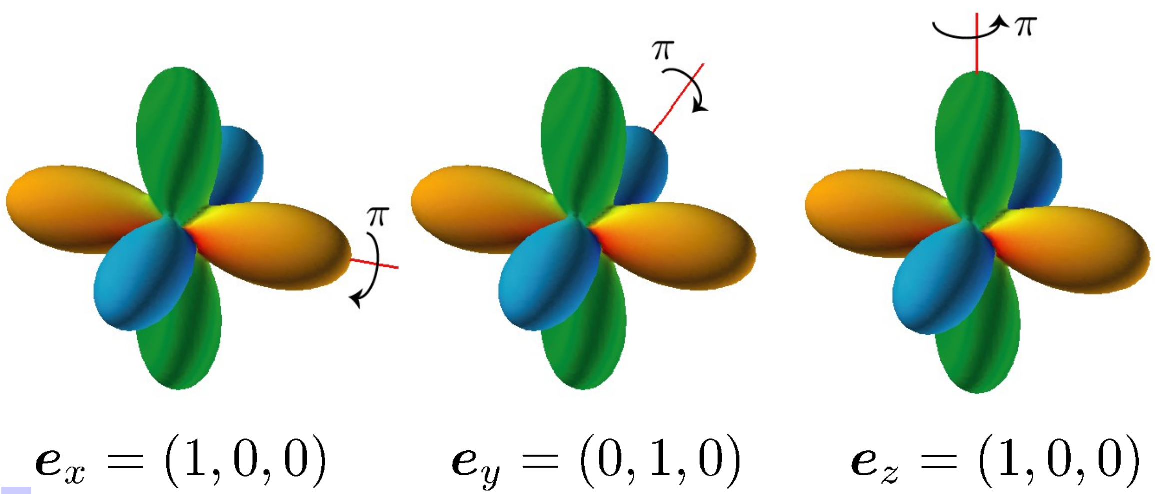
$$\boldsymbol{e}_3 = (-1,1,-1)/\sqrt{3}, \quad \boldsymbol{e}_4 = (-1,-1,1)/\sqrt{3}$$

Vortices in Spin-2 BEC

vortices	mass circulation	core structure
gauge	1	density core
Integer spin	0	polar core
1/2 spin	0	polar core
1/3	1/3	ferromagnetic core
2/3	2/3	ferromagnetic core

Topological Charge of Vortices is Non-Abelian

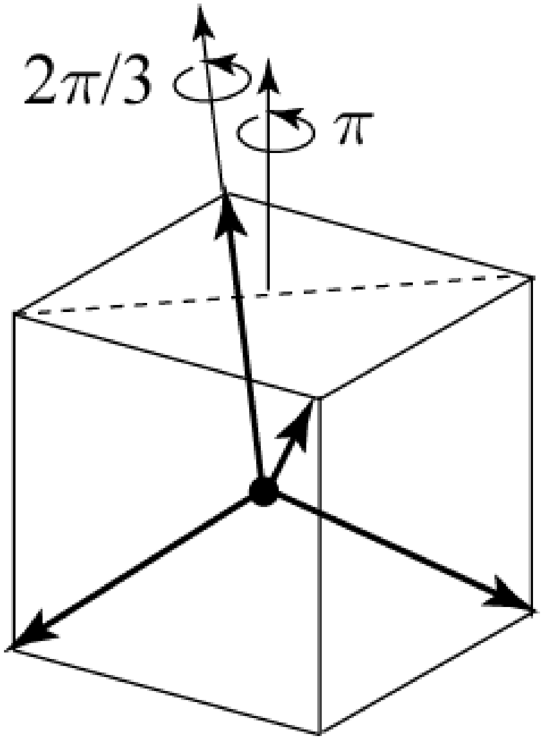




There are 12 rotations for vortices

Non-Abelian Vortices

12 rotations makes non-Abelian tetrahedral group T



Topological charge can be expressed by non-Abelian algebra which includes tetrahedral symmetry
→ non-Abelian vortex

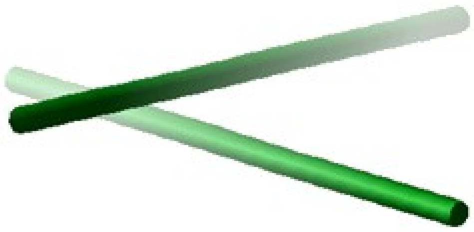
$$\pi_1(G/H) = (Z_2 \ltimes T) \ltimes Z$$

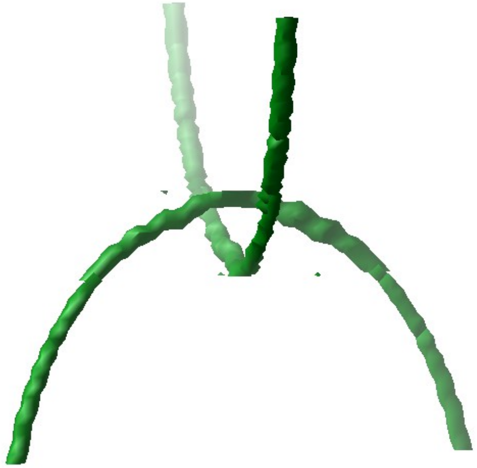
Collision Dynamics of Vortices

"**Non-Abelian**" character becomes remarkable when two vortices collide with each other

→Numerical simulation of the Gross-Pitaevskii equation

Initial state：two straight vortices in oblique angle, linked vortices





Gross-Pitaevskii Equation

$$i\hbar\frac{\partial \Psi_m}{\partial t} = \frac{\delta H}{\delta \Psi_m^*}$$

$$i\hbar\frac{\partial \Psi_2}{\partial t} = -\frac{\hbar^2}{2M}\nabla^2\Psi_2 + c_0 n_{\mathrm{tot}}\Psi_2 + c_1(F_-\Psi_1 + 2F_z\Psi_2) + \frac{c_2}{\sqrt{5}}A_{00}\Psi^*_{-2}$$

$$i\hbar\frac{\partial \Psi_1}{\partial t} = -\frac{\hbar^2}{2M}\nabla^2\Psi_1 + c_0 n_{\mathrm{tot}}\Psi_1 + c_1\left(\frac{\sqrt{6}}{2}F_-\Psi_0 + F_+\Psi_2 + F_z\Psi_1\right) - \frac{c_2}{\sqrt{5}}A_{00}\Psi^*_{-1}$$

$$i\hbar\frac{\partial \Psi_0}{\partial t} = -\frac{\hbar^2}{2M}\nabla^2\Psi_0 + c_0 n_{\mathrm{tot}}\Psi_0 + \frac{\sqrt{6}}{2}c_1(F_-\Psi_{-1} + F_+\Psi_1) + \frac{c_2}{\sqrt{5}}A_{00}\Psi^*_0$$

$$i\hbar\frac{\partial \Psi_{-1}}{\partial t} = -\frac{\hbar^2}{2M}\nabla^2\Psi_{-1} + c_0 n_{\mathrm{tot}}\Psi_{-1} + c_1\left(\frac{\sqrt{6}}{2}F_+\Psi_0 + F_-\Psi_{-2} - F_z\Psi_{-1}\right) - \frac{c_2}{\sqrt{5}}A_{00}\Psi^*_1$$

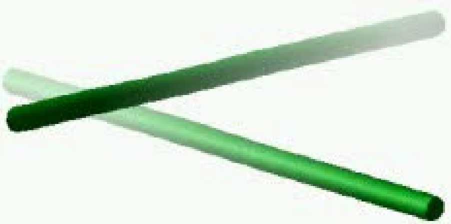
$$i\hbar\frac{\partial \Psi_{-2}}{\partial t} = -\frac{\hbar^2}{2M}\nabla^2\Psi_{-2} + c_0 n_{\mathrm{tot}}\Psi_{-2} + c_1(F_+\Psi_{-1} - 2F_z\Psi_{-2}) + \frac{c_2}{\sqrt{5}}A_{00}\Psi^*_2$$

Used Pair of Vortices

1, same vortices	1/3 vortex ($\boldsymbol{e}_1$)	1/3 vortex ($\boldsymbol{e}_1$)
2, different commutative vortices	1/3 vortex ($\boldsymbol{e}_1$)	2/3 vortex ($\boldsymbol{e}_1$)
3, different non-commutative vortices	1/3 vortex ($\boldsymbol{e}_1$)	2/3 vortex ($\boldsymbol{e}_2$)
	1/3 vortex ($\boldsymbol{e}_1$)	1/3 vortex ($\boldsymbol{e}_2$)

Collision Dynamics of Vortices

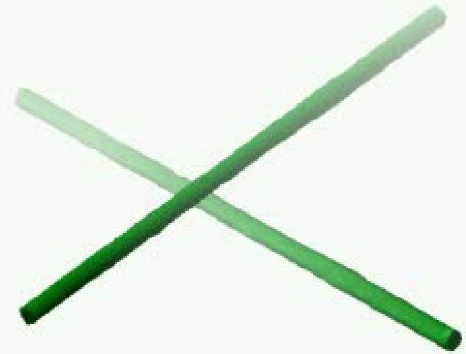
Commutative topological charge



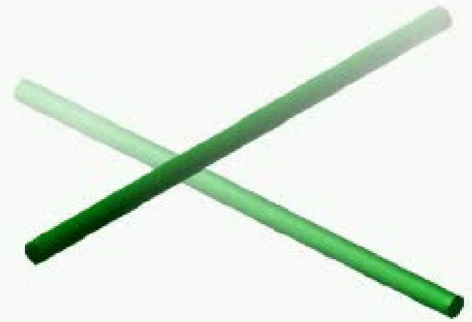
reconnection

passing through

Non-commutative topological charge



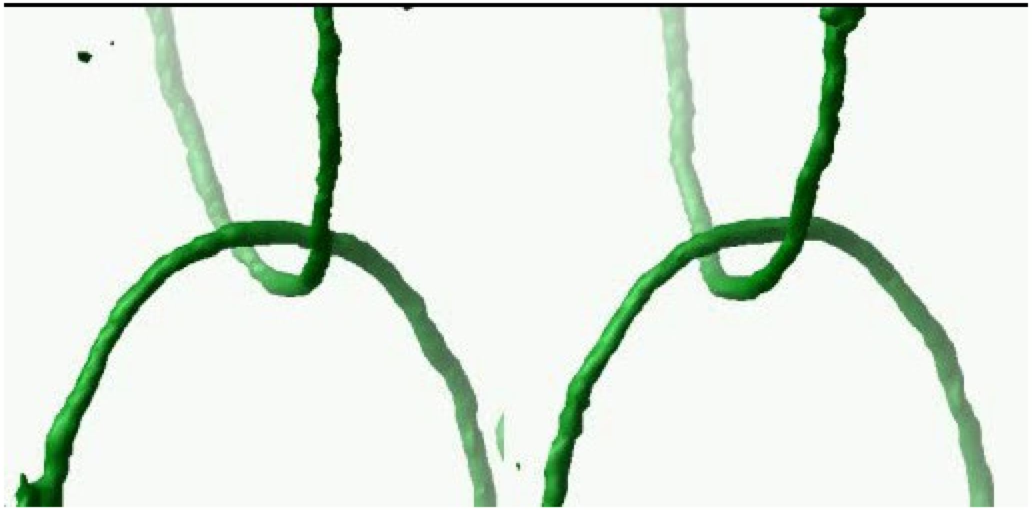
polar rung



ferromagnetic rung

Collision Dynamics of Linked Vortices

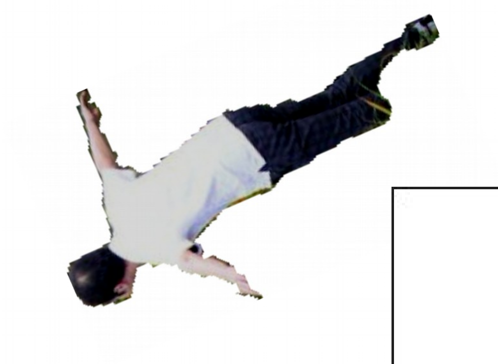
Commutative Non-commutative

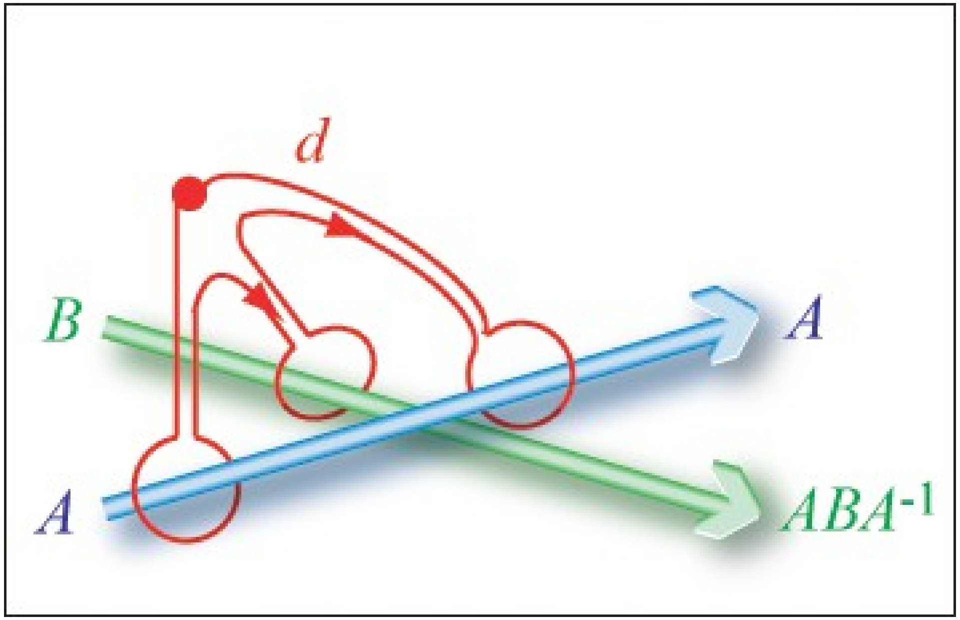


untangle not untangle

Algebraic Approach

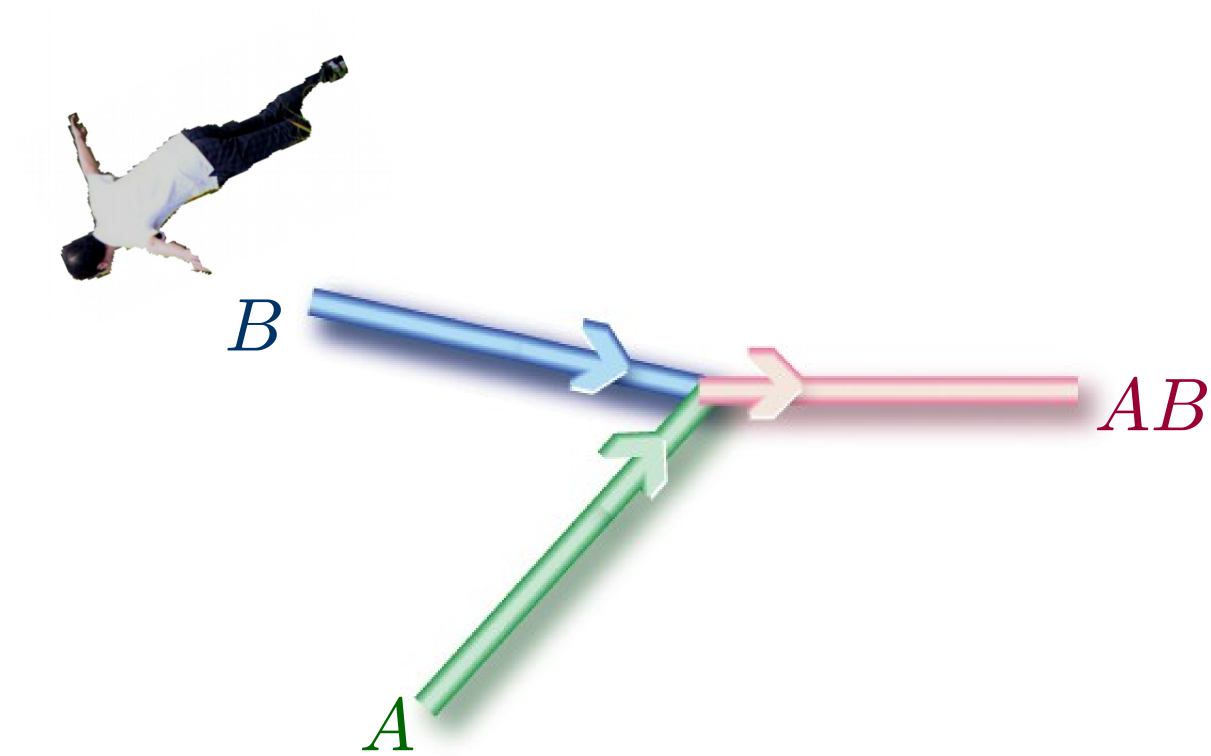
Consider 4 closed paths encircling two vortices



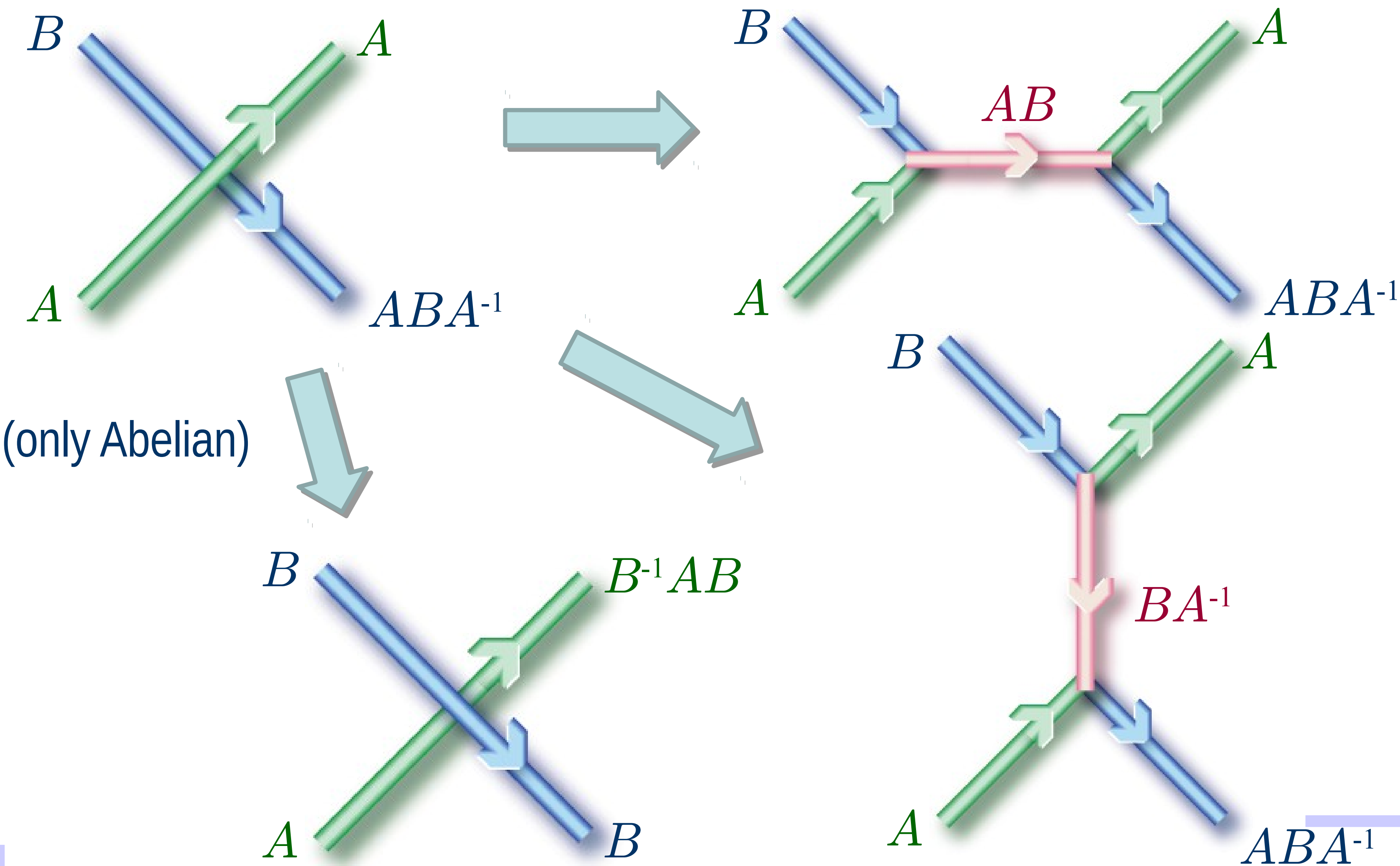


Path d defines vortex B as ABA^{-1} (same conjugacy class)

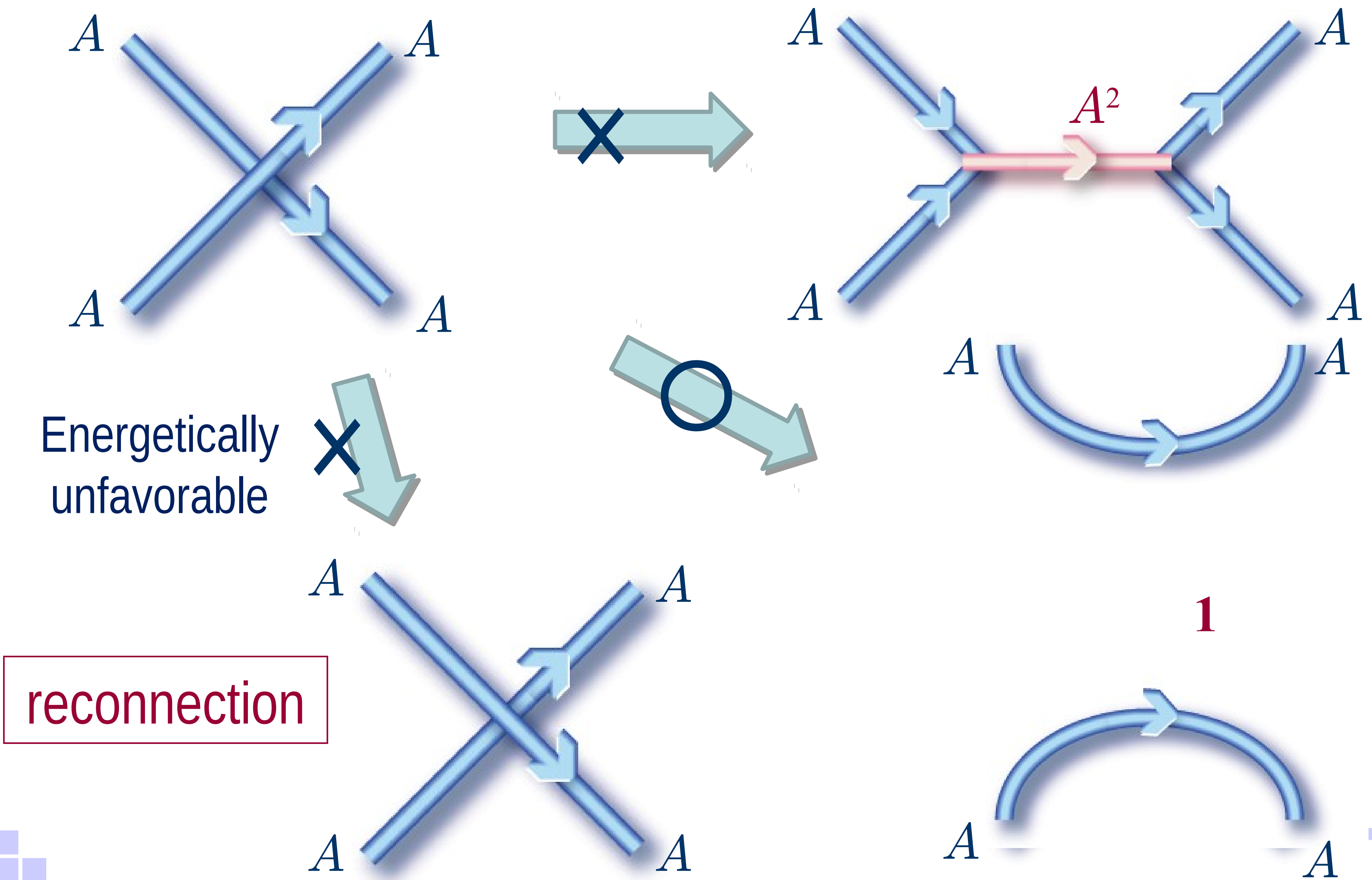
Y-shape Junction



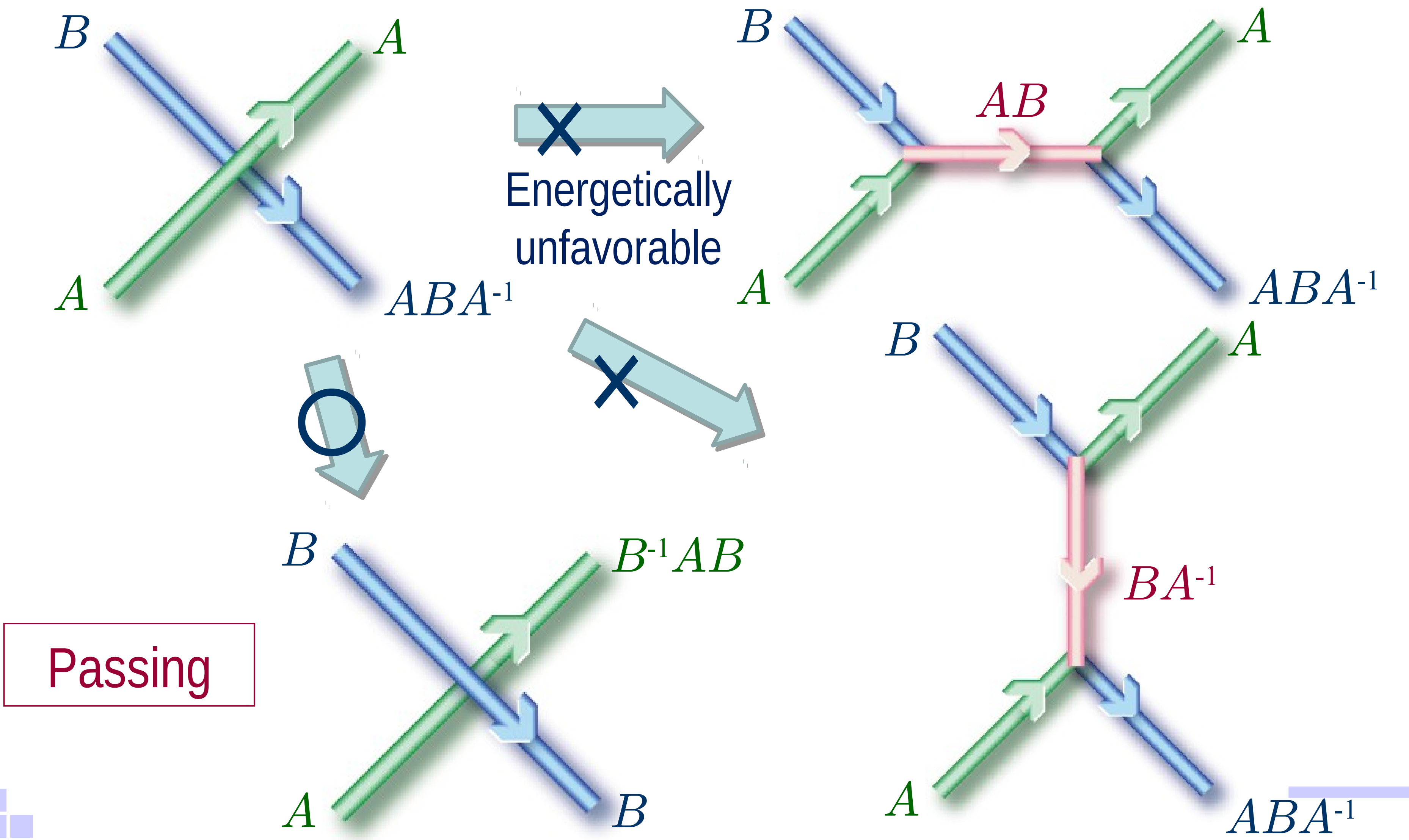
Collision of Vortices



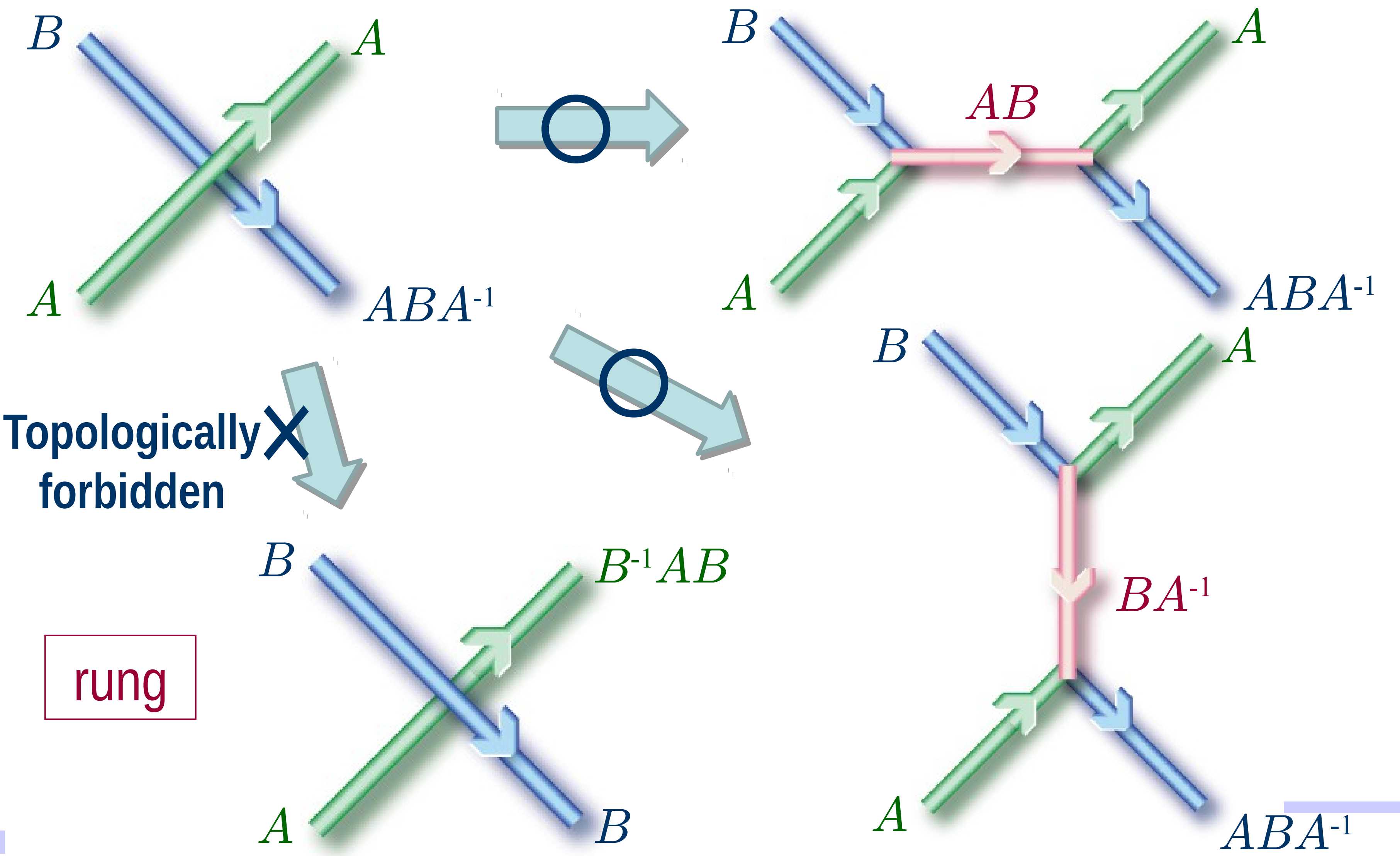
Collision of Same Vortices



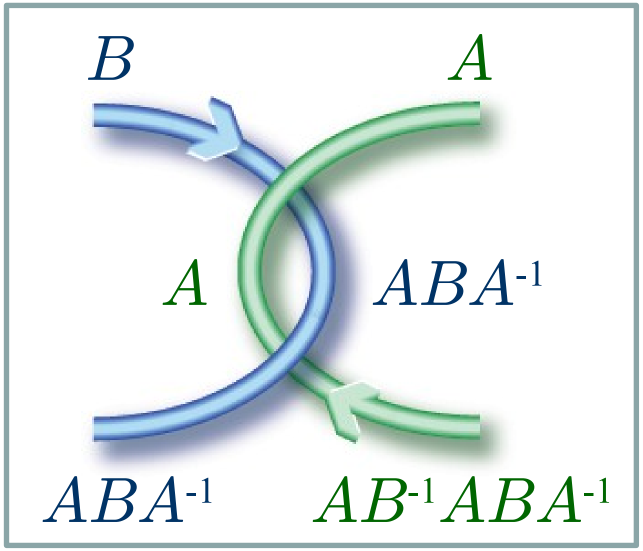
Collision of Different Commutative Vortices

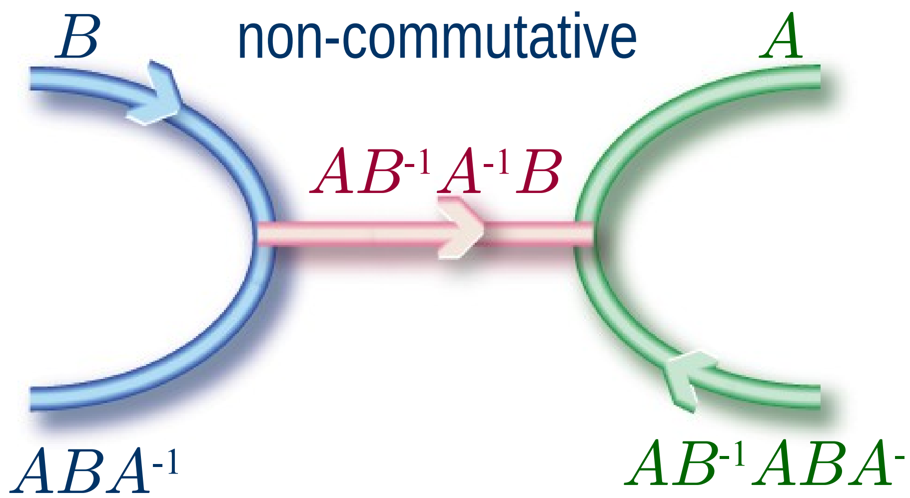


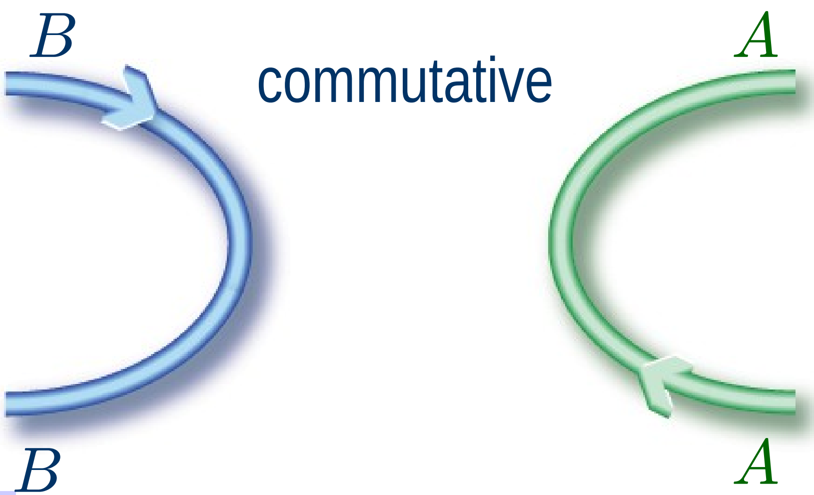
Collision of Different Non-commutative Vortices



Linked Vortices





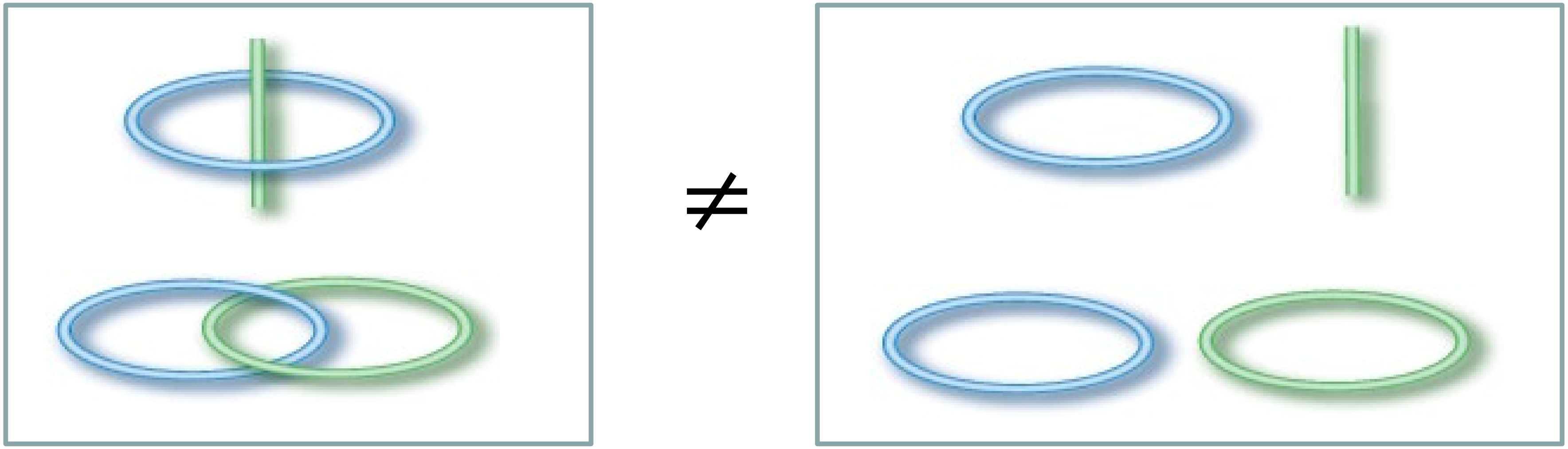


Linked vortices cannot untangle

Summary

1. Vortices with non-commutative circulations are defined as non-Abelian vortices.
2. Non-Abelian vortices can be realized in the cyclic phase of spin-2 BEC
3. Collision of two non-Abelian vortices create a new vortex between them as a rung (networking structure).

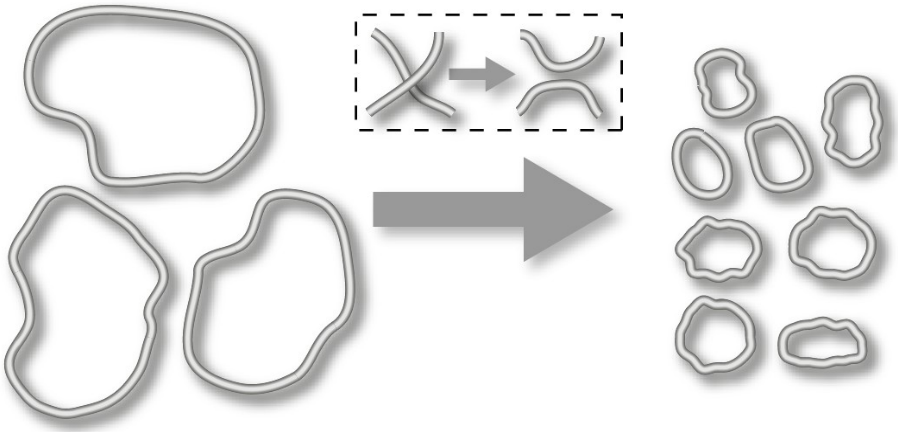
Future: Topological Charge of Linked Vortices



Linked vortex itself has another topological charge
→Searching and applying new homotopy theories

Poster-11, S. Kobayashi “Classification of topological defects by Fox homotopy group”

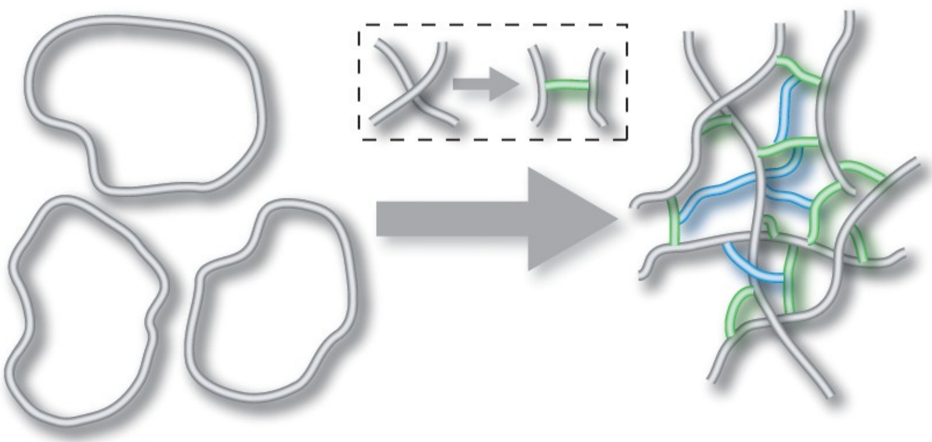
Future: Network Structure in Quantum Turbulence



Turbulence with Abelian vortices

↓

•Cascade of vortices



Turbulence with non-Abelian vortices

↓

•Large-scale networking structures among vortices with rungs

•Non-cascading turbulence

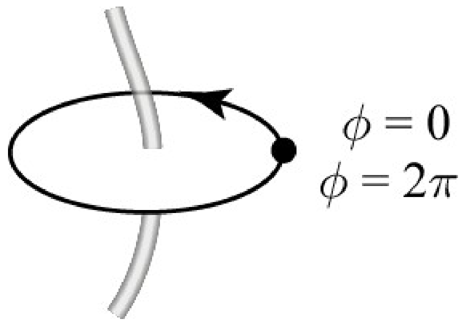
New turbulence!

Quantized Vortices in Multi-component BEC

Scalar BEC
^{4}He

$e^{i\phi}$

gauge

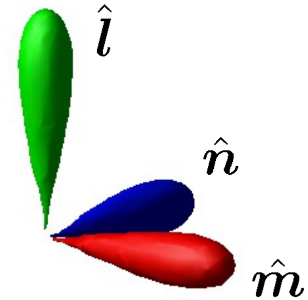


integer vortex

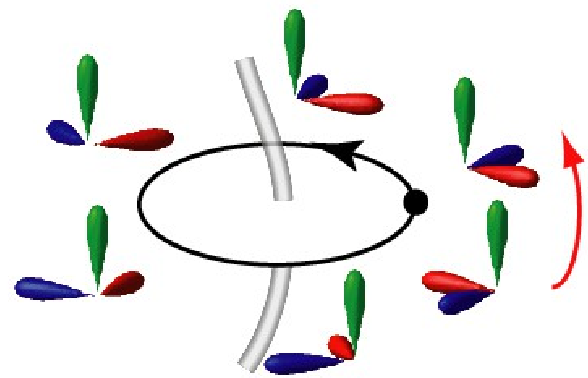
^{3}He-A

$\hat{\boldsymbol{d}}(\hat{\boldsymbol{m}} + i\hat{\boldsymbol{n}})_j$

$\boldsymbol{d}$ vector + triad



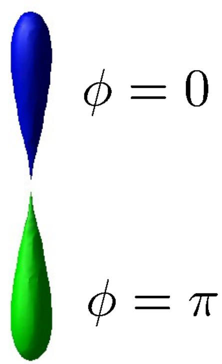
1/2 vortex



reverse of $\boldsymbol{d}$ vector

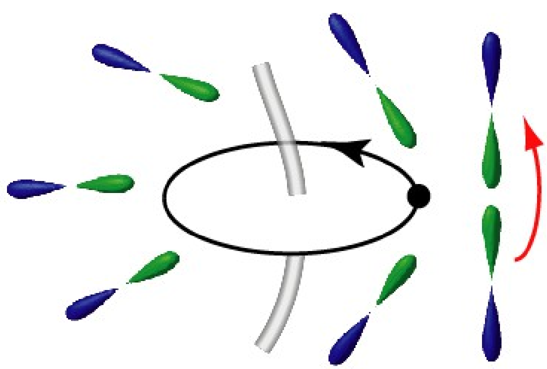
Polar in S = 1 BEC

$e^{i\phi} \cos\theta$



gauge + headless vector

1/2 vortex

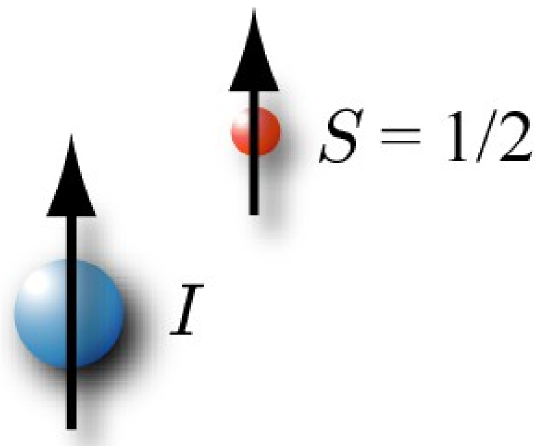


π gauge transformation

Spin-2 BEC

Bose-Einstein condensate in optical trap
(spin degrees of freedom is alive)

Hyperfine coupling
($\mathrm{F} = I + \mathrm{S}$)



^{87}Rb ($I = 3/2$)

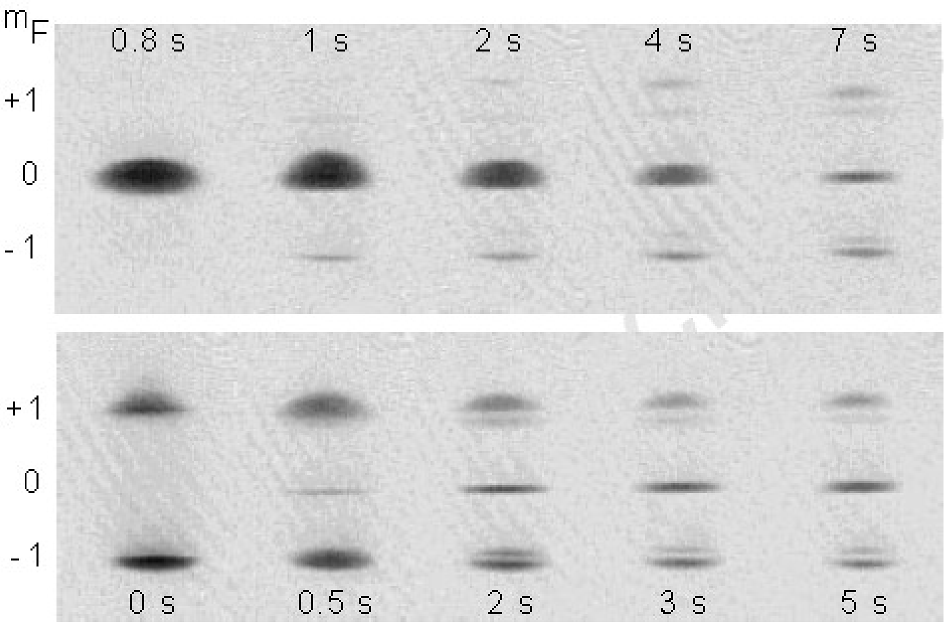
$$F = 2 \begin{cases} m_F = 2 \\ m_F = 1 \\ m_F = 0 \\ m_F = -1 \\ m_F = -2 \end{cases} \qquad F = 1 \begin{cases} m_F = 1 \\ m_F = 0 \\ m_F = -1 \end{cases}$$

BEC characterized by m_F

Spin dynamics of BEC

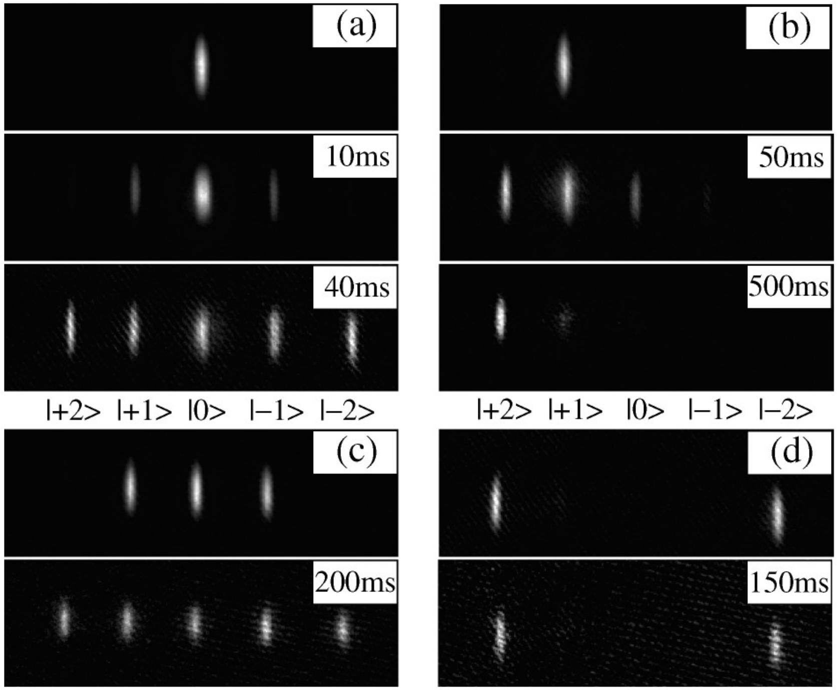
Stern-Gerlach experiment

$F = 1$



J. Stenger et al. Nature **396**, 345 (1998)

$F = 2$



H. Schmaljohann et al. PRL **92**, 040402 (2004)

Spin-2 BEC

$$H \simeq \int d\boldsymbol{x} \left[-\Psi_m^* \frac{\hbar^2}{2M} \nabla^2 \Psi_m + \frac{c_0}{2} n_{\rm tot}^2 + \frac{c_1}{2} \boldsymbol{F}^2 + \frac{c_2}{2} |A_{00}|^2 \right]$$

1. $c_1 < 0 \rightarrow$ ferromagnetic phase : $\boldsymbol{F} \neq 0$
2. $c_1 > 0,\ c_2 < 0 \rightarrow$ polar phase : $\boldsymbol{F} = 0,\ A_{00} \neq 0$
3. $c_1 > 0,\ c_2 > 0 \rightarrow$ cyclic phase : $\boldsymbol{F} = A_{00} = 0$

ferromagnetic

$$e^{i\phi} e^{-i\boldsymbol{e}\cdot\hat{\boldsymbol{F}}\alpha} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

polar

$$e^{i\phi} e^{-i\boldsymbol{e}\cdot\hat{\boldsymbol{F}}\alpha} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

cyclic

$$e^{i\phi} e^{-i\boldsymbol{e}\cdot\hat{\boldsymbol{F}}\alpha} \begin{pmatrix} i/2 \\ 0 \\ 1/\sqrt{2} \\ 0 \\ i/2 \end{pmatrix}$$

Spin-2 BEC

$$H \simeq \int d\boldsymbol{x} \left[-\Psi_m^* \frac{\hbar^2}{2M} \nabla^2 \Psi_m + (mp + m^2 q) n_{\text{tot}} + \frac{c_0}{2} n_{\text{tot}}^2 + \frac{c_1}{2} \boldsymbol{F}^2 + \frac{c_2}{2} |A_{00}|^2 \right]$$

1. $c_1 < 0 \rightarrow$ ferromagnetic phase : $\boldsymbol{F} \neq 0$
2. $c_1 > 0,\ c_2 < 0 \rightarrow$ polar phase : $\boldsymbol{F} = 0,\ A_{00} \neq 0$
3. $c_1 > 0,\ c_2 > 0 \rightarrow$ cyclic phase : $\boldsymbol{F} = A_{00} = 0$

Experimental observation for ^{87}Rb

$c_1 / (4\pi h^2 / M) = (0.99 \pm 0.06)\ a_B$

$c_2 / (4\pi h^2 / M) = (-0.53 \pm 0.58)\ a_B$

Whether the system is in polar or cyclic has not decided yet

A. Widera et al. New J. Phys **8**, 152 (2006)

Phase Diagram

$$H \simeq \int d\boldsymbol{x} \left[-\Psi_m^* \frac{\hbar^2}{2M} \nabla^2 \Psi_m + (mp + m^2 q) n_{\text{tot}} + \frac{c_0}{2} n_{\text{tot}}^2 + \frac{c_1}{2} \boldsymbol{F}^2 + \frac{c_2}{2} |A_{00}|^2 \right]$$

$$\frac{E_{\text{f}}}{N} = 2p + 4q + \frac{c_0 n_{\text{tot}}}{2} + 2c_1 n_{\text{tot}}$$

$$\frac{E_{\text{pu}}}{N} = \frac{c_0 n_{\text{tot}}}{2} + \frac{c_2 n_{\text{tot}}}{10}$$

$$\frac{E_{\text{pb}}}{N} = 4q + \frac{c_0 n_{\text{tot}}}{2} + \frac{c_2 n_{\text{tot}}}{10}$$

$$\frac{E_{\text{c}}}{N} = 2q + \frac{c_0 n_{\text{tot}}}{2}$$

Phase diagram with neglecting linear Zeeman

$$B_{\text{f-pu}} : q = -\frac{c_1 n_{\text{tot}}}{2} + \frac{c_2 n_{\text{tot}}}{40}$$

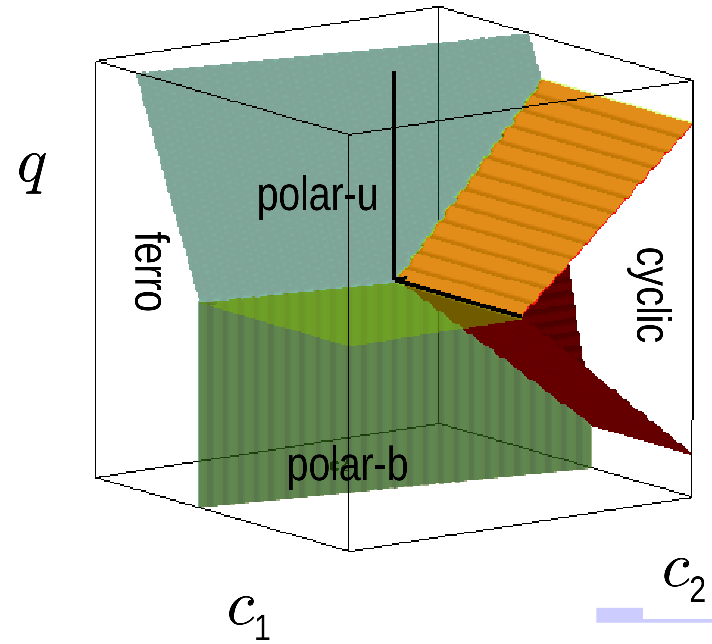
$$B_{\text{f-pb}} : c_1 n_{\text{tot}} = \frac{c_2 n_{\text{tot}}}{20}$$

$$B_{\text{f-c}} : q = -c_1 n_{\text{tot}}$$

$$B_{\text{pu-pb}} : q = 0$$

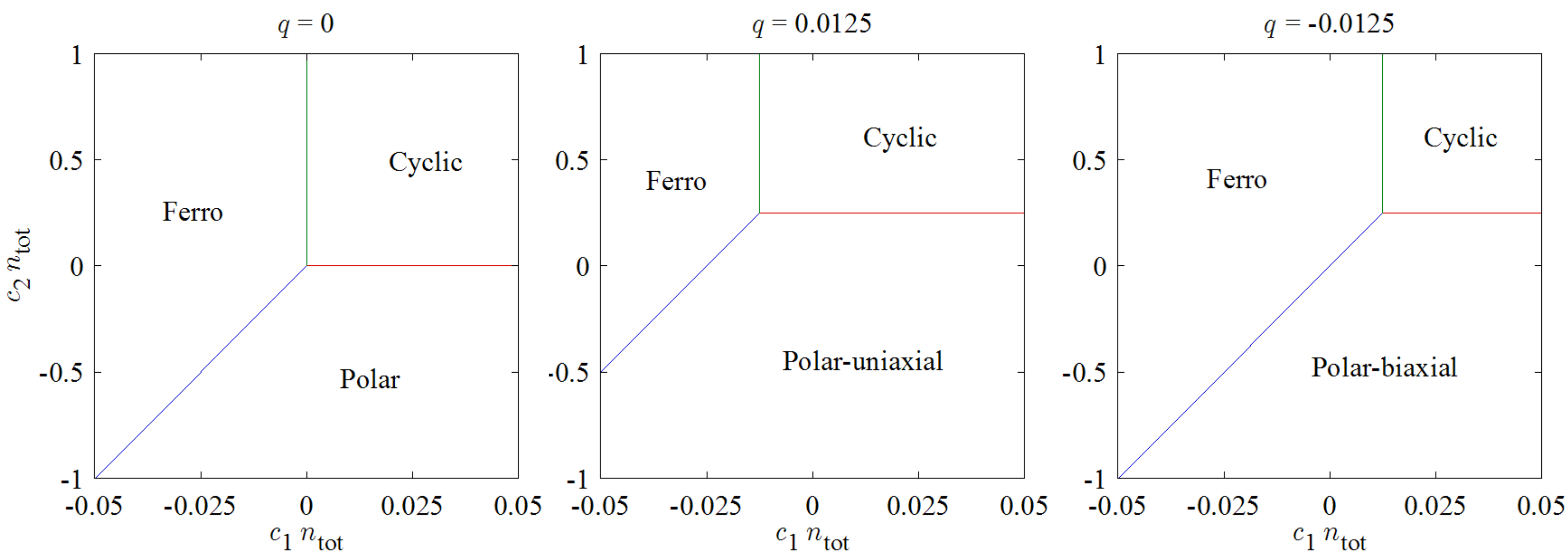
$$B_{\text{pu-c}} : q = \frac{c_2 n_{\text{tot}}}{20}$$

$$B_{\text{pb-c}} : q = -\frac{c_2 n_{\text{tot}}}{20}$$



Phase Diagram

$$H \simeq \int d\boldsymbol{x} \left[-\Psi_m^* \frac{\hbar^2}{2M} \nabla^2 \Psi_m + (mp + m^2 q) n_{\rm tot} + \frac{c_0}{2} n_{\rm tot}^2 + \frac{c_1}{2} \boldsymbol{F}^2 + \frac{c_2}{2} |A_{00}|^2 \right]$$



Phase Diagram

Estimation of number density : TF

$$\mu = c_0 n_{\rm tot} + \frac{1}{2} M(\omega_r^2 r^2 + \omega_z^2 z^2)$$

$$n_{\rm tot} = \frac{2\mu - M(\omega_r^2 r^2 + \omega_z^2 z^2)}{2c_0}$$

$$N = \int d^3r\, n_{\rm tot} = \frac{16\sqrt{2}\pi\mu^{5/2}}{15 c_0 M^{3/2} \omega_r^2 \omega_z}$$

$$n_{\rm tot}(\boldsymbol{r}=0) = \frac{\mu}{c_0} = \sqrt[5]{\frac{225 M^3 N^2 \omega_r^4 \omega_z^2}{512\pi^2 c_0^3}}$$

cyclic vs ferro

$$\frac{(\mu_{\rm B} B)^2}{4\Delta_{\rm hf}} = c_1 n_{\rm tot}$$

$$B = \frac{2\sqrt{c_1 n_{\rm tot} \Delta_{\rm hf}}}{\mu_{\rm B}} = 0.481\text{Gauss}$$

cyclic vs polar

$$\frac{(\mu_{\rm B} B)^2}{4\Delta_{\rm hf}} = \frac{c_2 n_{\rm tot}}{20}$$

$$B = \frac{\sqrt{c_2 n_{\rm tot} \Delta_{\rm hf}}}{\sqrt{5}\mu_{\rm B}} = 0.024\text{Gauss}$$

Assuming cyclic phase

$$c_0/(4\pi\hbar^2 a_{\rm B}/M) = 112.54$$

$$c_1/(4\pi\hbar^2 a_{\rm B}/M) = 0.99$$

$$c_2/(4\pi\hbar^2 a_{\rm B}/M) = 0.05$$

- $q = -(\mu_{\rm B} B)^2/(4\Delta_{\rm hf}) = 4.746 \times 10^{-32} B(\text{Gauss})^2\ \text{J}$
- $\omega_z = 21 \times 2\pi\ \text{Hz}$
- $\omega_r = 141 \times 2\pi\ \text{Hz}$
- $M = 1.46 \times 10^{-25}\ \text{kg}$
- $N = 2.5 \times 10^5$
- $c_0 = 5.703 \times 10^{-51}\ \text{Jm}^3$
- $c_1 = 5.017 \times 10^{-53}\ \text{Jm}^3$
- $c_2 = 2.534 \times 10^{-54}\ \text{Jm}^3$
- $n_{\rm tot} = 2.19 \times 10^{20}\ \text{m}^{-3}$

渦状態

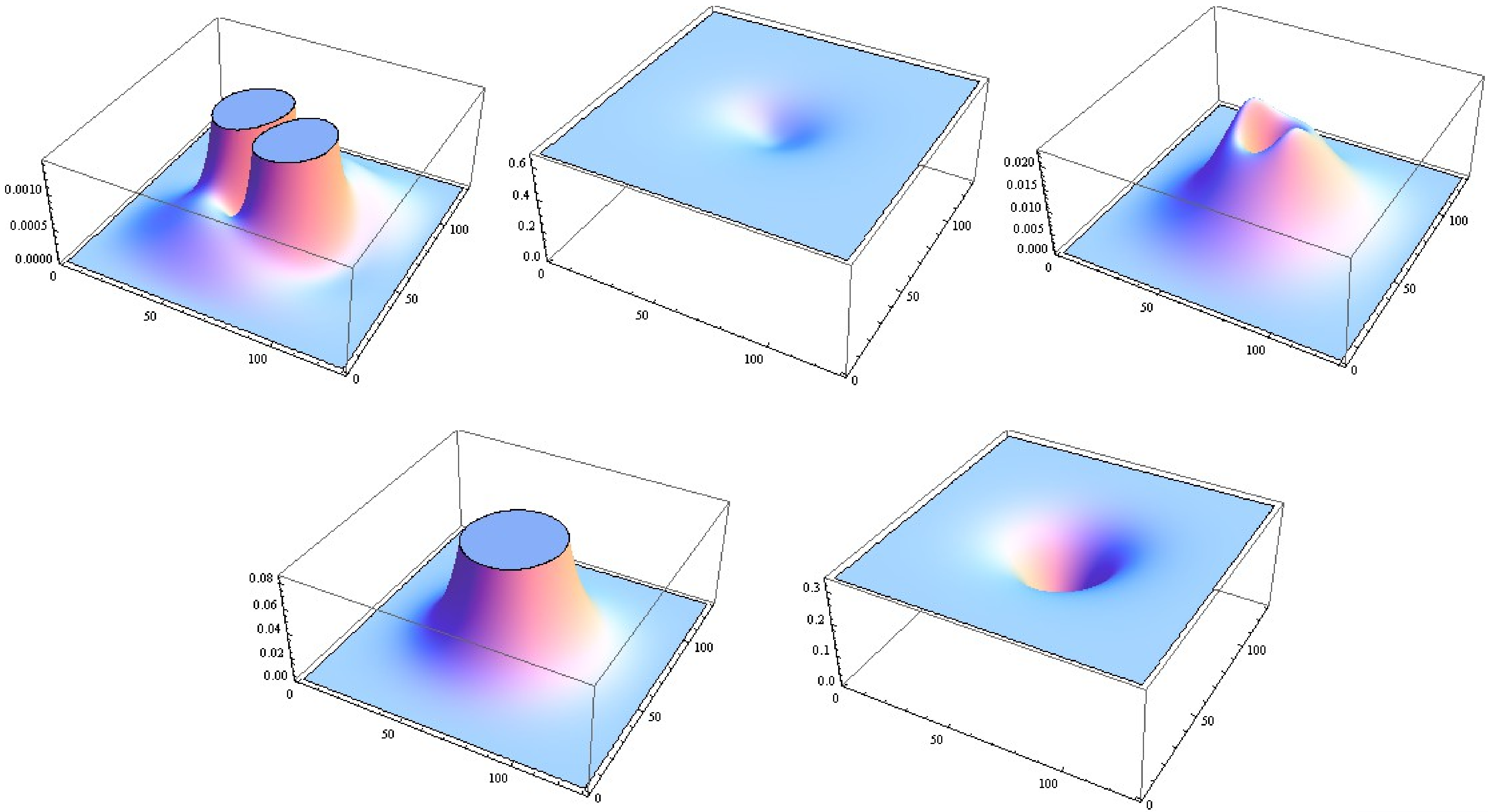
最も低エネルギーだと思われる（有限mass circulationの）渦

- Cyclic : 1/3 vortex
- Polar : 1/4 vortex

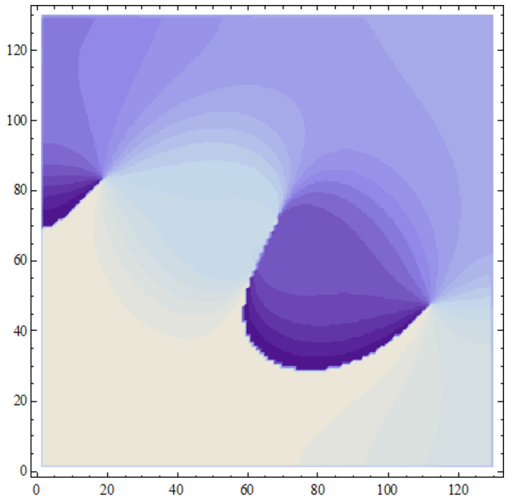
実はどちらも非可換量子渦の１つ

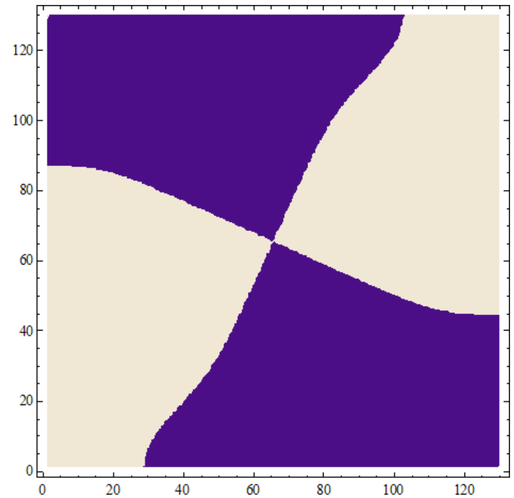
$$\Psi_{1/3} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 \\ \sqrt{2} \\ 0 \\ 0 \\ e^{i\theta} \end{pmatrix} \qquad \Psi_{1/4} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ e^{i\theta} \end{pmatrix}$$

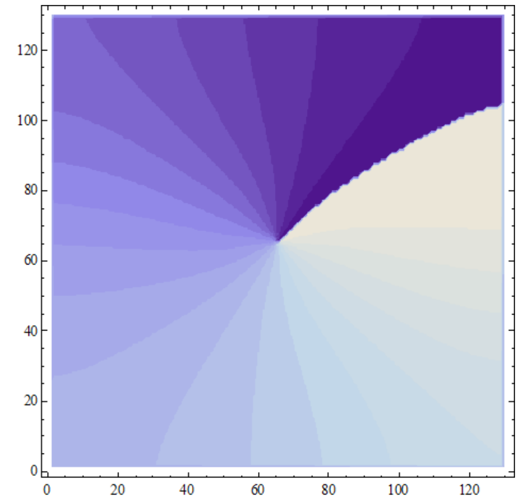
渦状態(1/3 vortex)

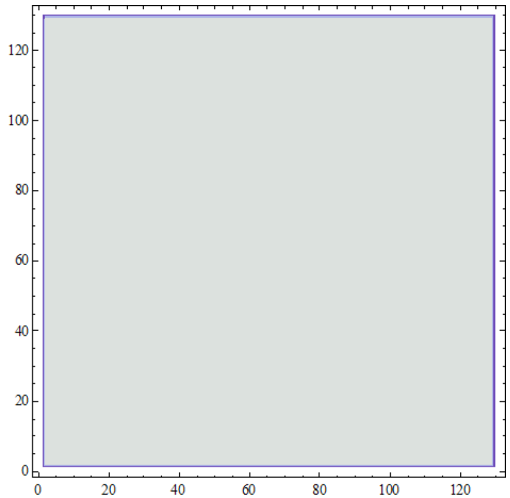


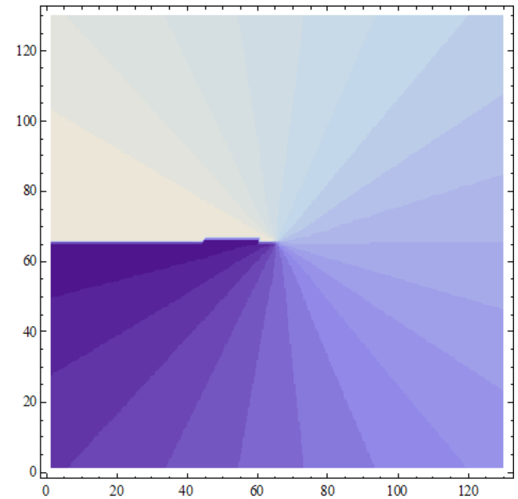
渦状態(1/3 vortex)



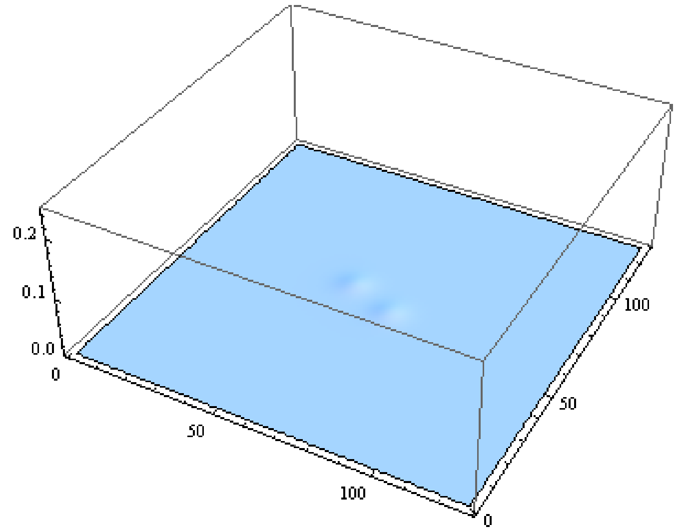


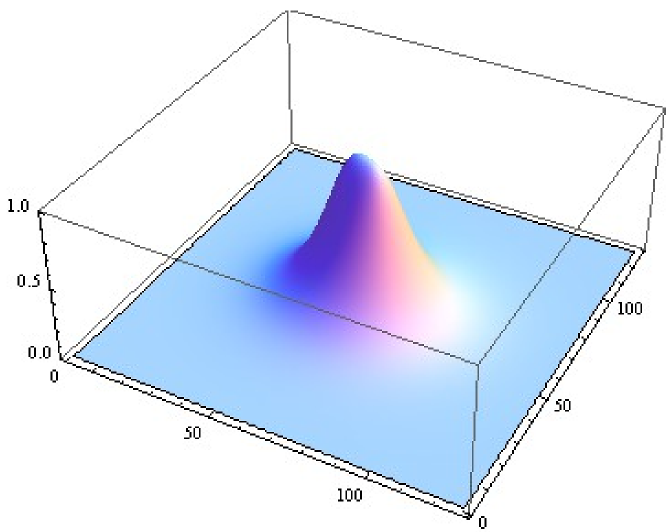


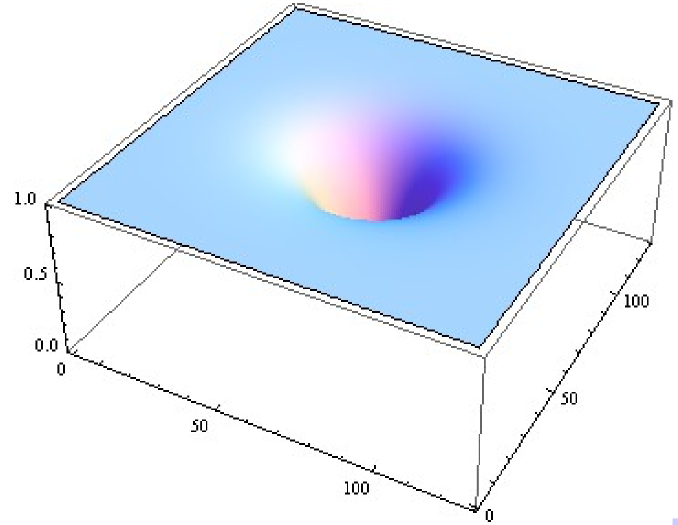




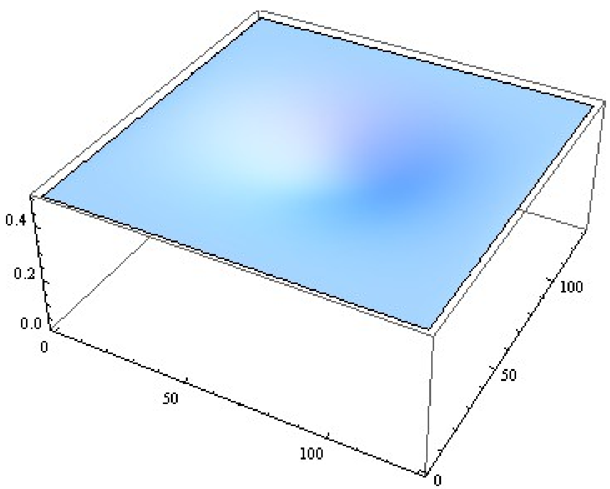
渦状態(1/3 vortex)

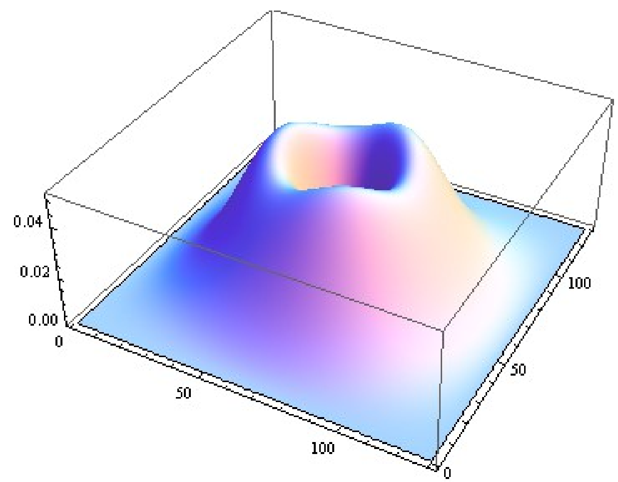


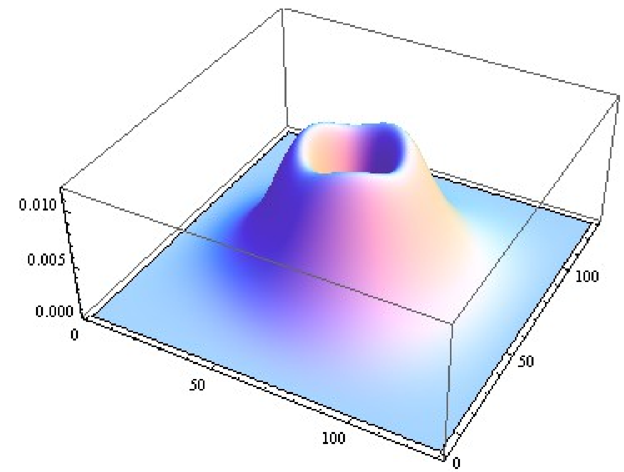


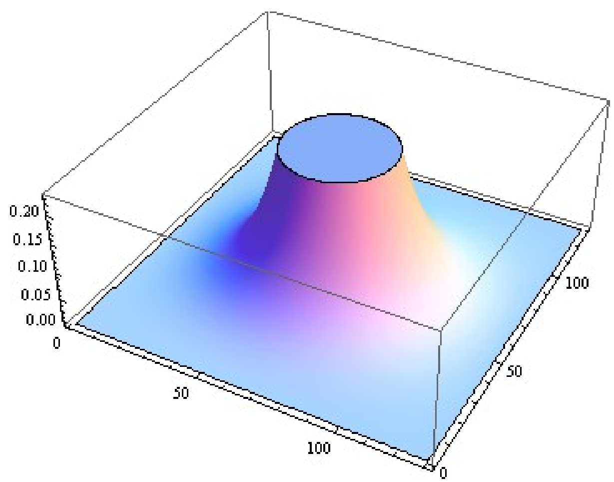


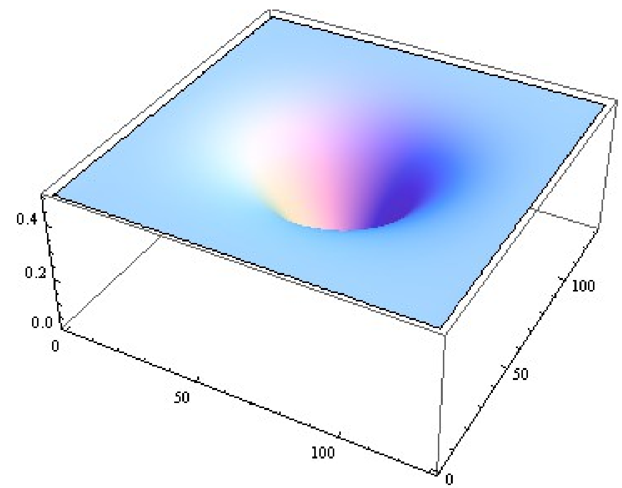
渦状態(1/4 vortex)



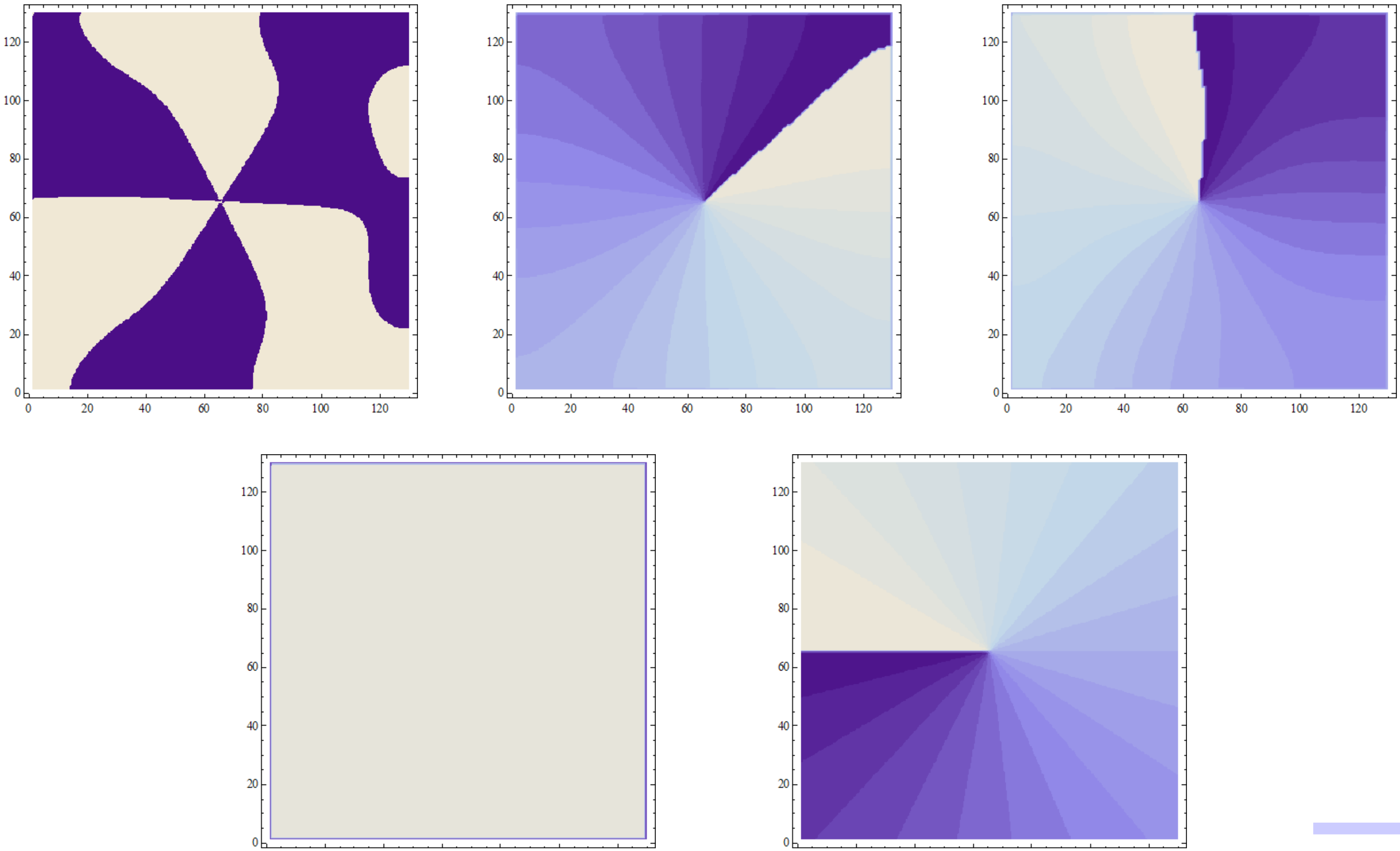




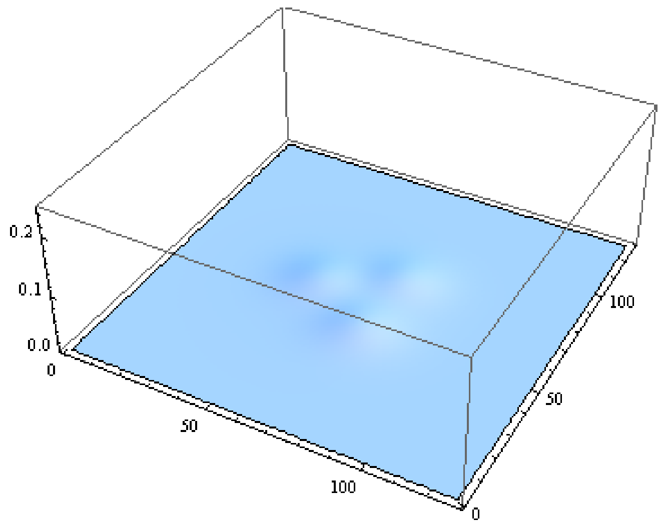


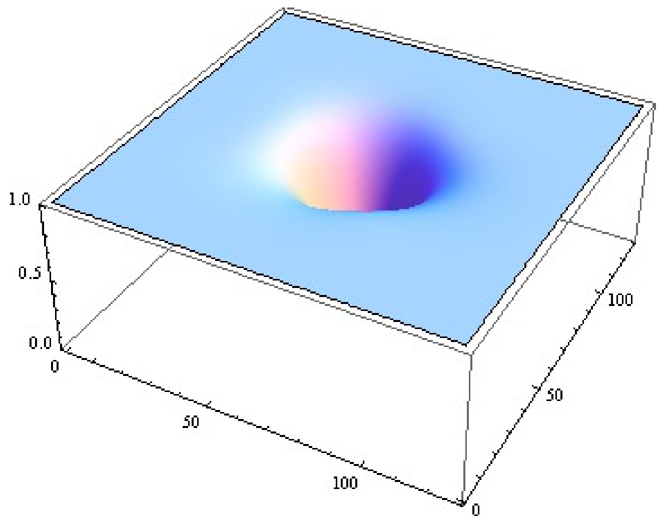


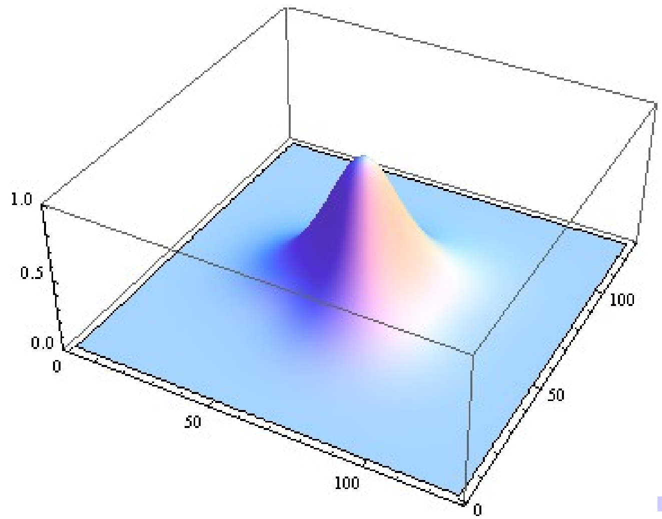
渦状態(1/4 vortex)



渦状態(1/4 vortex)







まとめ

1. cyclicではpolarコアの、polarではcyclicの渦が入る。
2. polarコアは2回軸対称を、cyclicコアは3回軸対称性を自発的に破る（入った渦の対称性が見えれば相を同定できる？）
3. 以上の結果から、局所密度近似が敗れるような状況ではpolar相は2回軸対称性の破れをcyclic相は3回軸対称性の破れを好む可能性がある（3角形のトラップや3角格子を作れば$c_2 < 0$でもcyclicが増強される可能性がある）。