Non-Abelian Vortices in Spinor Bose-Einstein Condensates

Michikazu Kobayashi[a]

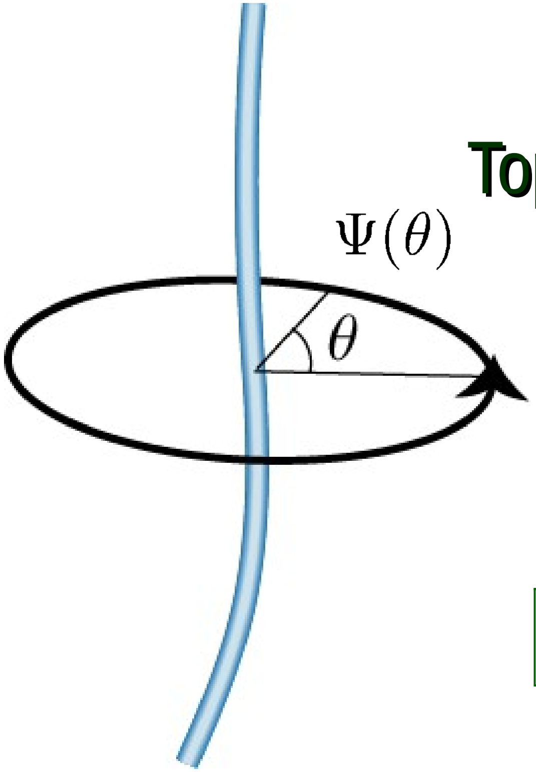
Collaborators: Yuki Kawaguchi[a], Muneto Nitta[b], and Masahito Ueda[a]

University of Tokyo[a] and Keio University[b]

July. 27 – Aug. 7, 2009, Eleventh J. J. Gianbiagi Winter School: The Quantum Mechanics of the XXI Century: Manipulation of Coherent Atomic Matter

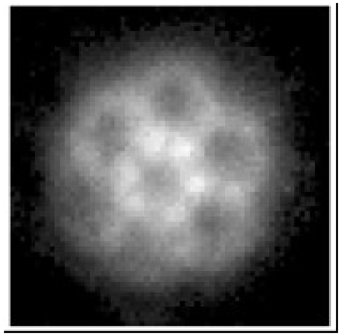
Quantized Vortex and Topological Charge

Topological charge of a vortex can be considered how order parameter changes around the vortex core



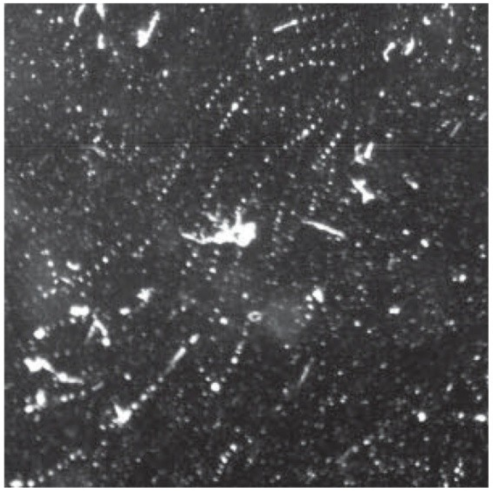
Single component BEC : $\Psi(\theta) \propto \exp[in\theta]$

Topological charge can be expressed by integer n



vortex in ^{87}Rb BEC

K. W. Madison et al.
PRL **86**, 4443 (2001)



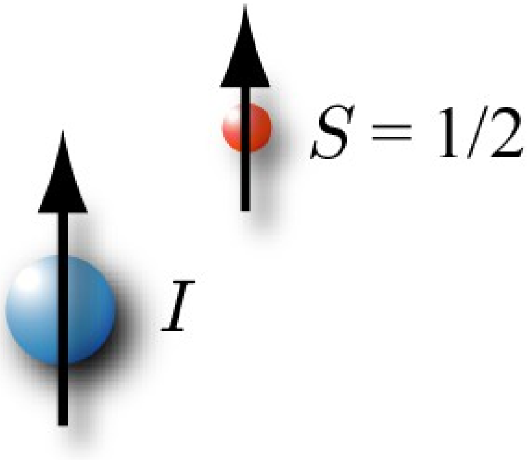
vortex in ^{4}He

G. P. Bewley et al.
Nature **441**, 588 (2006)

Spin-2 BEC

Bose-Einstein condensate in optical trap
(spin degrees of freedom is alive)

Hyperfine coupling
(F $= I +$ S)



^{87}Rb$(I = 3/2)$

$$F = 2 \begin{cases} m_F = 2 \\ m_F = 1 \\ m_F = 0 \\ m_F = -1 \\ m_F = -2 \end{cases} \qquad F = 1 \begin{cases} m_F = 1 \\ m_F = 0 \\ m_F = -1 \end{cases}$$

BEC characterized by m_F

Mean Field Approximation for BEC at $T = 0$

Case of Spin-2

$$H \simeq \int d\boldsymbol{x} \left[-\Psi_m^* \frac{\hbar^2}{2M} \nabla^2 \Psi_m + \frac{c_0}{2} n_{\rm tot}^2 + \frac{c_1}{2} \boldsymbol{F}^2 + \frac{c_2}{2} |A_{00}|^2 \right]$$

$$c_0 = \frac{4g_2 + 3g_4}{7}, \quad c_1 = \frac{g_4 - g_2}{7}, \quad c_2 = \frac{7g_0 - 10g_2 + 3g_4}{7}$$

$$n_{\rm tot}(\boldsymbol{x}) = \Psi_m^*(\boldsymbol{x}) \Psi_m(\boldsymbol{x}), \quad \boldsymbol{F}(\boldsymbol{x}) = \Psi_m^*(\boldsymbol{x}) \hat{\boldsymbol{F}}_{mm'}(\boldsymbol{x}) \Psi_{m'}(\boldsymbol{x})$$

$$A_{00}(\boldsymbol{x}) = \frac{1}{\sqrt{5}} [2\Psi_2(\boldsymbol{x})\Psi_{-2}(\boldsymbol{x}) - 2\Psi_1(\boldsymbol{x})\Psi_{-1}(\boldsymbol{x}) + \Psi_0(\boldsymbol{x})^2]$$

$n_{\rm tot}$: total density

$\boldsymbol{F}$: magnetization

A_{00} : singlet pair amplitude

Spin-2 BEC

$$H \simeq \int d\boldsymbol{x} \left[-\Psi_m^* \frac{\hbar^2}{2M} \nabla^2 \Psi_m + \frac{c_0}{2} n_{\rm tot}^2 + \frac{c_1}{2} \boldsymbol{F}^2 + \frac{c_2}{2} |A_{00}|^2 \right]$$

1. $c_1 < 0$, $c_2 > 20c_1$ → ferromagnetic phase : $\boldsymbol{F} \neq 0$
2. $c_1 > c_2/20$, $c_2 < 0$ → nematic phase : $\boldsymbol{F} = 0$, $A_{00} \neq 0$
3. $c_1 > 0$, $c_2 > 0$ → cyclic phase : $\boldsymbol{F} = A_{00} = 0$

ferromagnetic

$$e^{i\phi} e^{-i\boldsymbol{e}\cdot\hat{\boldsymbol{F}}\alpha} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

nematic

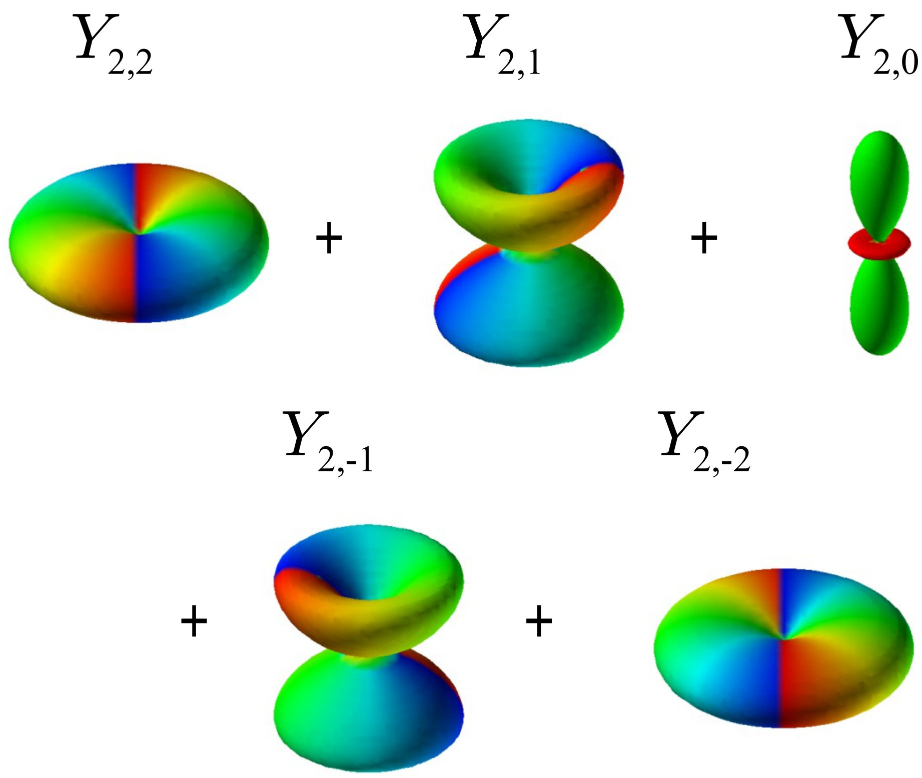
$$e^{i\phi} e^{-i\boldsymbol{e}\cdot\hat{\boldsymbol{F}}\alpha} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 0 \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$$

cyclic

$$e^{i\phi} e^{-i\boldsymbol{e}\cdot\hat{\boldsymbol{F}}\alpha} \begin{pmatrix} i/2 \\ 0 \\ 1/\sqrt{2} \\ 0 \\ i/2 \end{pmatrix}$$

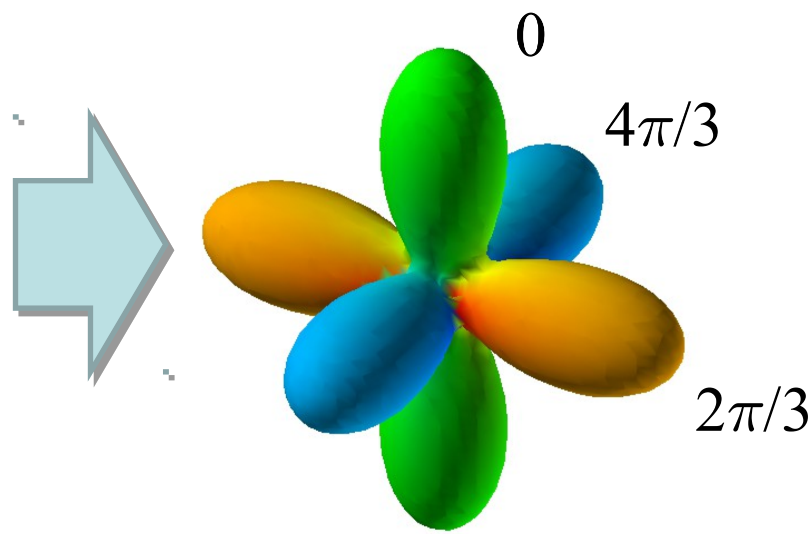
Spin-2 BEC

$$\sum_{m=-2}^{2} \Psi_m Y_{2,m}$$



Cyclic phase

$$e^{i\phi}(3\cos^2\theta + \sqrt{3}i\sin^2\theta\cos 2\varphi - 1)$$



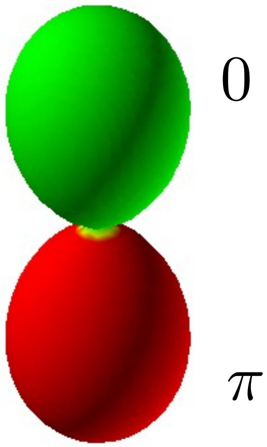
headless triad

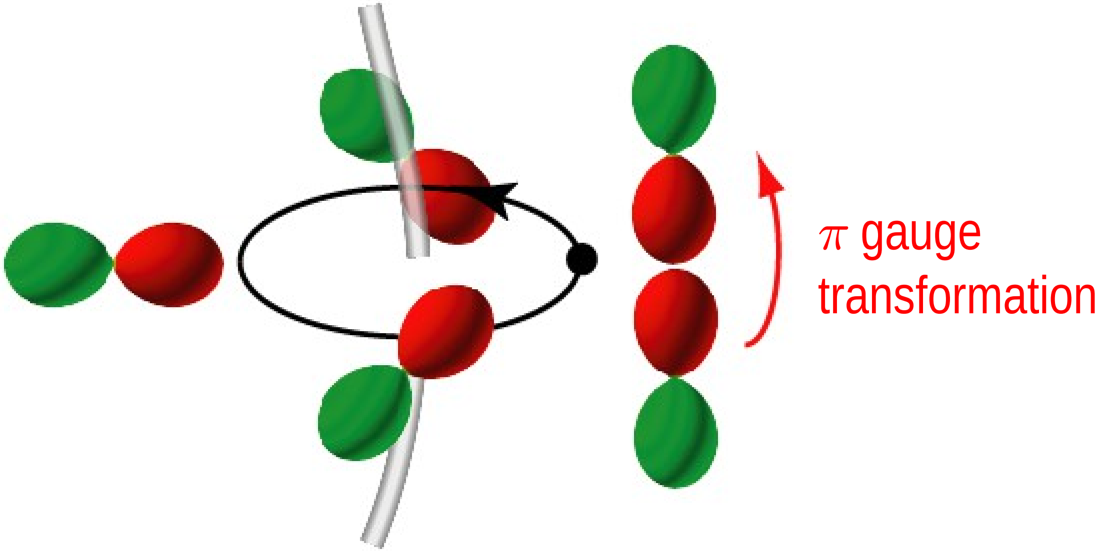
Vortices in Spinor BEC

$S = 1$ Polar phase

$$e^{i\phi} e^{-i\boldsymbol{e}\cdot\hat{\boldsymbol{F}}\alpha} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

headless vector





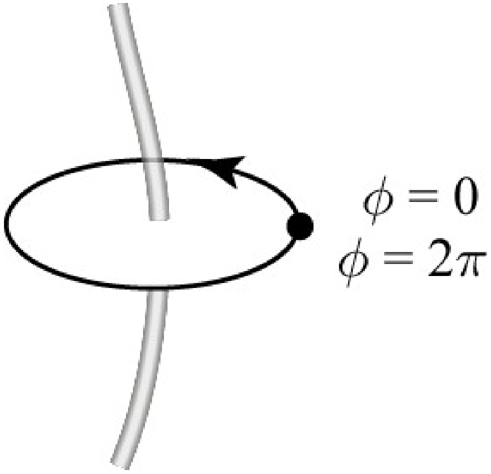
Half quantized vortex : spin & gauge rotate by π around vortex core

Topological charge can be expressed by integer and half integer (Abelian vortex)

Vortices in Spin-2 BEC

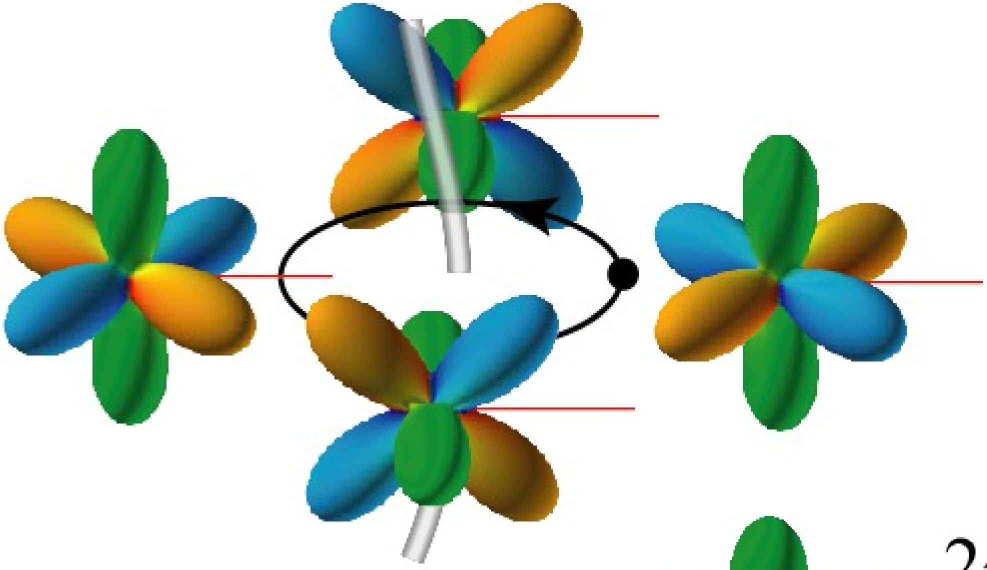
There are 5 types of vortices in the cyclic phase

gauge vortex

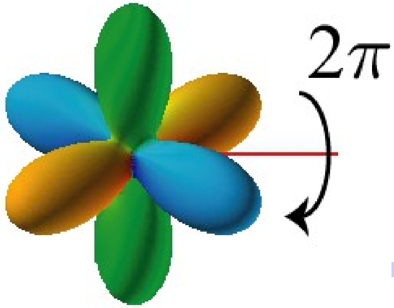


mass circulation : 1
spin circulation : 0

integer spin vortex

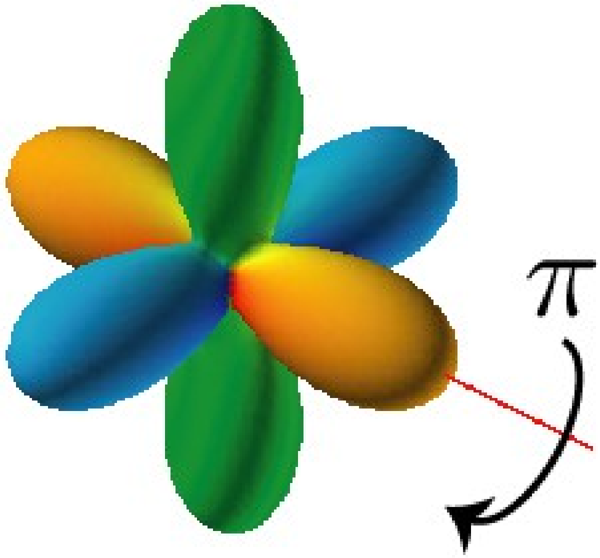


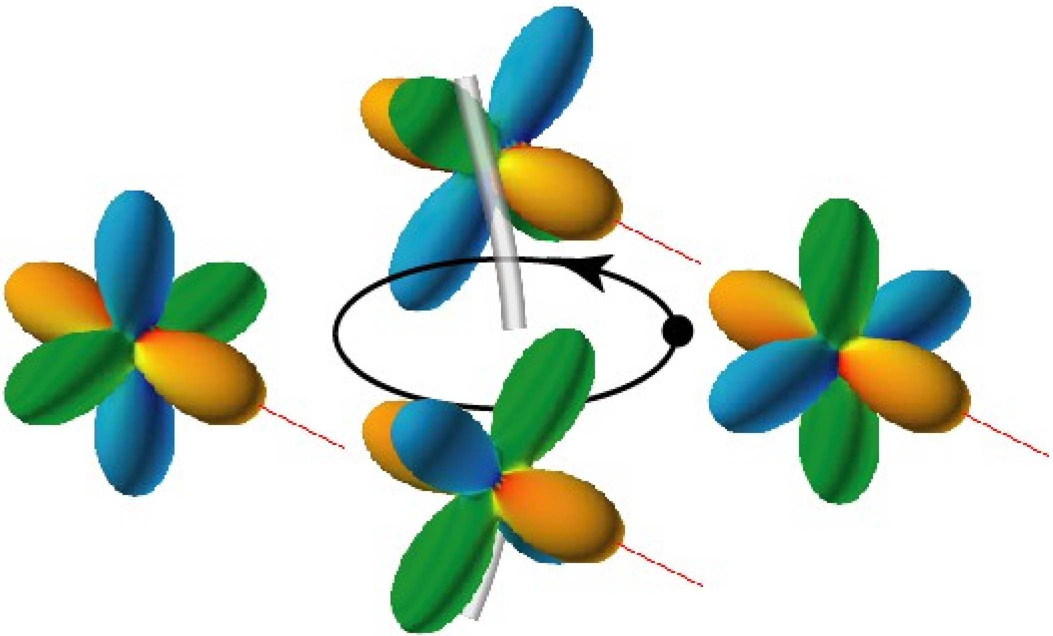
mass circulation : 0
spin circulation : 1



Vortices in Spin-2 BEC

1/2-spin vortex : triad rotate by π around three axis $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$



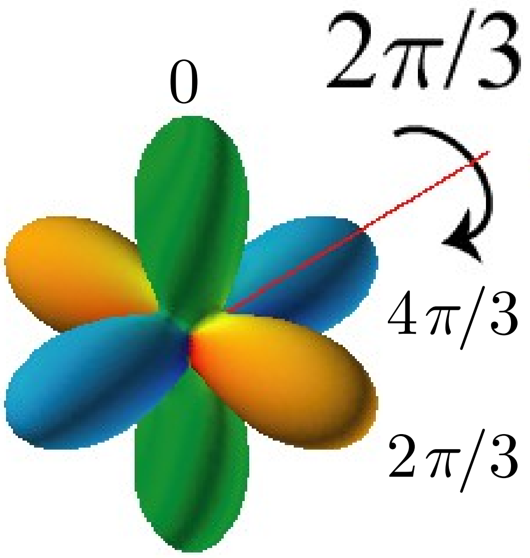


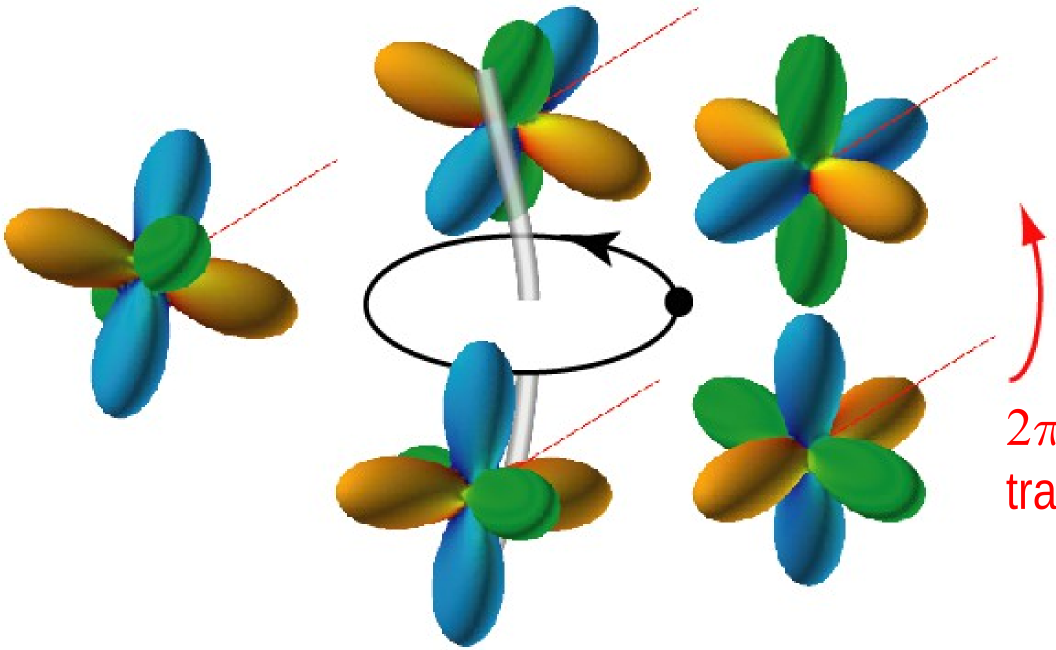
mass circulation : 0

spin circulation : 1/2

Vortices in Spin-2 BEC

1/3 vortex : triad rotate by $2\pi/3$ around four axis $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$, $\boldsymbol{e}_4$ and $2\pi/3$ gauge transformation



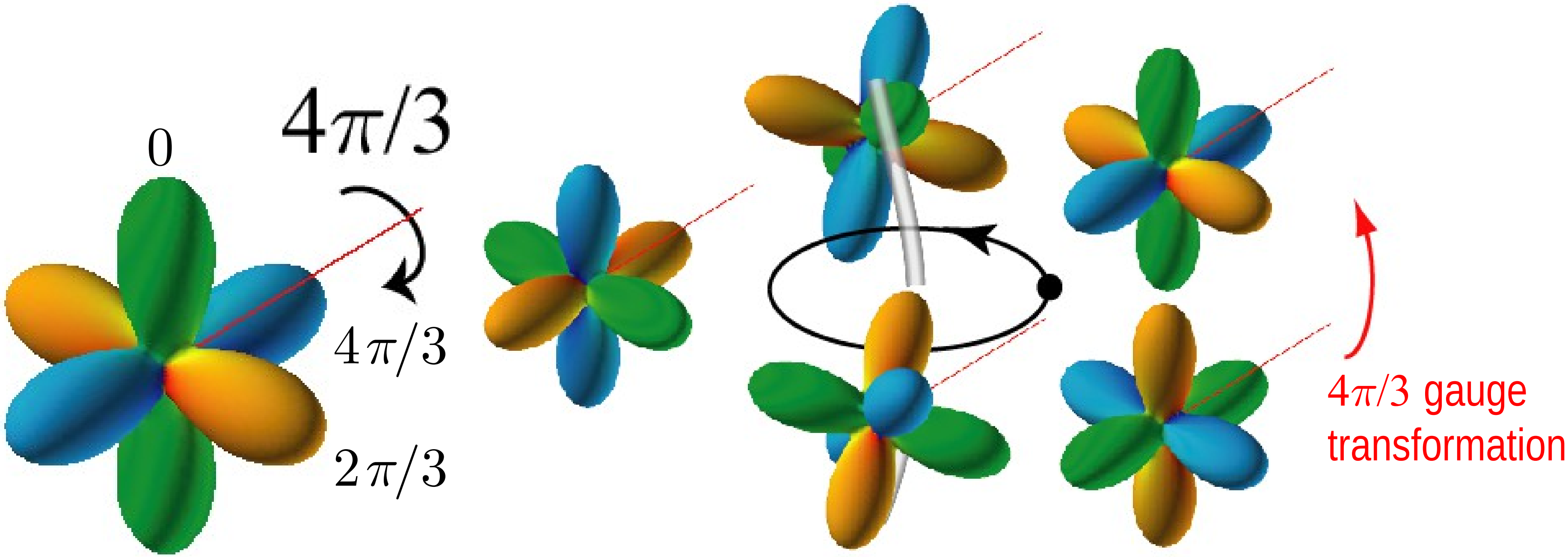


mass circulation : 1/3
spin circulation : 1/3

$$\boldsymbol{e}_1 = (1,1,1)/\sqrt{3}, \quad \boldsymbol{e}_2 = (1,-1,-1)/\sqrt{3}$$
$$\boldsymbol{e}_3 = (-1,1,-1)/\sqrt{3}, \quad \boldsymbol{e}_4 = (-1,-1,1)/\sqrt{3}$$

Vortices in Spin-2 BEC

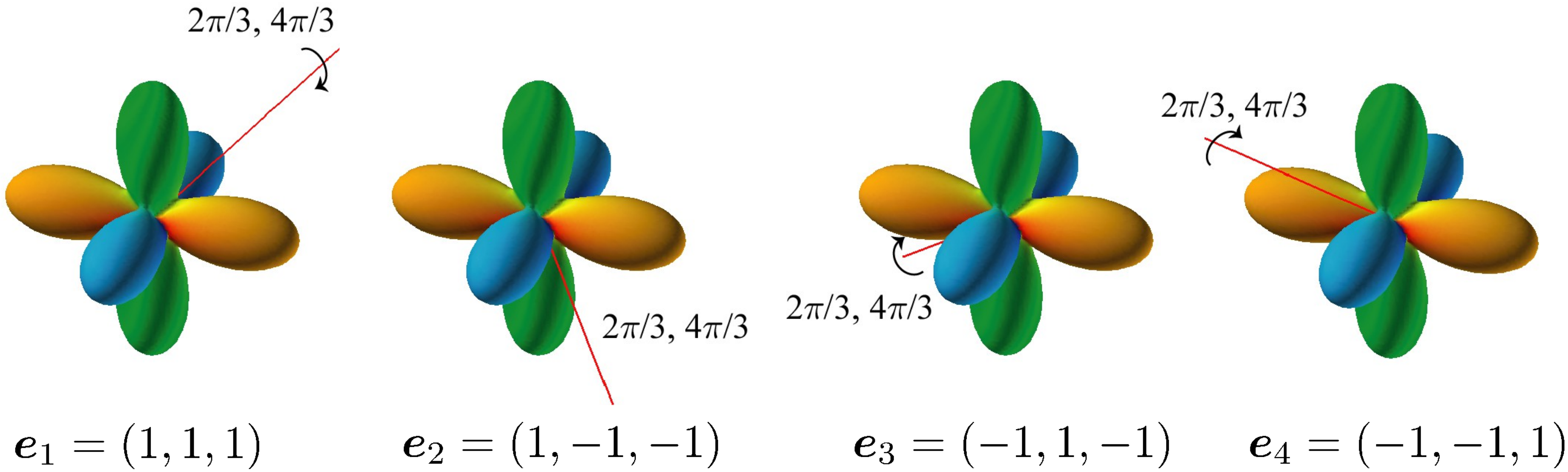
4, 2/3 vortex : triad rotate by $4\pi/3$ around four axis $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$, $\boldsymbol{e}_4$
and $4\pi/3$ gauge transformation

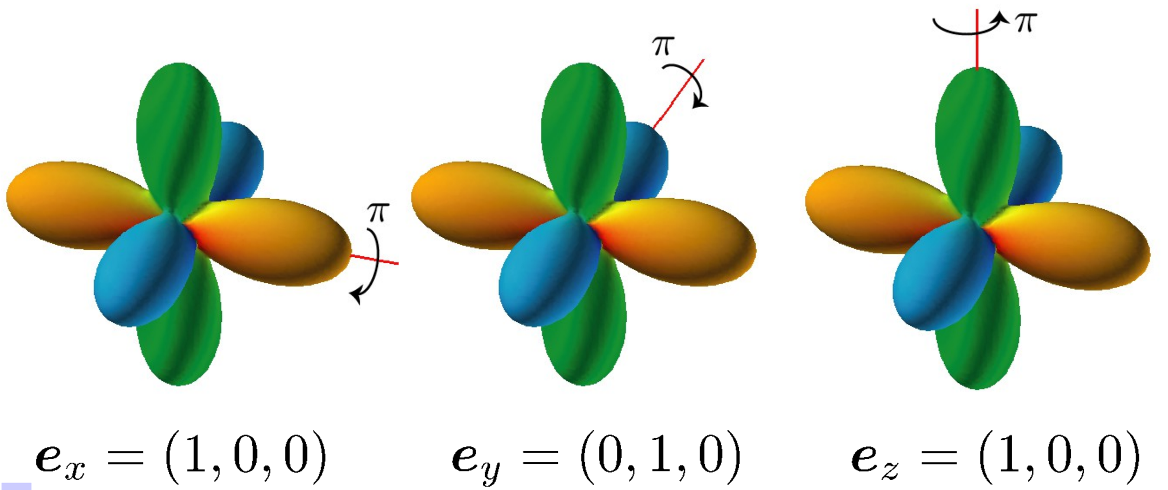


mass circulation : 2/3
spin circulation : 2/3

$$\boldsymbol{e}_1 = (1,1,1)/\sqrt{3}, \quad \boldsymbol{e}_2 = (1,-1,-1)/\sqrt{3}$$
$$\boldsymbol{e}_3 = (-1,1,-1)/\sqrt{3}, \quad \boldsymbol{e}_4 = (-1,-1,1)/\sqrt{3}$$

Topological Charge of Vortices is Non-Abelian





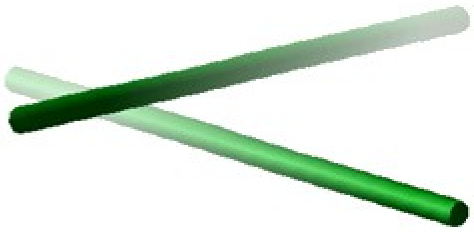
There are 12 rotations for vortices

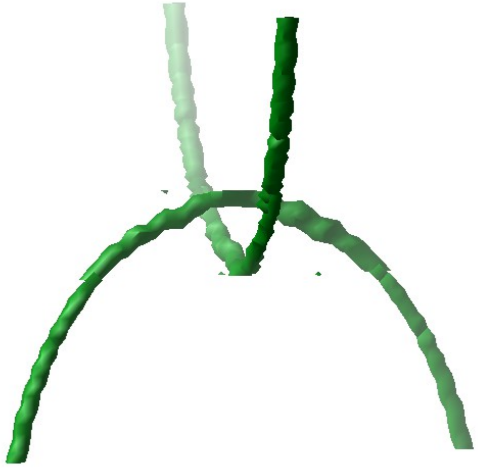
Collision Dynamics of Vortices

“**Non-Abelian**” character becomes remarkable when two vortices collide with each other

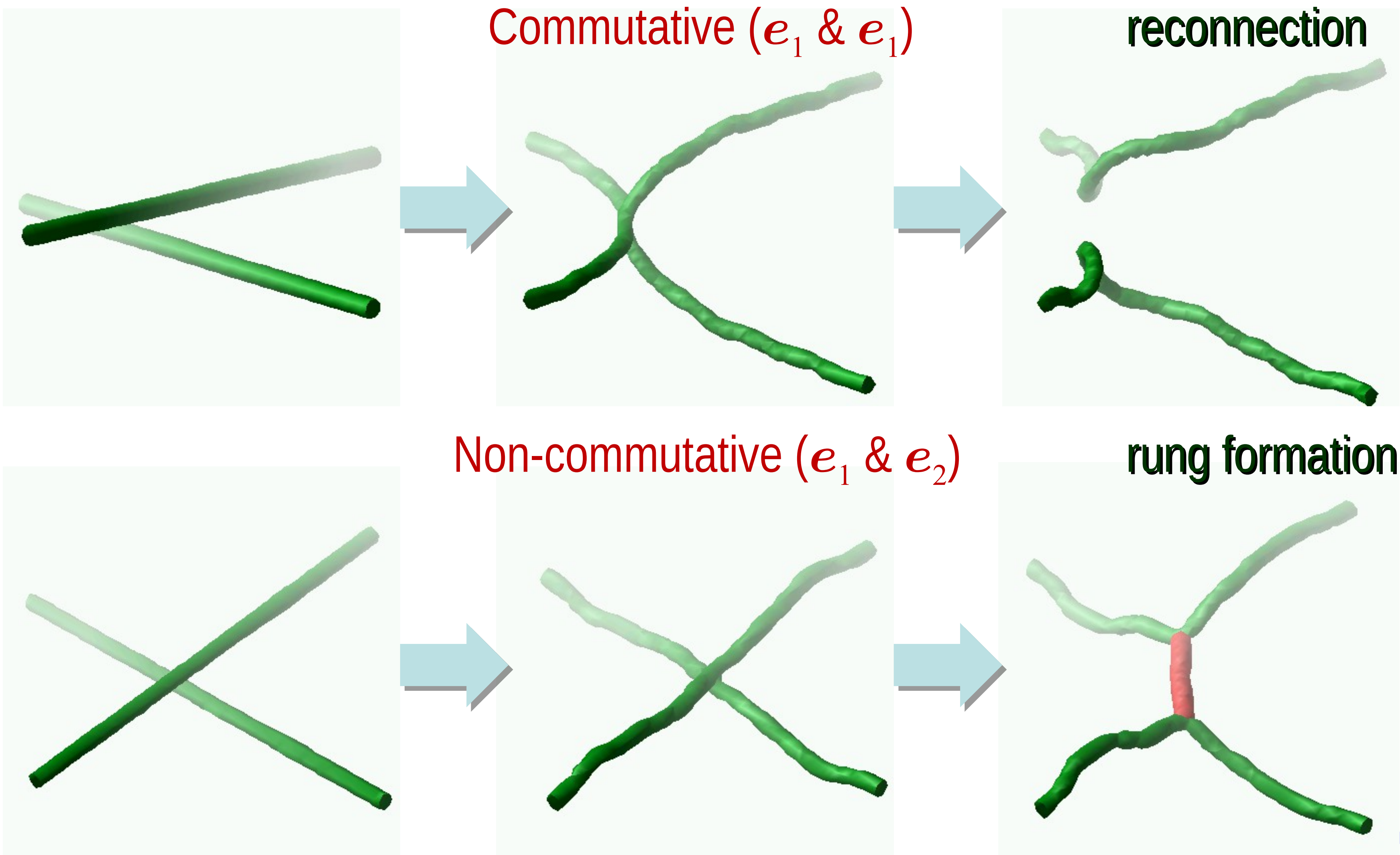
→Numerical simulation of the Gross-Pitaevskii equation

Initial state：two straight vortices in oblique angle, linked vortices



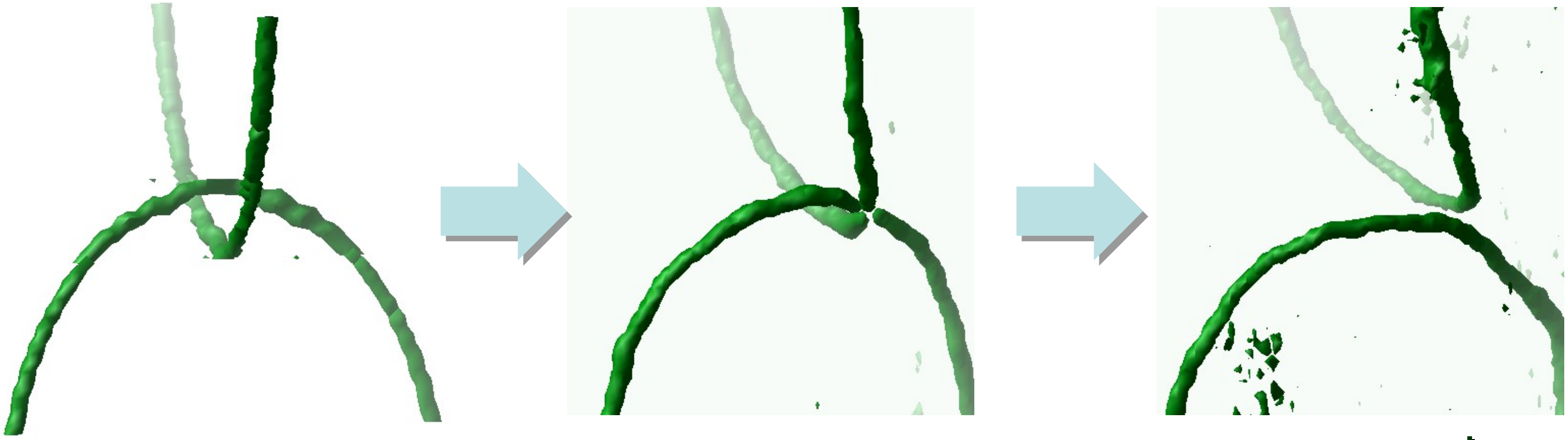


Collision Dynamics of Vortices

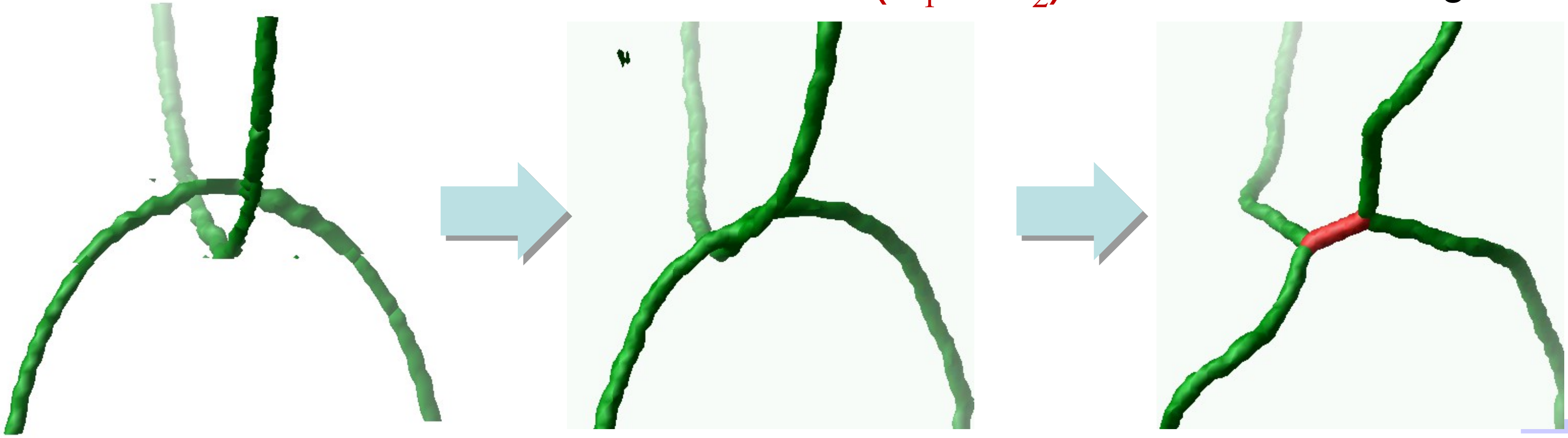


Collision Dynamics of Linked Vortices

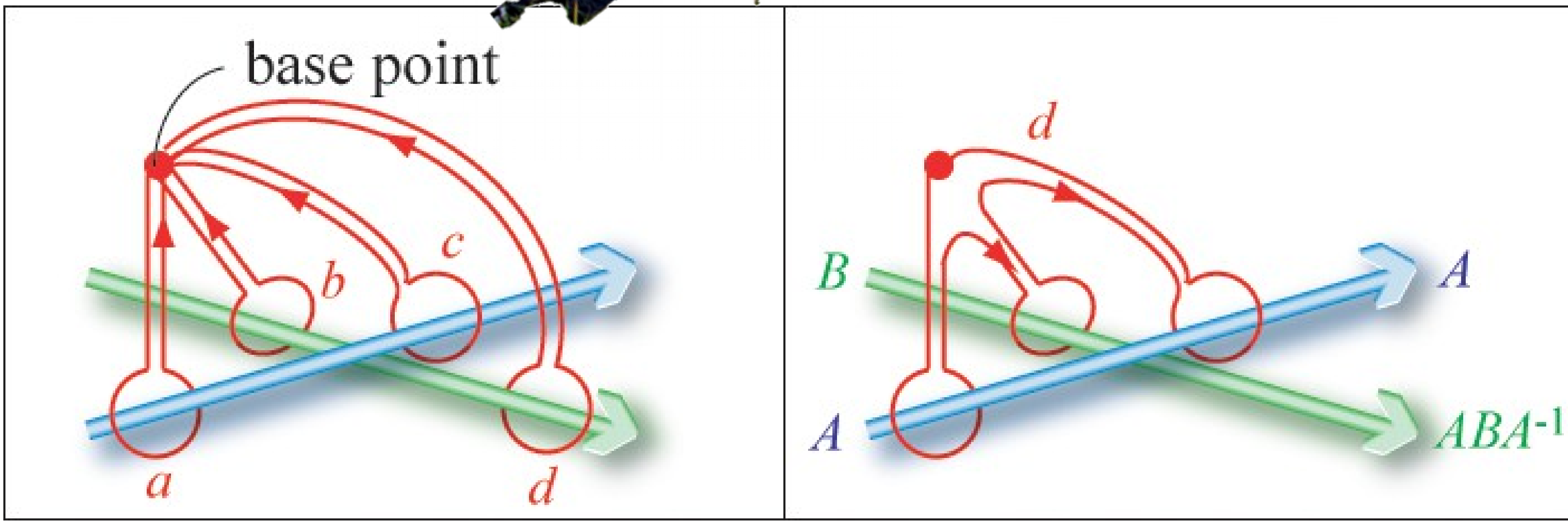
Commutative (e_1 & e_1) untangle



Non-commutative (e_1 & e_2) not untangle

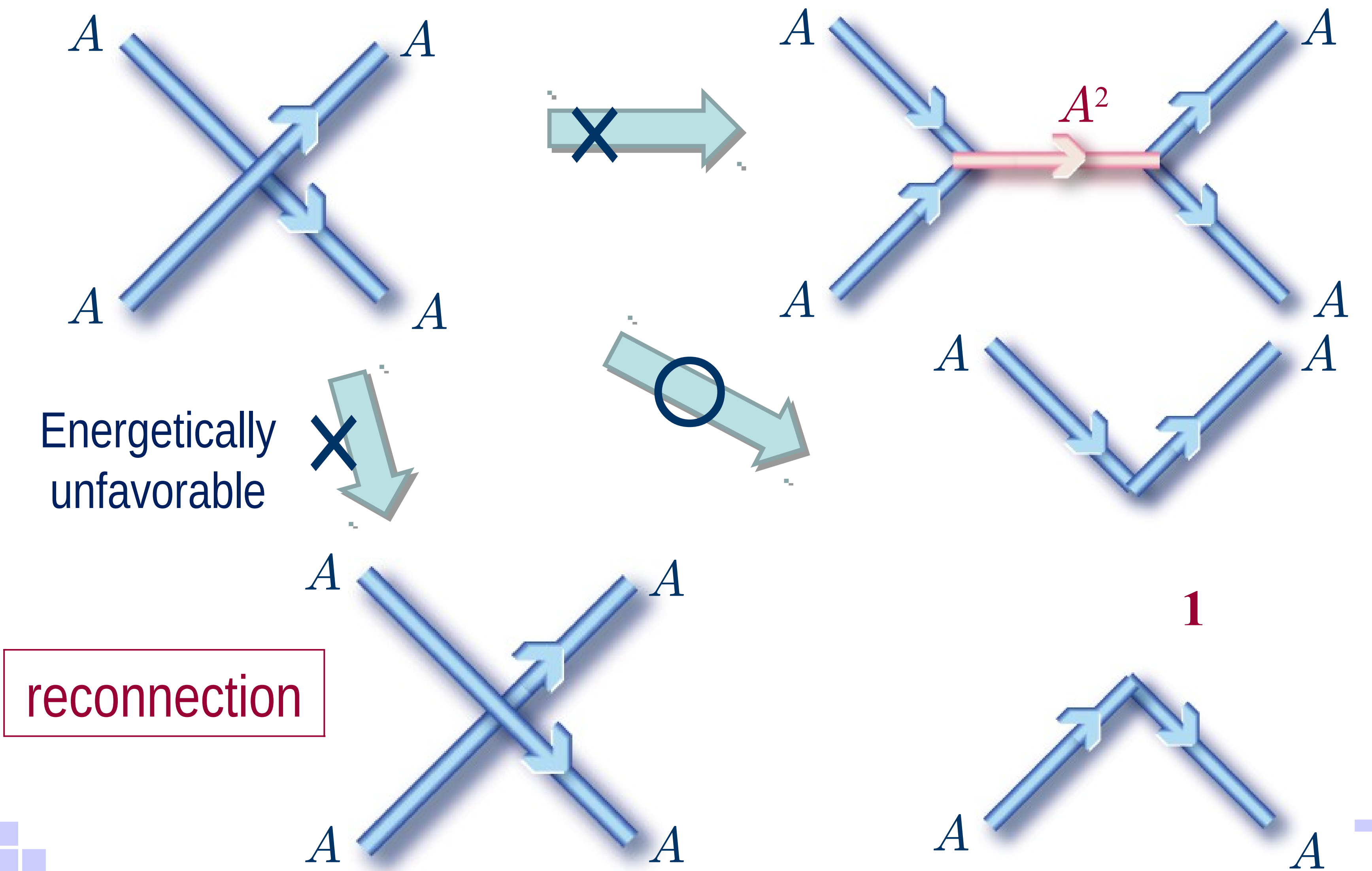


Algebraic Approach

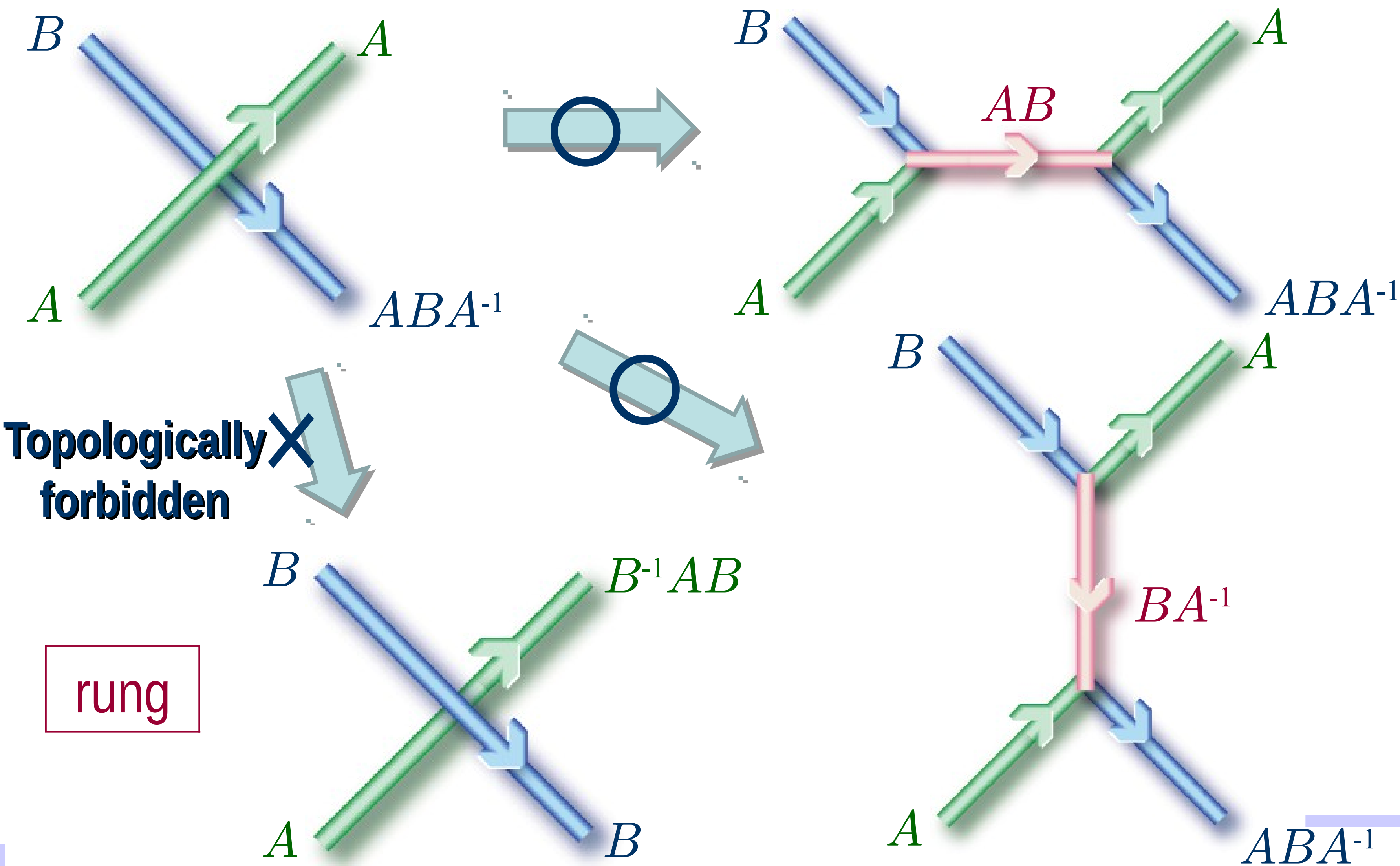


Path d defines vortex B as ABA^{-1} (same conjugacy class)

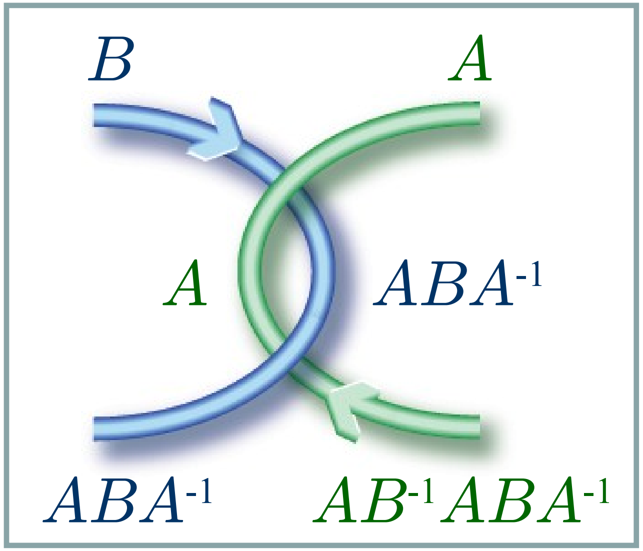
Collision of Same Vortices

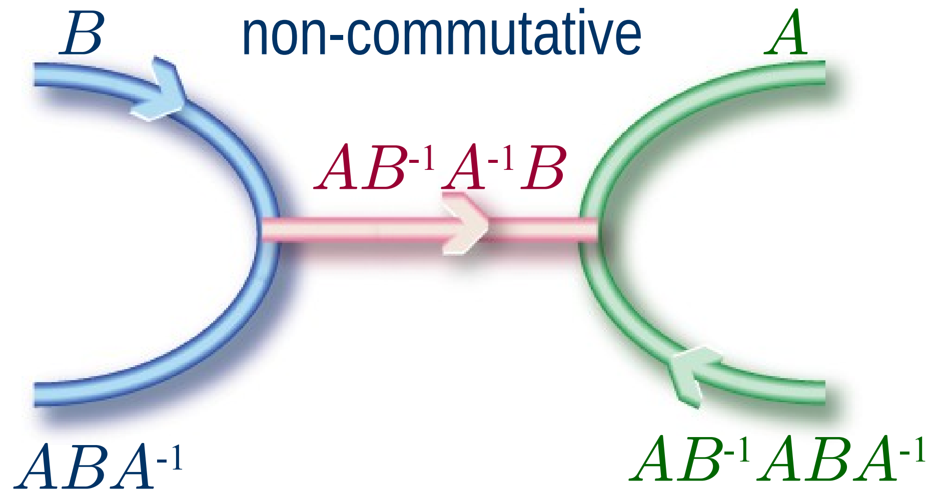


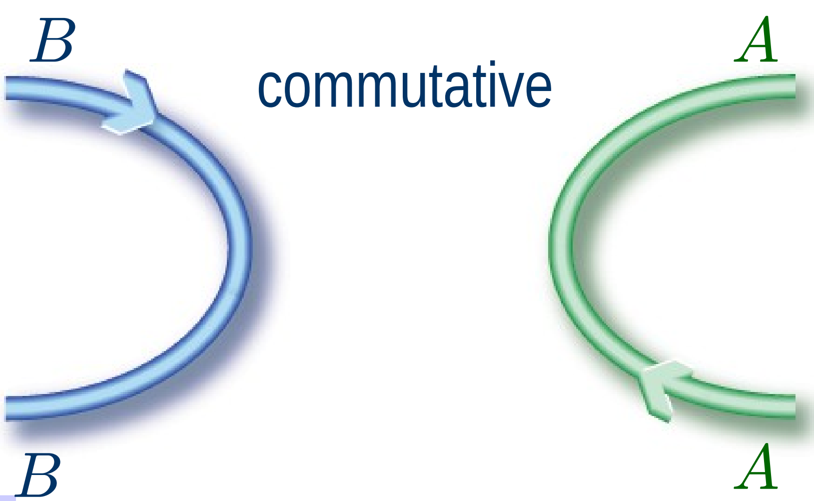
Collision of Different Non-commutative Vortices



Linked Vortices





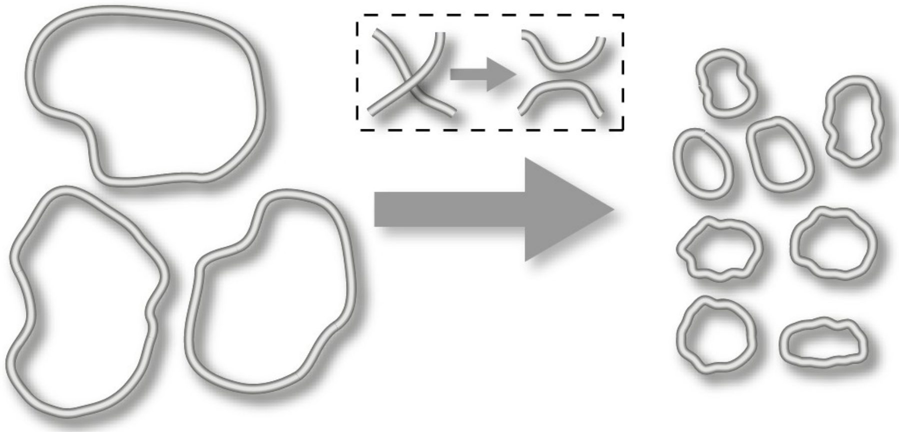


Linked vortices cannot untangle

Summary

1. Vortices with non-commutative circulations are defined as non-Abelian vortices.
2. Non-Abelian vortices can be realized in the cyclic phase of spin-2 BEC
3. Collision of two non-Abelian vortices create a new vortex between them as a rung (networking structure).

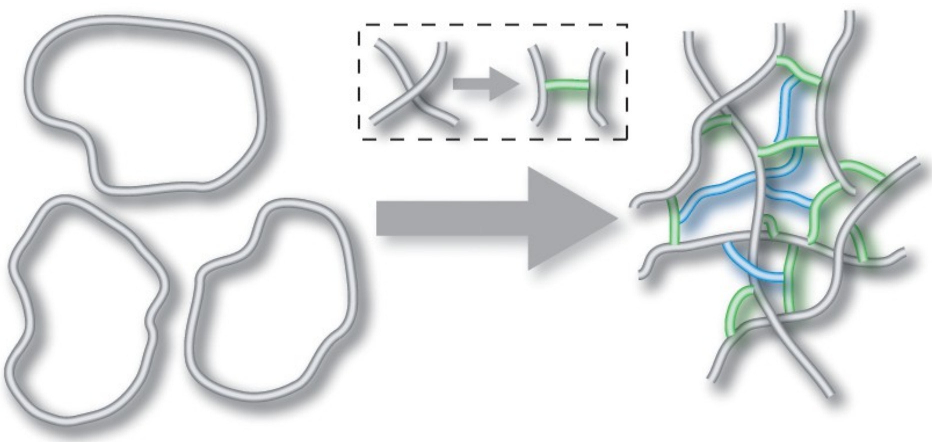
Future: Network Structure in Quantum Turbulence



Turbulence with Abelian vortices

↓

•Cascade of vortices



Turbulence with non-Abelian vortices

↓

•Large-scale networking structures among vortices with rungs

•Non-cascading turbulence

New turbulence!