Non-Abelian Vortices and Their Non-equilibrium Dynamics in Bose-Einstein Condensates with Spins

Michikazu Kobayashi[a]

University of Tokyo

November 18th, 2011 at Keio University
"2nd Workshop on Quarks and Hadrons under Extreme Conditions
- Lattice QCD, Holography, Topology, and Physics at RHIC / LHC -"

Special Thanks to

- Condensed matter system & Bose-Einstein condensate

Shingo Kobayashi (University of Tokyo)

Yuki Kawaguchi (University of Tokyo)

Masahito Ueda (University of Tokyo)

- Cosmology

Shun Uchino (Kyoto University)

Muneto Nitta (Keio University)

- Statistical physics

Shin-ichi Sasa (University of Tokyo)

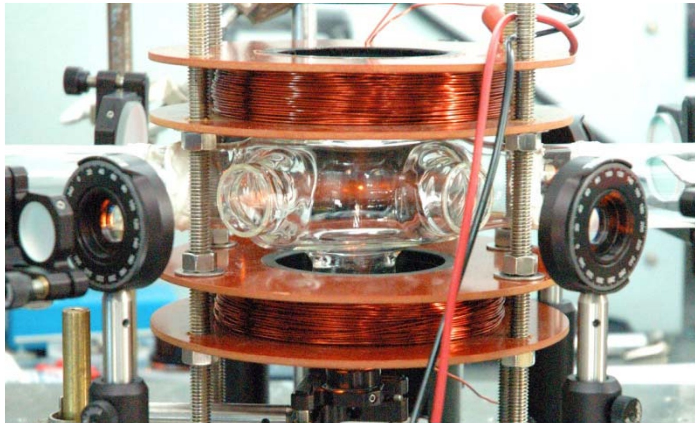
Leticia F. Cugliandolo (Université Pierre et Marie Curie)

Contents

Contents

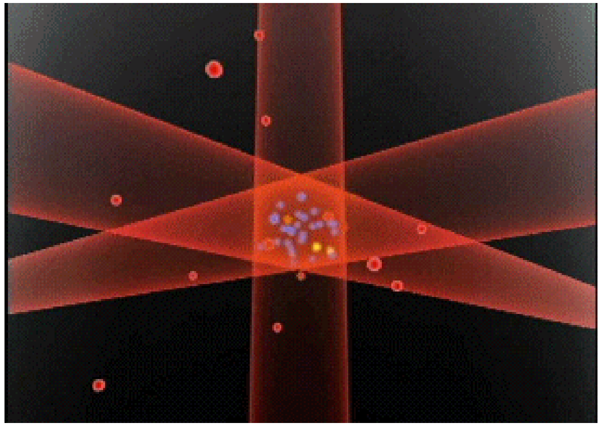
Ultracold Atomic Bose-Einstein Condensate

Dilute alkali atomic BEC has been succeeded in 1997

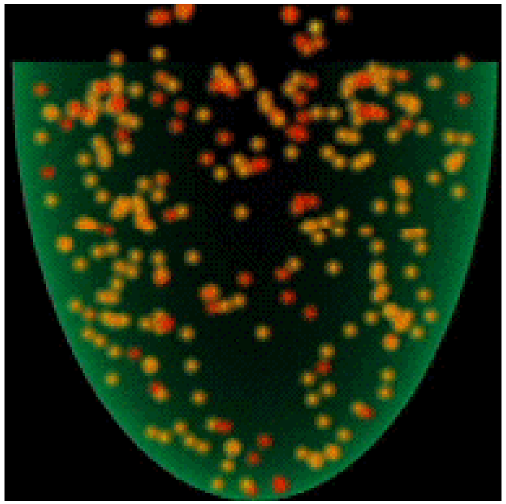


Trap of atoms

^{87}Rb, ^{23}Na, ^{7}Li, ^{1}H, ^{85}Rb, ^{41}K, ^{4}He, ^{133}Cs, ^{174}Yb, ^{52}Cr, ^{40}Ca, ^{84}Sr, ^{164}Dy



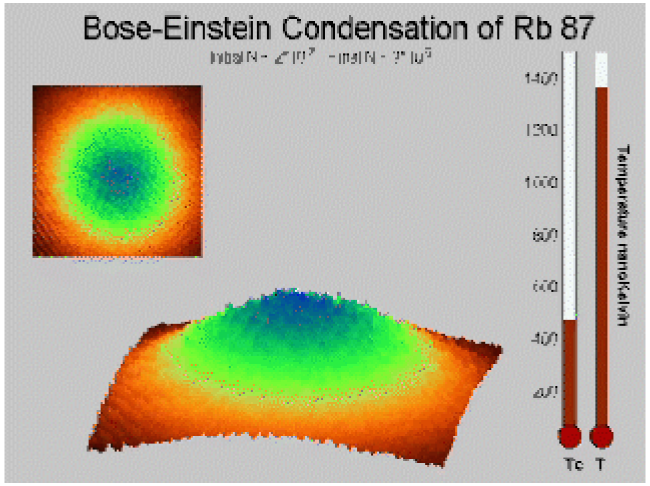
Laser cooling

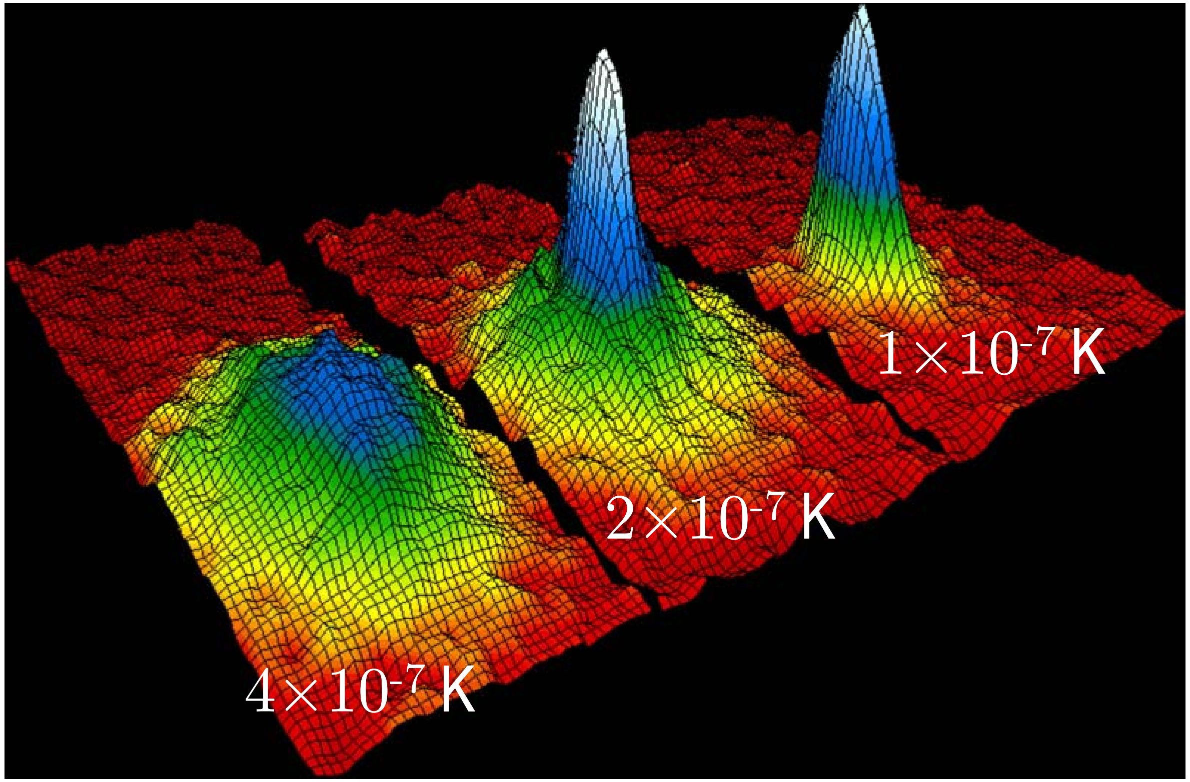


Evaporative cooling

Atomic Bose-Einstein Condensate

BEC of ^{87}Rb





Bose-Einstein Condensation

Essence of BEC : Broken $U(1)$ - gauge symmetry

$$\rho(\boldsymbol{x},\boldsymbol{y}) = \langle \hat{\psi}(\boldsymbol{x})\hat{\psi}^\dagger(\boldsymbol{y})\rangle \xrightarrow{|\boldsymbol{x}-\boldsymbol{y}|\to\infty} \psi(\boldsymbol{x})\psi(\boldsymbol{y})^*$$

$\psi(\boldsymbol{x})=|\psi(\boldsymbol{x})| \exp[i\varphi(\boldsymbol{x})]$: $\varphi(\boldsymbol{x})$ is fixed
$\rightarrow$ broken $U(1)$ - gauge symmetry

condensate density : $n_c(\boldsymbol{x}) = |\psi(\boldsymbol{x})|^2$

condensate current : $\boldsymbol{j}_c(\boldsymbol{x}) = (\hbar/M)\mathrm{Im}[\psi^*(\boldsymbol{x})\nabla\psi(\boldsymbol{x})]$

superfluid velocity : $\boldsymbol{v}_c(\boldsymbol{x}) \equiv \boldsymbol{j}_c(\boldsymbol{x})/n_c(\boldsymbol{x}) = (\hbar/M)\nabla\varphi(\boldsymbol{x})$

Vortex in BEC

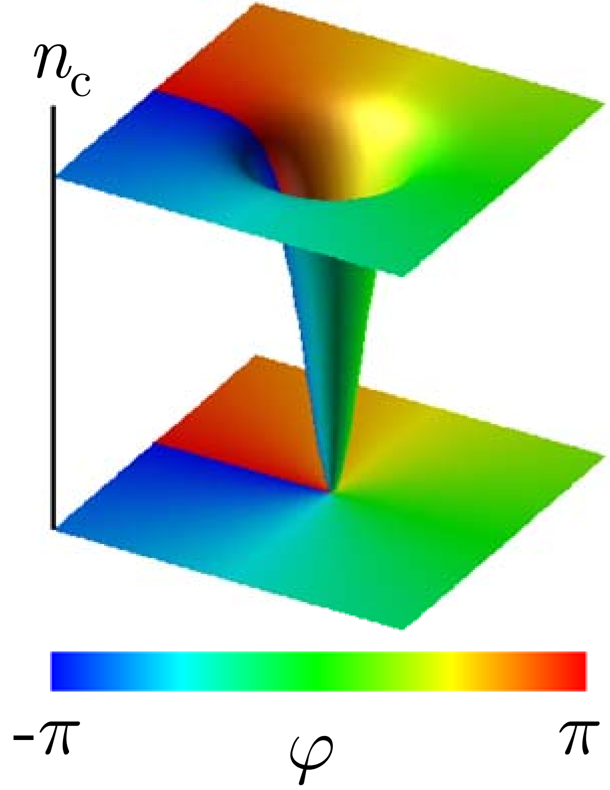
condensate density : $n_c = |\psi|^2$

superfluid velocity : $\boldsymbol{v}_c = (\hbar/M)\nabla\varphi$

Phase φ of the wave function shifts by $2\pi m$ (m : integer) around the vortex core where the wave function vanishes : $\psi = 0$ as a topological defect.

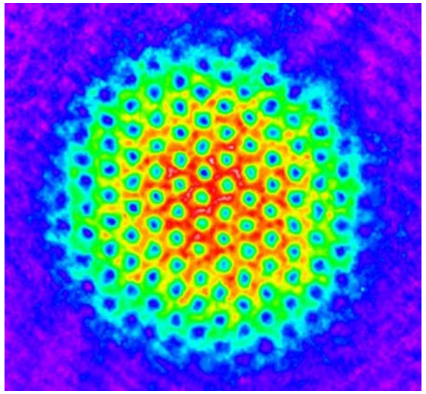
Each vortex can be characterized by m (additive group of integers)

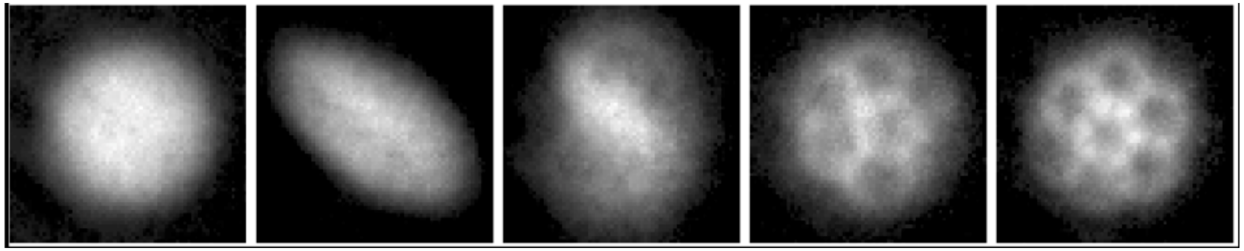
Quantized vortex for $m = +1$



Experimental Observation of Vortices

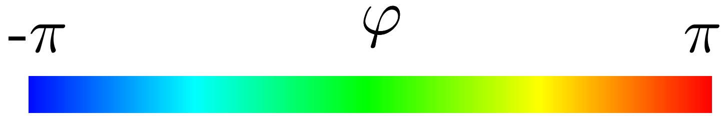
Vortex lattice and its formation in ^{87}Rb BEC

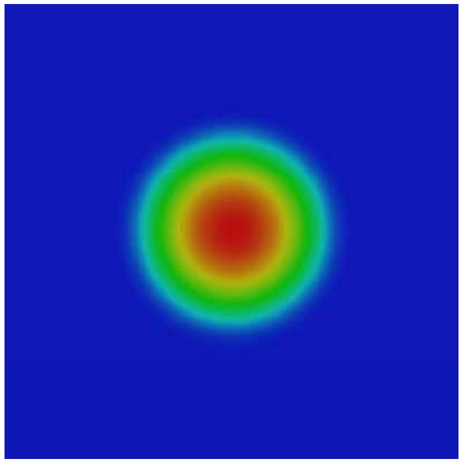


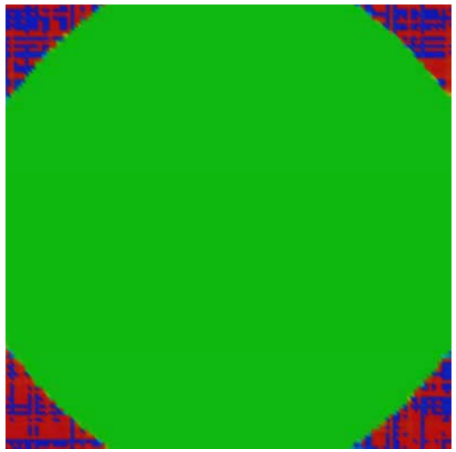


K. W. Madison et al. PRL 86, 4443 (2001)

Numerical simulation







Contents

Spinor Bose-Einstein Condensate

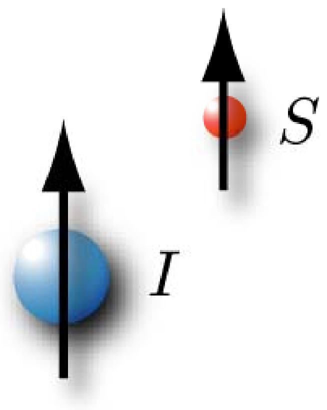
There are two ways to trap BECs : magnetic trap and laser trap

magnetic trap : spin degrees of freedom is frozen $\longrightarrow$ scalar BEC
laser trap : spin degrees of freedom is alive $\longrightarrow$ spinor BEC

Hyperfine interaction between electron spin and orbital and nuclear spin cannot be negligible for cold atoms

Hyperfine spin : $F = I + S + L$

I : nuclear spin
S : electron spin
L : electron orbital



^{87}Rb, ^{23}Na, ^{7}Li, ^{41}K	F=1, 2
^{85}Rb	F=2, 3
^{133}Cs	F=3, 4
^{52}Cr	F=3

Spinor Bose-Einstein Condensate

For ^{87}Rb $(I = 3/2,\ S = 1/2,\ L = 0) \rightarrow F = 1$ or 2

$$F = 2 \begin{cases} m = 2 \\ m = 1 \\ m = 0 \\ m = -1 \\ m = -2 \end{cases} \qquad F = 1 \begin{cases} m = 1 \\ m = 0 \\ m = -1 \end{cases}$$

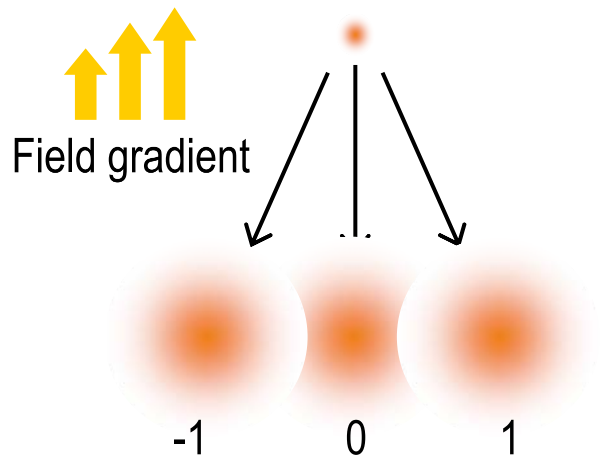
Multi-component BEC characterized by the quantum number m

Spin 1 : 3-component BEC

$$\psi = (\psi_1, \psi_0, \psi_{-1})$$

Spin 2 : 5-component BEC

$$\psi = (\psi_2, \psi_1, \psi_0, \psi_{-1}, \psi_{-2})$$



Effective Hamiltonian for BEC

$$H = \begin{cases} \displaystyle\int d\boldsymbol{x}\left[\frac{\hbar^2}{2M}|\nabla\psi_m|^2 + \frac{c_0}{2}n^2 + \frac{c_1}{2}\boldsymbol{F}^2\right] & \text{spin-1 BEC} \\ \displaystyle\int d\boldsymbol{x}\left[\frac{\hbar^2}{2M}|\nabla\psi_m|^2 + \frac{c_0}{2}n^2 + \frac{c_1}{2}\boldsymbol{F}^2 + \frac{c_2}{2}|A_{20}|^2\right] & \text{spin-2 BEC} \end{cases}$$

number density : $n(\boldsymbol{x}) = \psi_m^*(\boldsymbol{x})\psi_m(\boldsymbol{x})$

spin density : $\boldsymbol{F}(\boldsymbol{x}) = \psi_m^*(\boldsymbol{x})\hat{\boldsymbol{F}}_{mn}\psi_n(\boldsymbol{x})$

singlet-pair amplitude : $A_{20} = (-1)^m\psi_m\psi_{-m}$

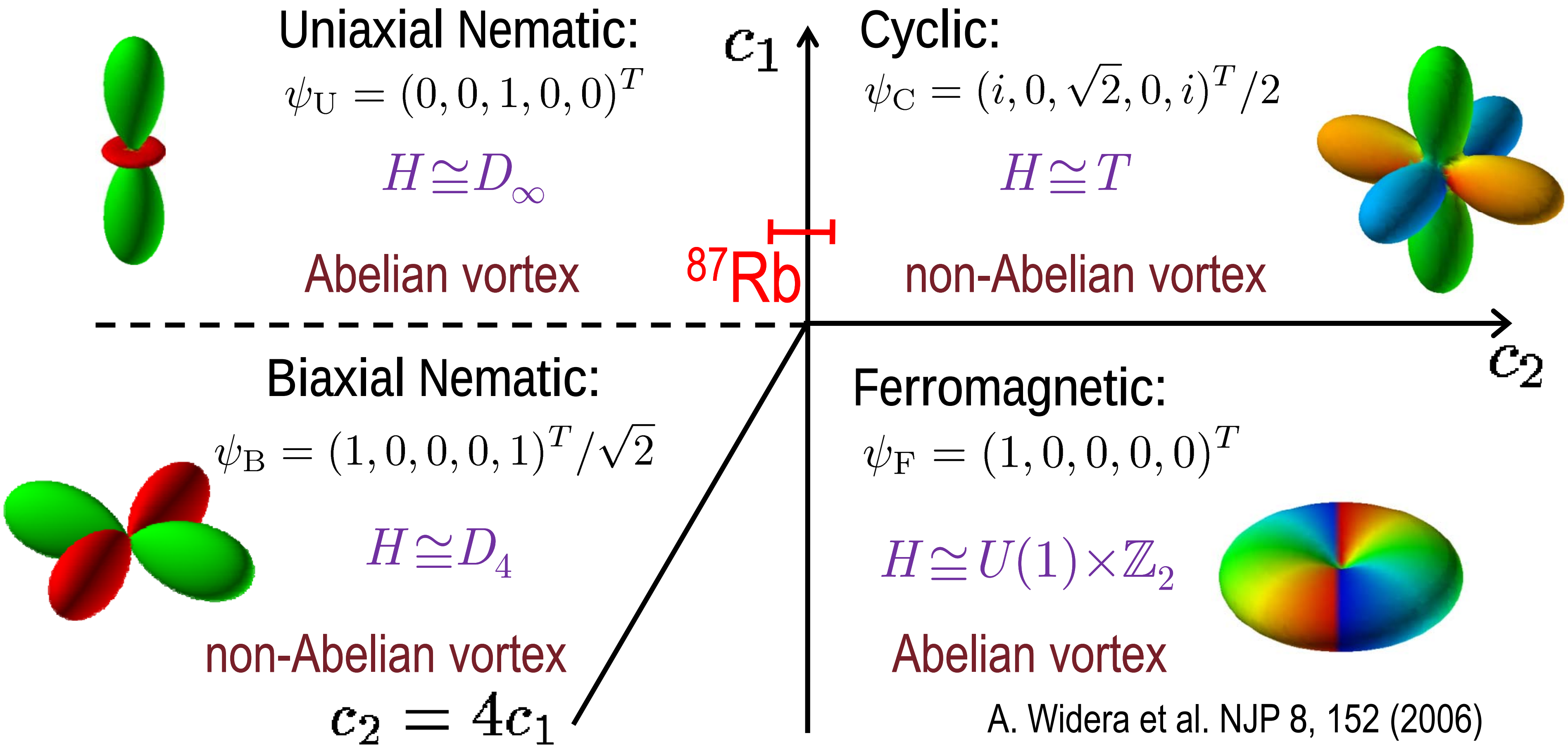
As well as $U(1)$ gauge symmetry, $SO(3)$ spin rotational symmetry is also broken : $G \cong U(1) \times SO(3)$

Remaining symmetry H can be (non-Abelian) subgroup of $SO(3)$

$\Rightarrow \pi_1[G/H]$ can be non-Abelian $\rightarrow$ non-Abelian vortex appears

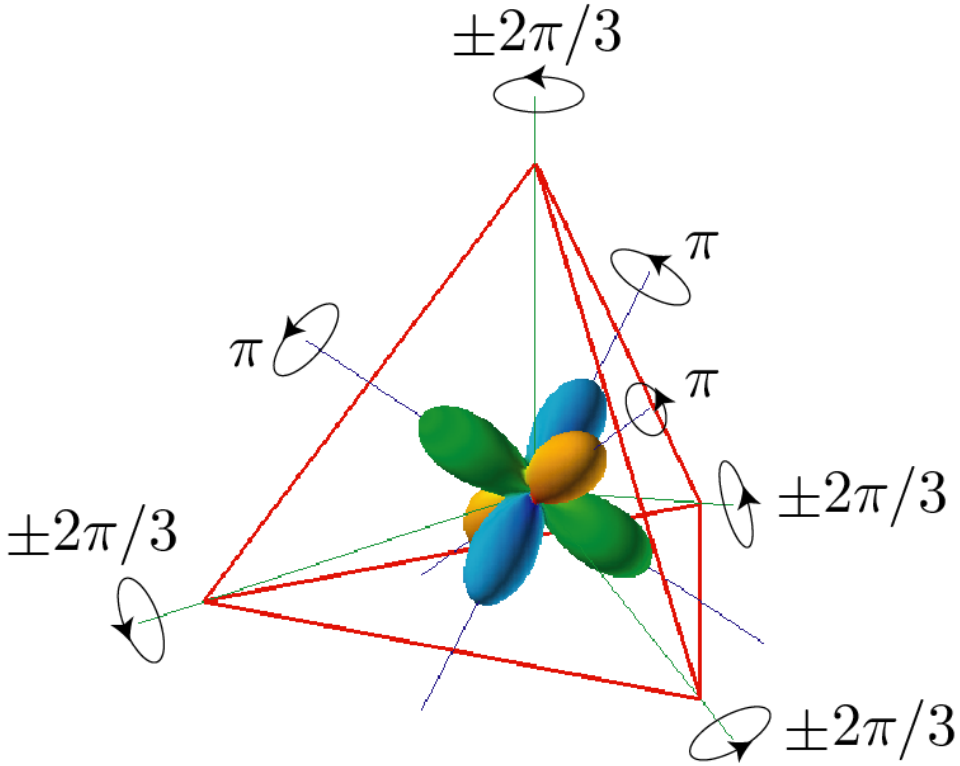
Spin-2 BEC

$$H = \int d\boldsymbol{x} \left[\frac{\hbar^2}{2M} |\nabla \psi_m|^2 + \frac{c_0}{2} n^2 + \frac{c_1}{2} \boldsymbol{F}^2 + \frac{c_2}{2} |A_{20}|^2 \right]$$



A. Widera et al. NJP 8, 152 (2006)

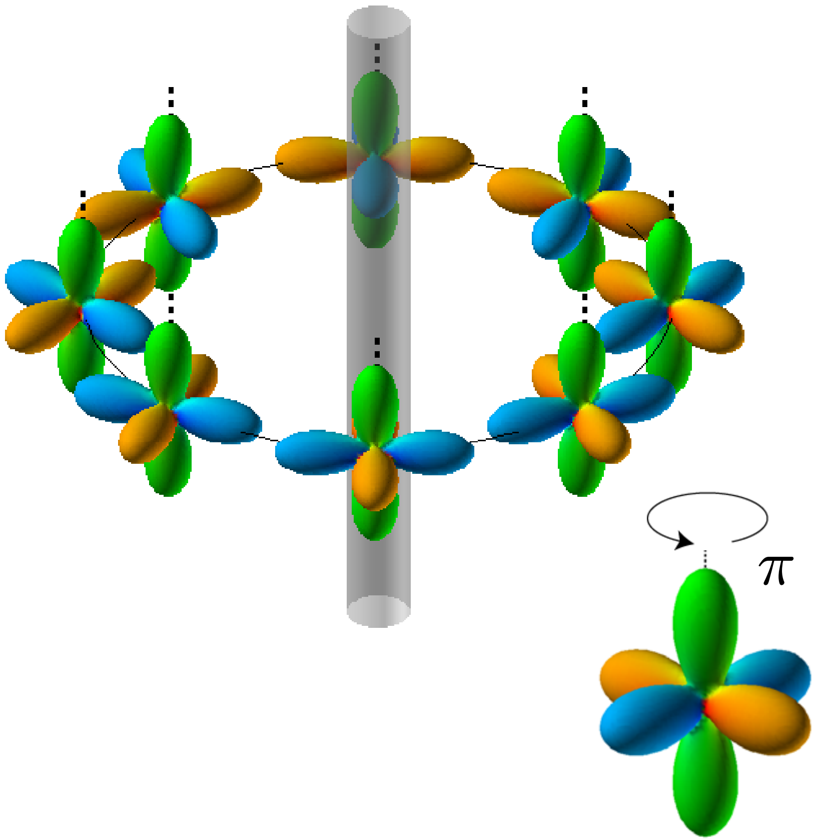
Symmetry of cyclic state



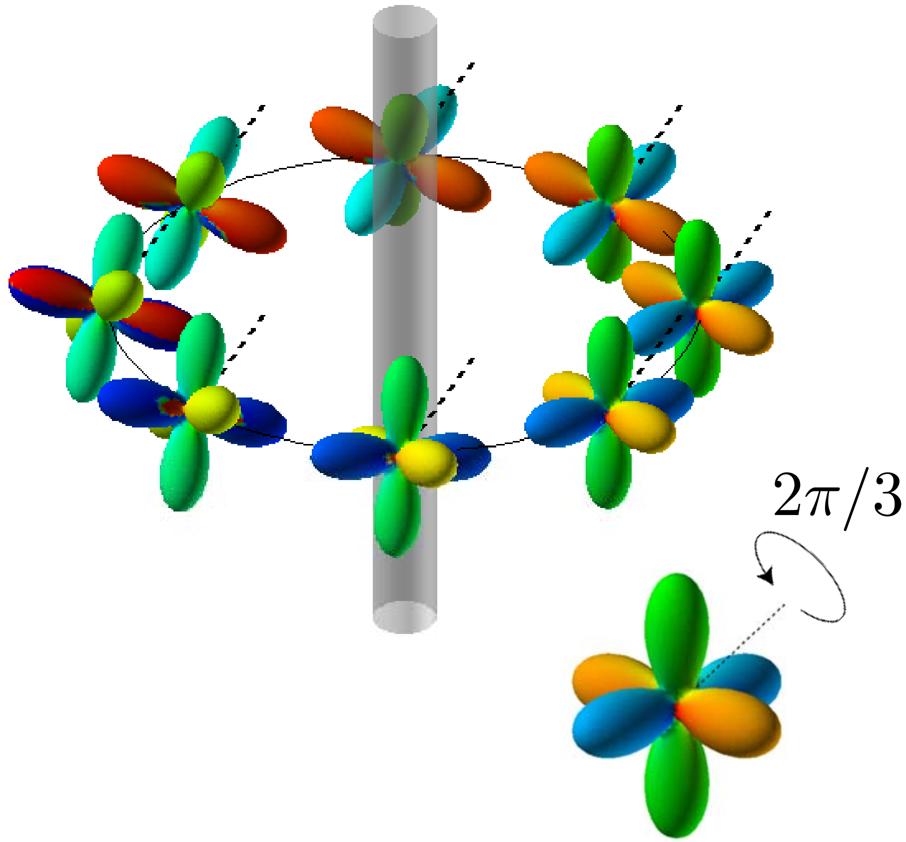
Spin rotations keeping cyclic state invariant form a non Abelian tetrahedral symmetry

Vortices in cyclic state

1/2 spin vortex



1/3 vortex



Interaction between two parallel vortices

Energetically obtained interaction between two parallel vortices

	1/3 (commutative)	1/3 (non-commutative)	1/2 (commutative)	1/2 (non-commutative)
1/3	$-(s_1 s_2/3)\log r_{12}$	$-(2/3)(\tanh(\alpha_1 r_1 2))^2$	$-\sqrt{1/3}(\tanh(\alpha_2 r_1 2))^2$	
1/2	$-\sqrt{1/3}(\tanh(\alpha_2 r_1 2))^2$		$-(s_1 s_2/2)\log r_{12}$	$-(1/2)(\tanh(\alpha_3 r_1 2))^2$

There is also “topological” interaction between vortices

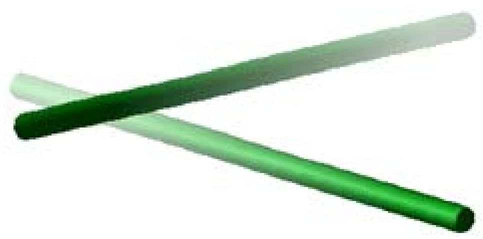
Contents

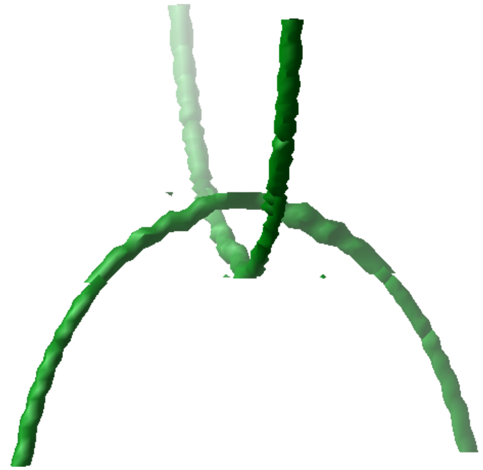
Collision Dynamics of Non-Abelian Vortices

"**Non-Abelian property**" becomes outstanding
for collision dynamics of vortices

→Simulation of Gross-Pitaevskii equation (GPE)

Initial state : two straight vortices & two linked vortices

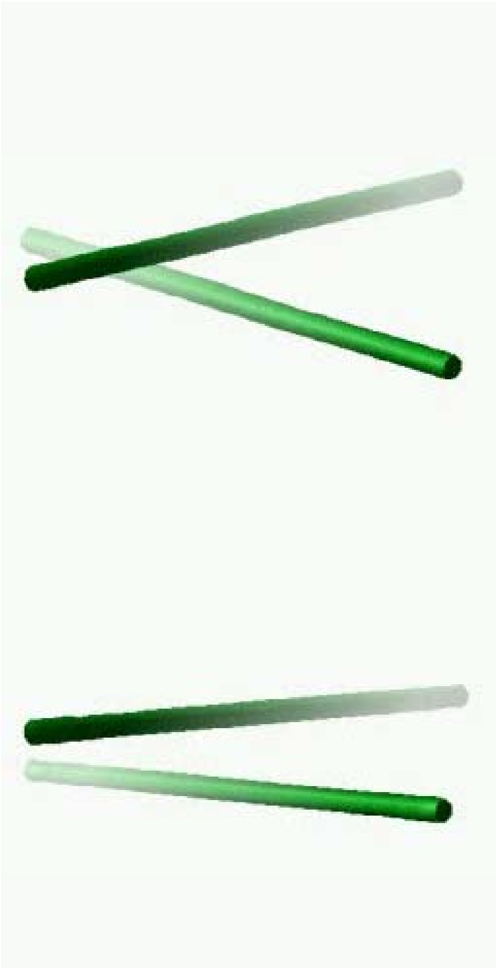




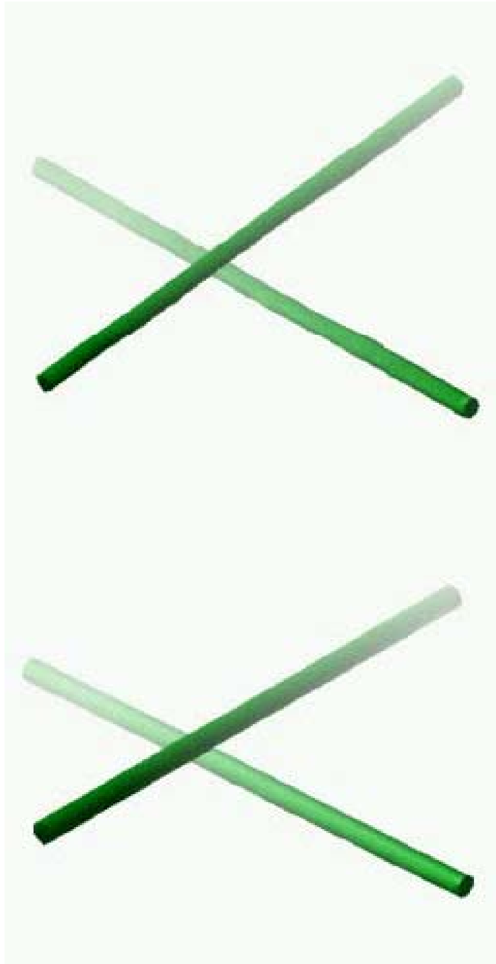
MK, Y. Kawaguchi, M, Nitta, and M. Ueda, PRL 103, 115301(2009).

Collision dynamics of vortices

Commutative pair

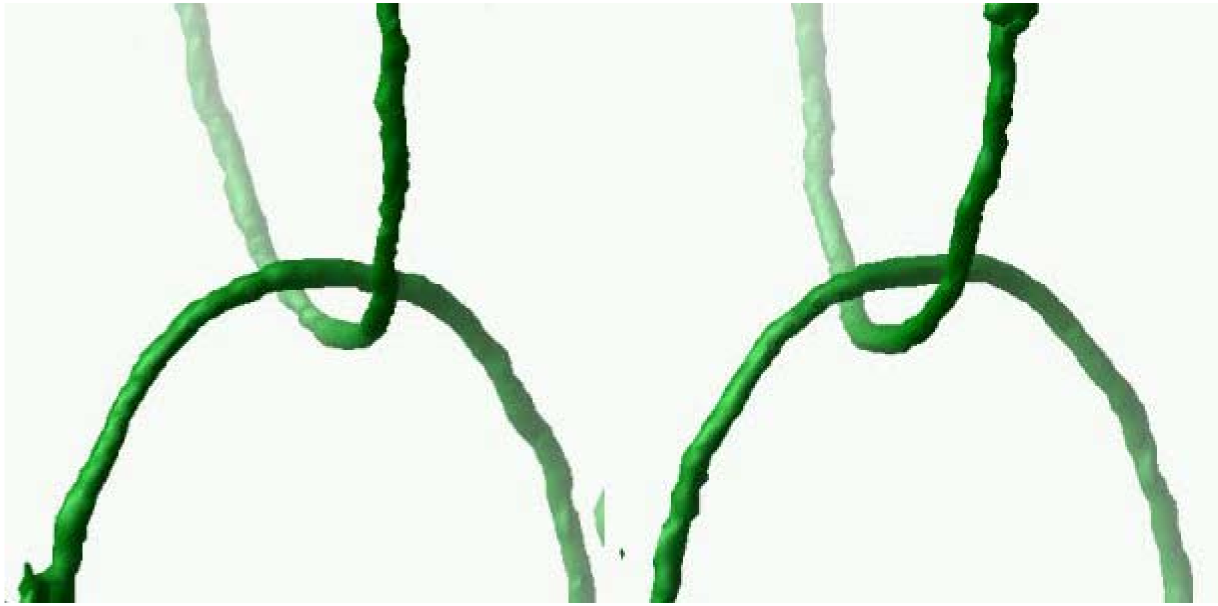


Non-commutative pair



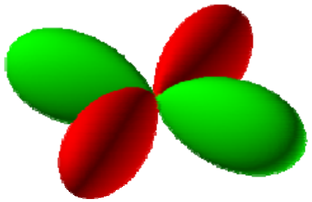
Collision dynamics of vortices

Commutative pair　　Non-commutative pair



Biaxial nematic state at finite temperature

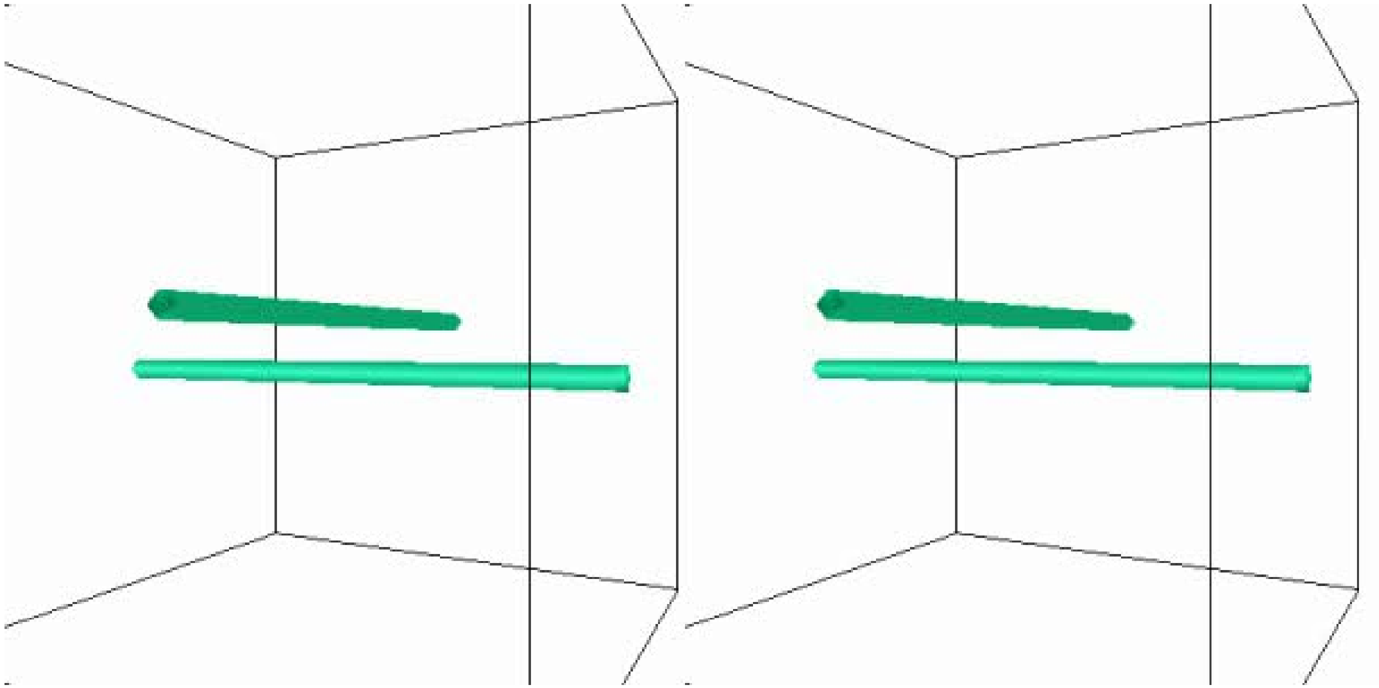
$H \cong D_4$



With stochastic GPE (SGPE)

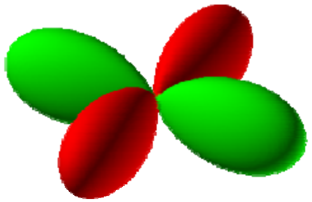
For commutative vortices

$T = 0.2\ T_c$ $T = 0.01\ T_c$

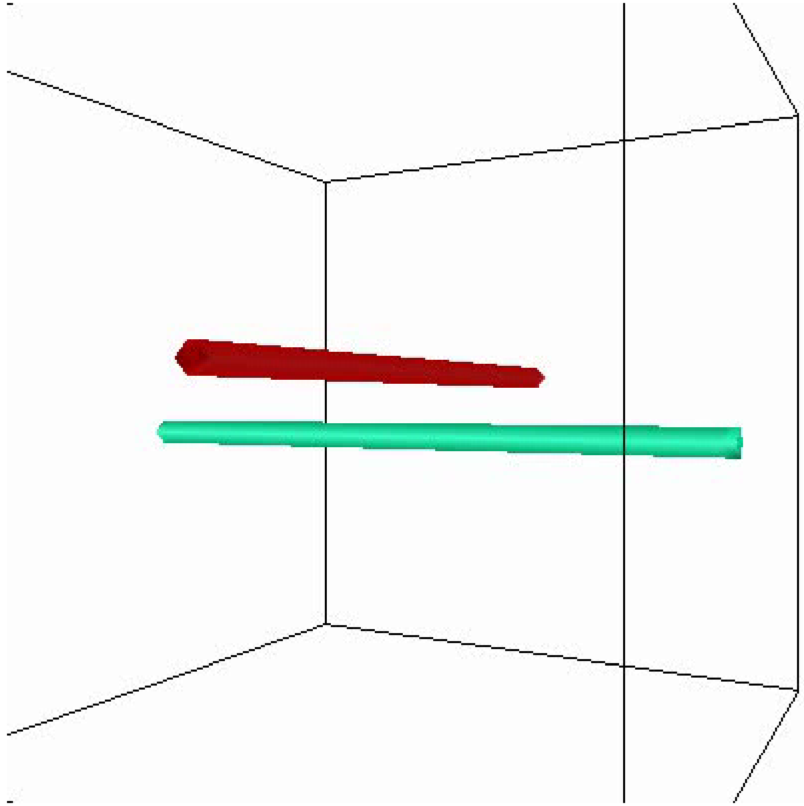


Biaxial nematic state at finite temperature

$H \cong D_4$

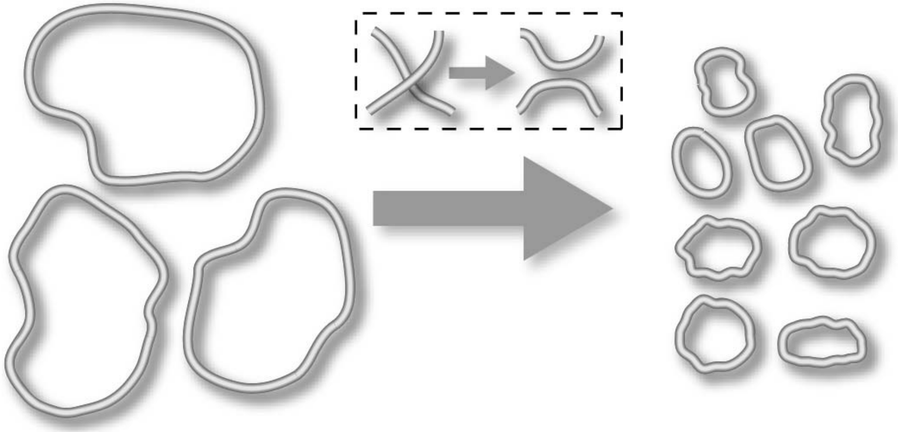


For non-commutative vortices



Contents

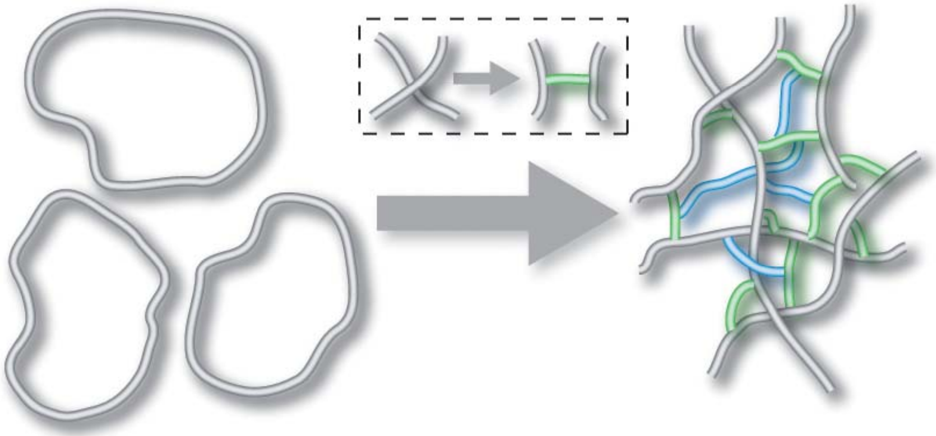
Non-equilibrium Dynamics of non-Abelian Vortices



Abelian vortices

↓

Cascade of vortices



non-Abelian vortices

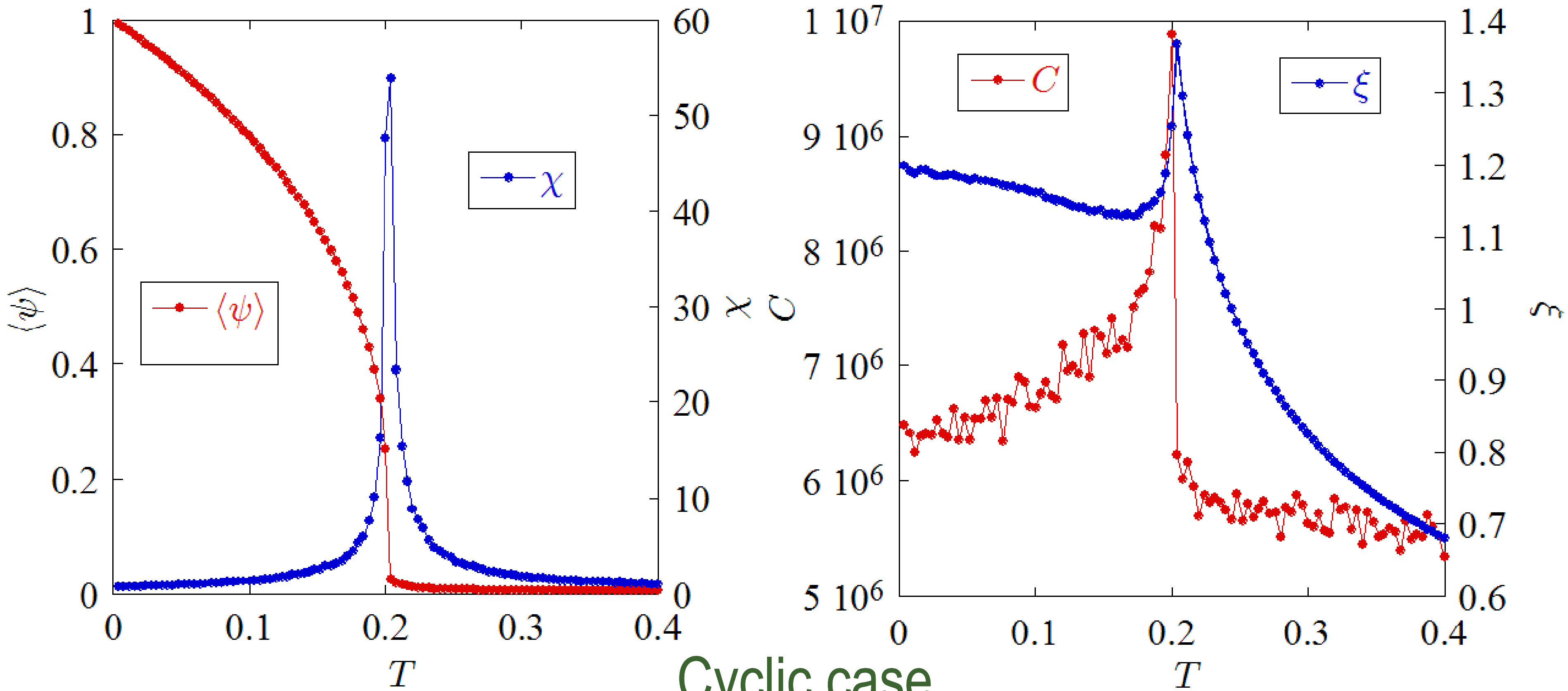
↓

Large-scale networking structures

Different non-equilibrium behavior is expected

Equilibrium property

Being independent of whether vortices are Abelian or non-Abelian, system shows the 2nd ordered phase transition



Cyclic case

Critical exponent

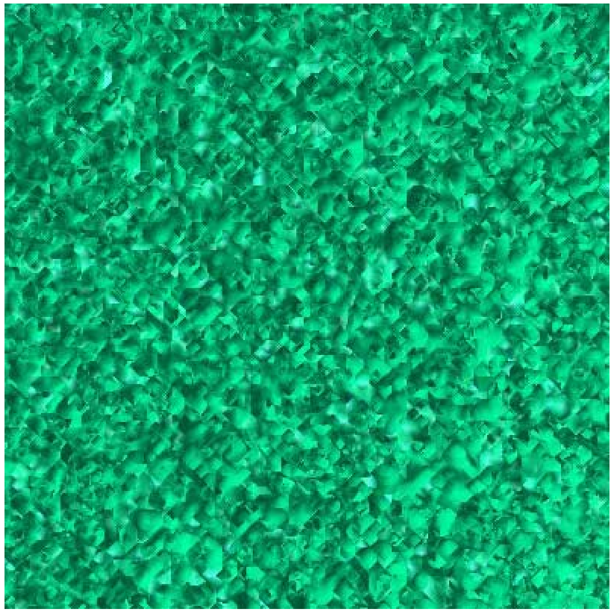
	α	$\beta(T < T_c)$	γ	ν
scalar BEC	-0.0080	0.35	1.3	0.67
cyclic	-0.04	0.37	1.3	0.68
biaxial nematic	0.5	0.26	0.99	0.50
S^9 $(c_1 = c_2 = 0)$	-0.37	0.43	1.3	0.71
$SO(2)$ cyclic	-0.18	0.49	1.51	0.79
mean field	0	1/2	1	1/2

Difference of the critical exponent shows the difference of topology of the order parameter

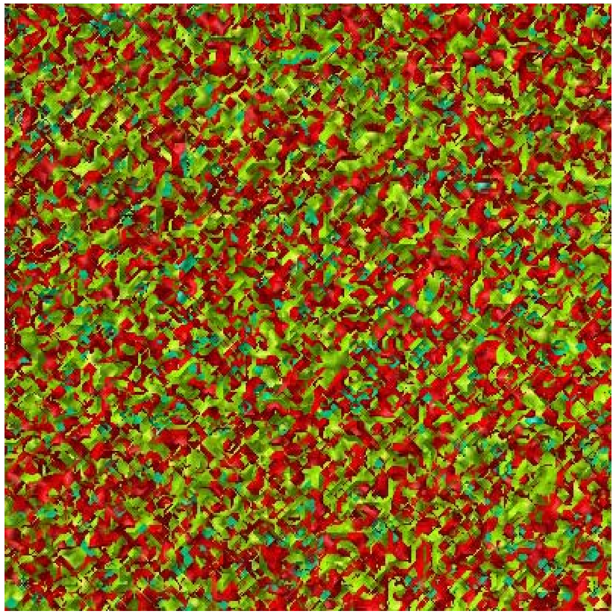
Phase-ordering dynamics

Rapid temperature quench from $T = 2T_c$ to $T \to 0$

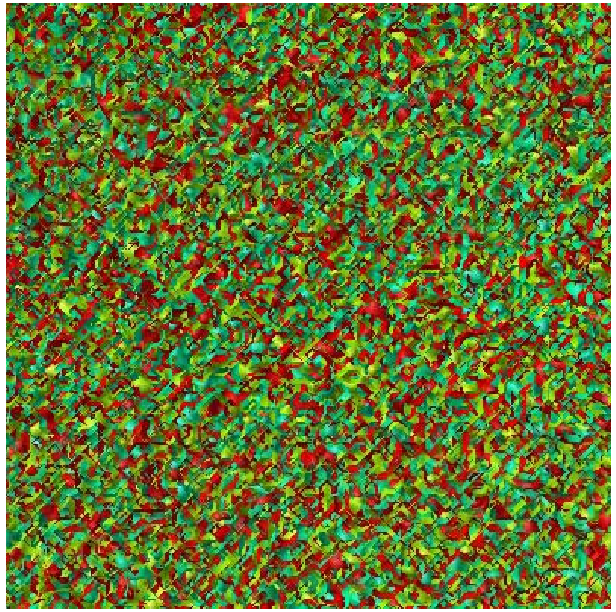
Scalar



Cyclic

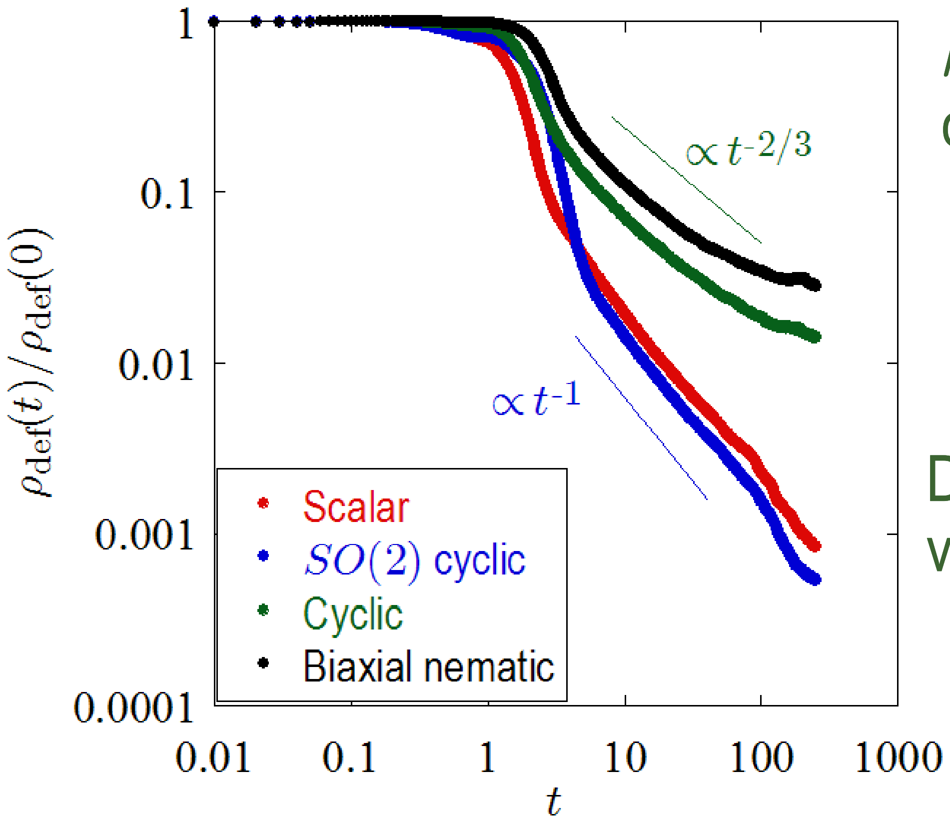


Biaxial nematic



Phase-ordering dynamics

Density of vortex line length



$\rho \propto t^{-1}$: Universal behavior for phase-ordering with Abelian vortices

Decay becomes slower for non-Abelian vortices : $\rho \propto t^{-2/3}$

Phase-ordering dynamics

Slower dynamics has also been observed for phase ordering of conserved Ising model : $\langle S \rangle = 0$ (total magnetization is fixed to 0)
$\Rightarrow$Decay of domain wall becomes slower in the phase ordering than that in non-conserved Ising model.

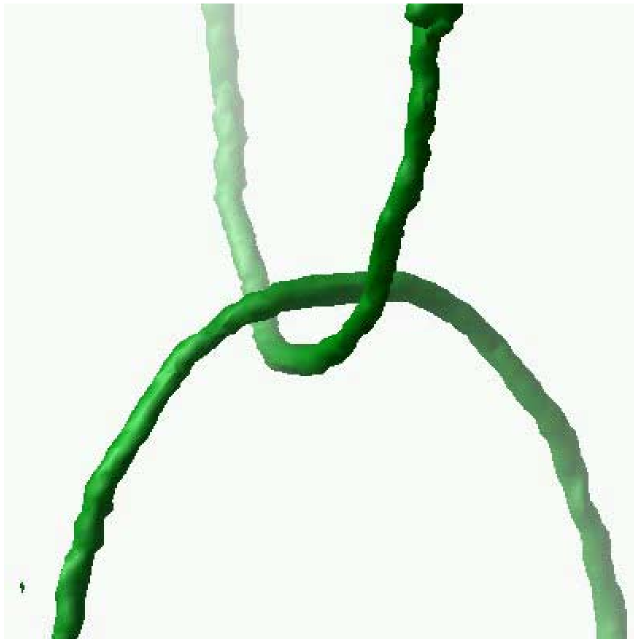
Conserved value in this case : linking number

Phase-ordering dynamics

Conserved value in this case : linking number

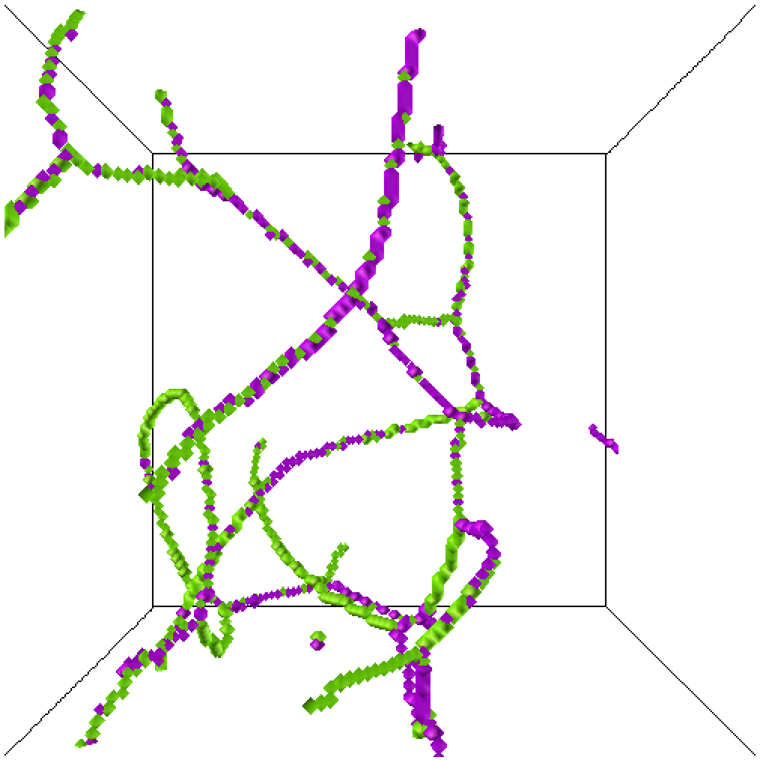
Non-commutative vortices cannot pass through each other, behaving like substantial string

⇒Linking number of vortices are conserved

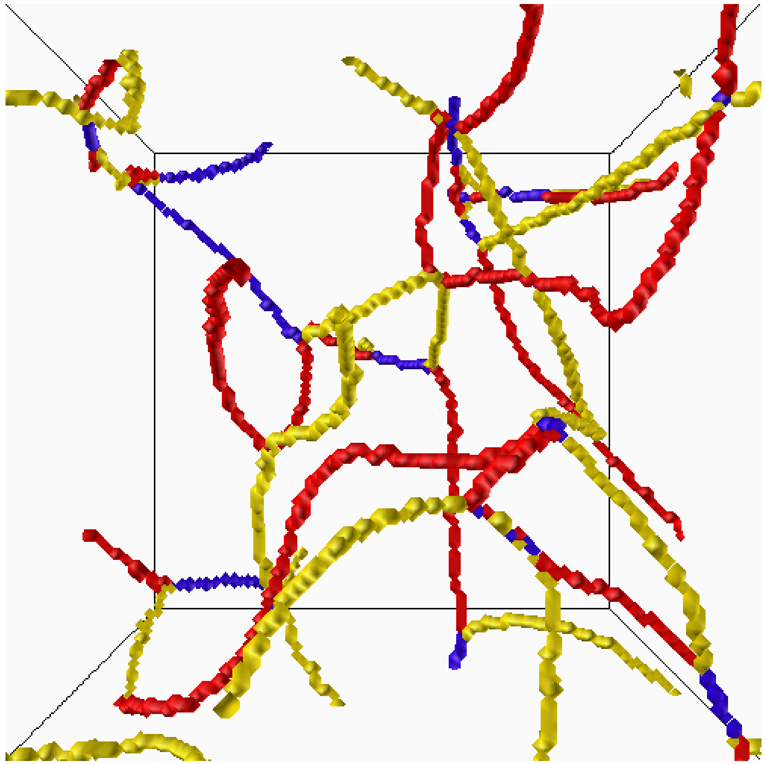


Final State

Sometimes, dynamics stops with finite number of vortices, never relaxing to equilibrium state



Cyclic

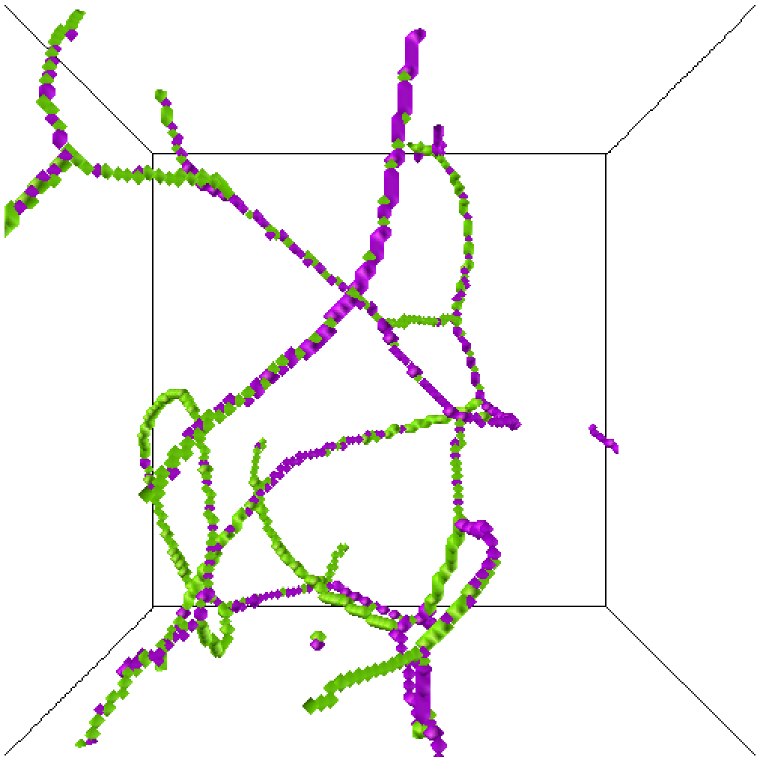


Biaxial nematic

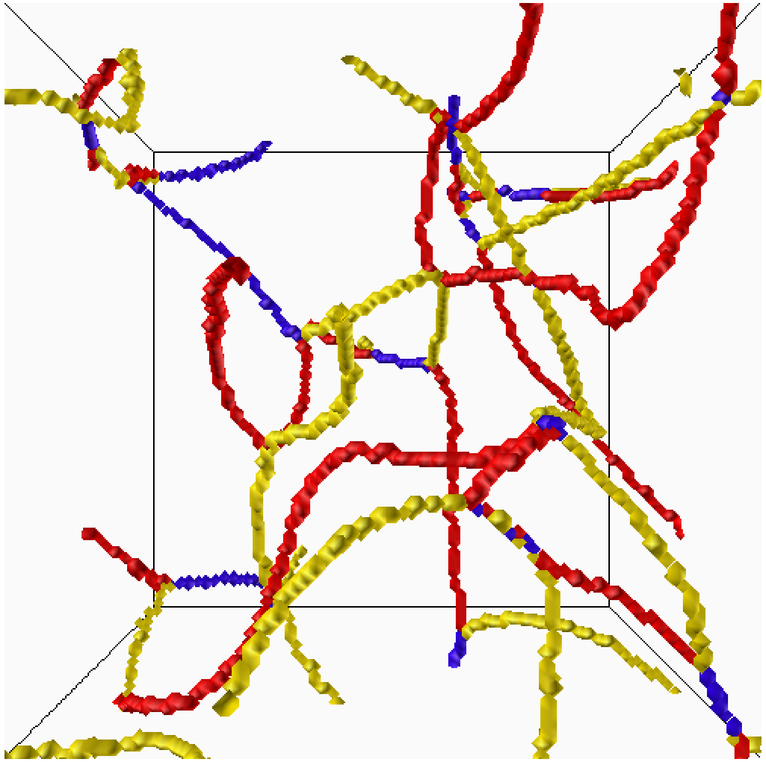
Final State

Probability for appearing non-equilibrium final state

Cyclic：~10%



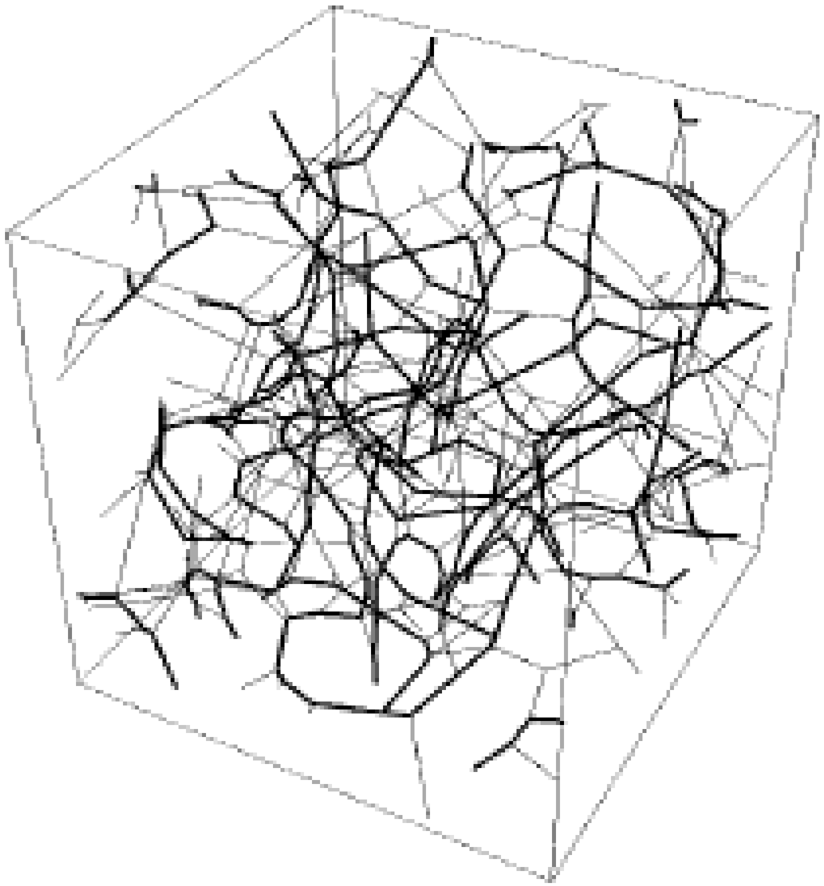
Biaxial nematic：~50%



Probability depends on the topology (the number of conjugacy class?)

Non-Abelian Cosmic Strings

Similar behavior has been reported in the context of cosmic string

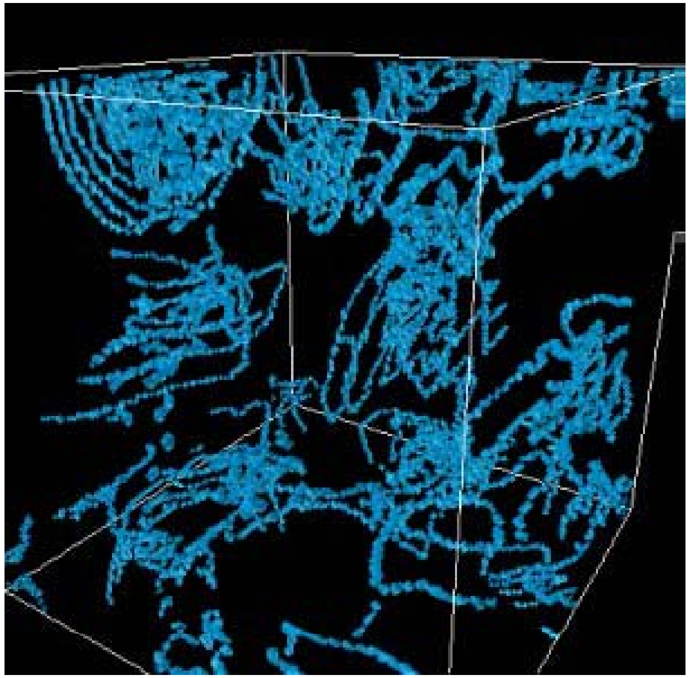


Networking structure of non-Abelian cosmic strings are predicted

P. McGraw, PRD 57, 3317 (1998)

Quantum turbulence

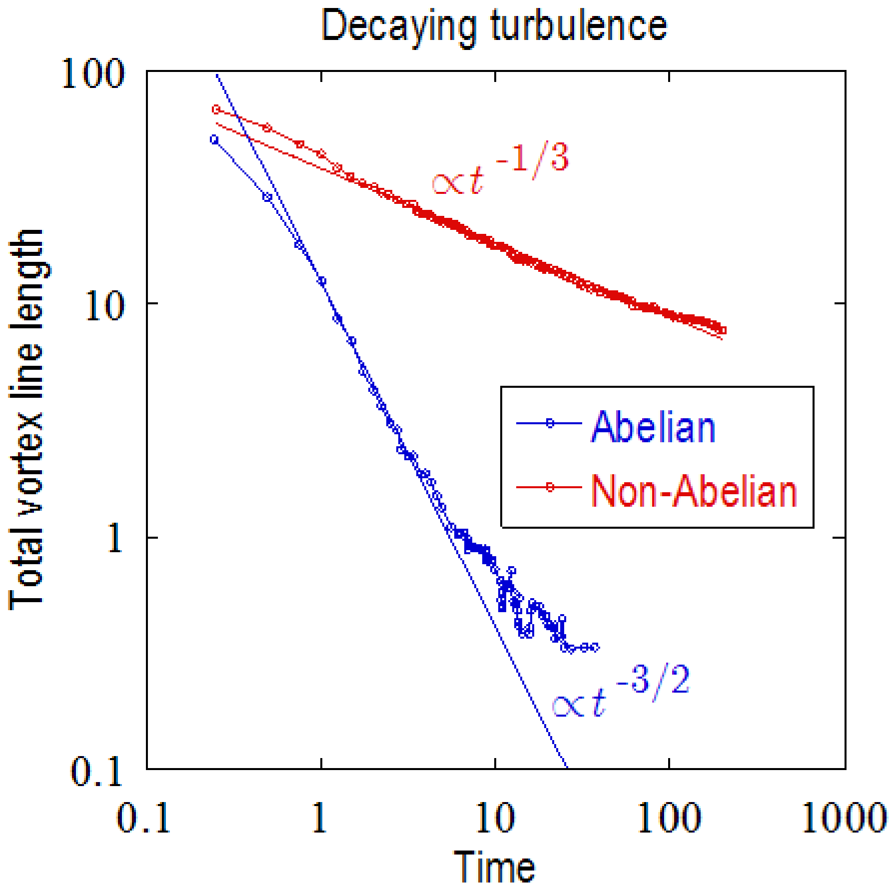
Starting from large-scale vortex loops
$\Rightarrow$Cascade of large to smaller vortices : turbulence

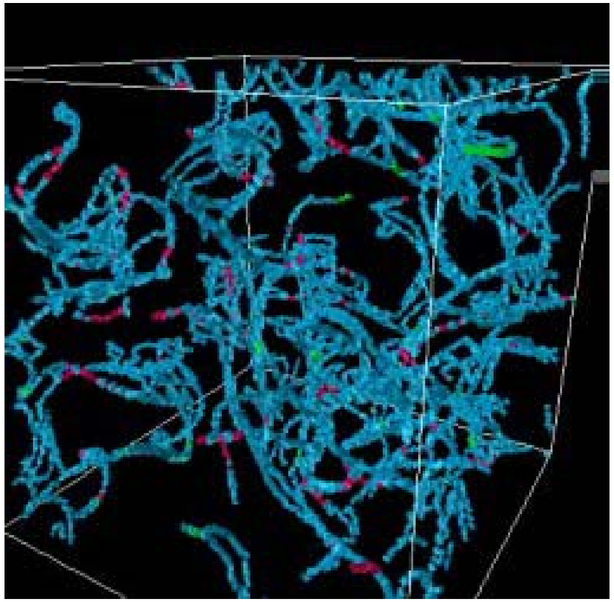


Turbulence of scalar BEC :
In large scales, the Kolmogorov spectrum (spectrum in classical turbulence) has been confirmed.

MK and M. Tsubota, PRL 94, 065302 (2005)

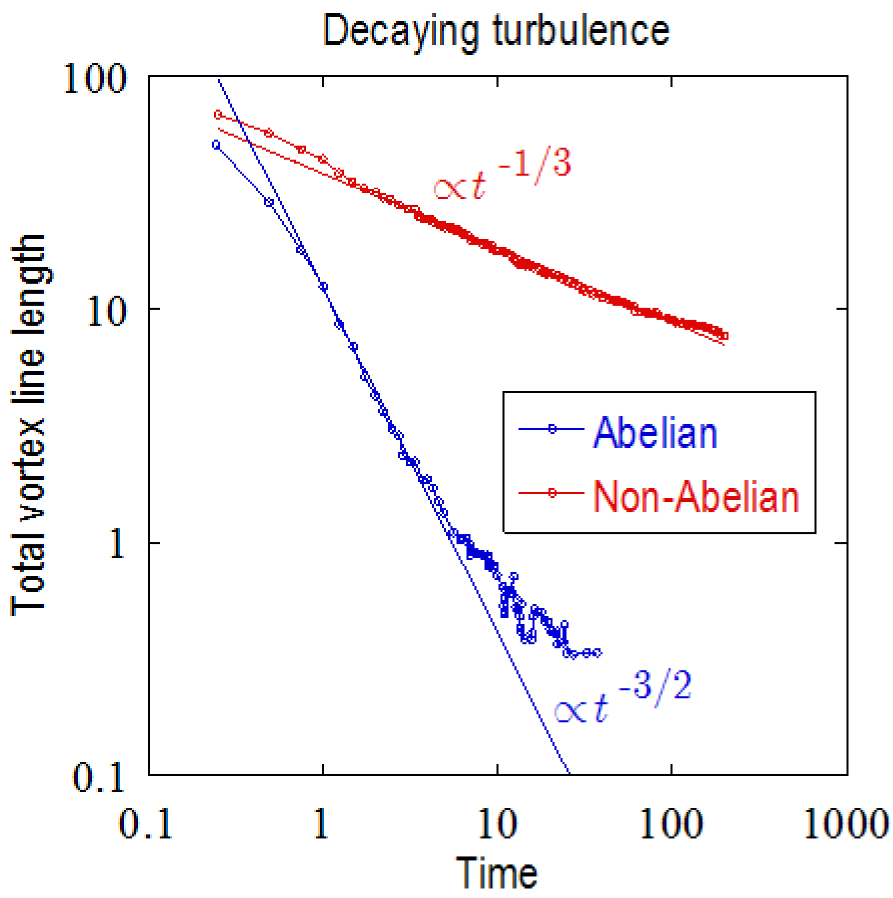
Quantum turbulence

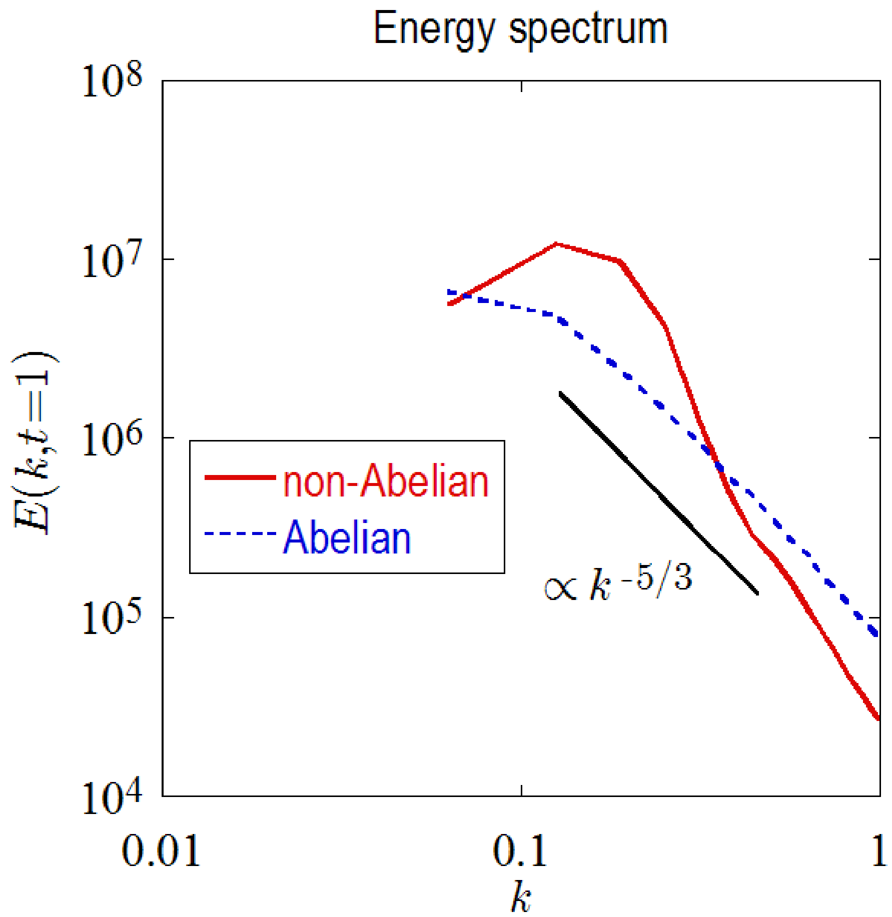




Turbulent behavior is strongly affected by topology

Quantum turbulence





Summary

1. Non-Abelian defects can be realized vortices in BEC with spin degrees of freedom
2. Collision of two defects are completely different between Abelian and non-Abelian vortices (creation of new defect bridging colliding defects)
3. Non-Abelian vortices also change various kind of dynamical behavior (especially makes dynamics slower due to tangling of vortices)