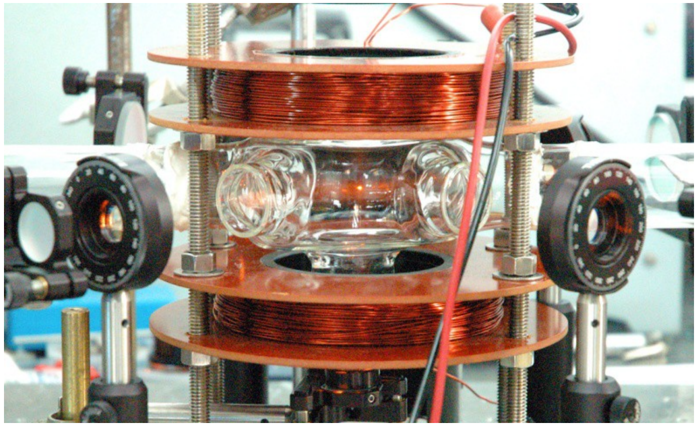
Vortices and other topological defects in ultracold atomic gases

Michikazu Kobayashi (Kyoto Univ.)

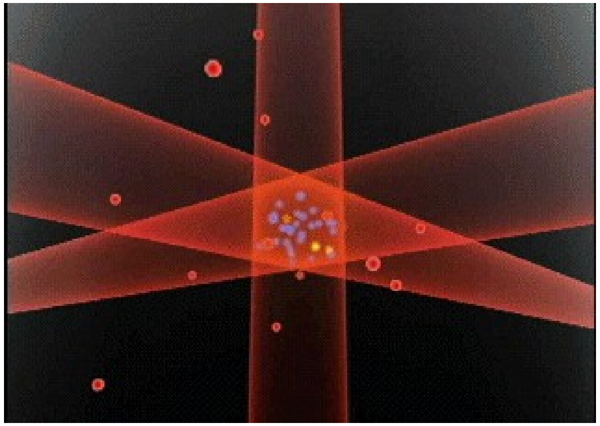
1. Introduction of topological defects in ultracold atoms
2. Kosterlitz-Thouless transition in spinor Bose gases
3. Non-abelian vortices

Feb. 21, 2016 “χ-Mag 2016 Symposium”

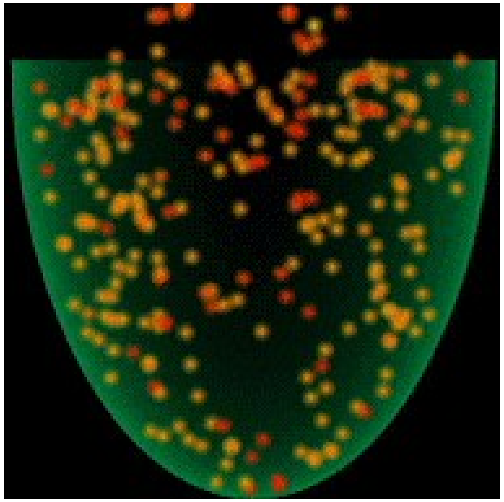
Ultracold atomic gases



Trapping atoms

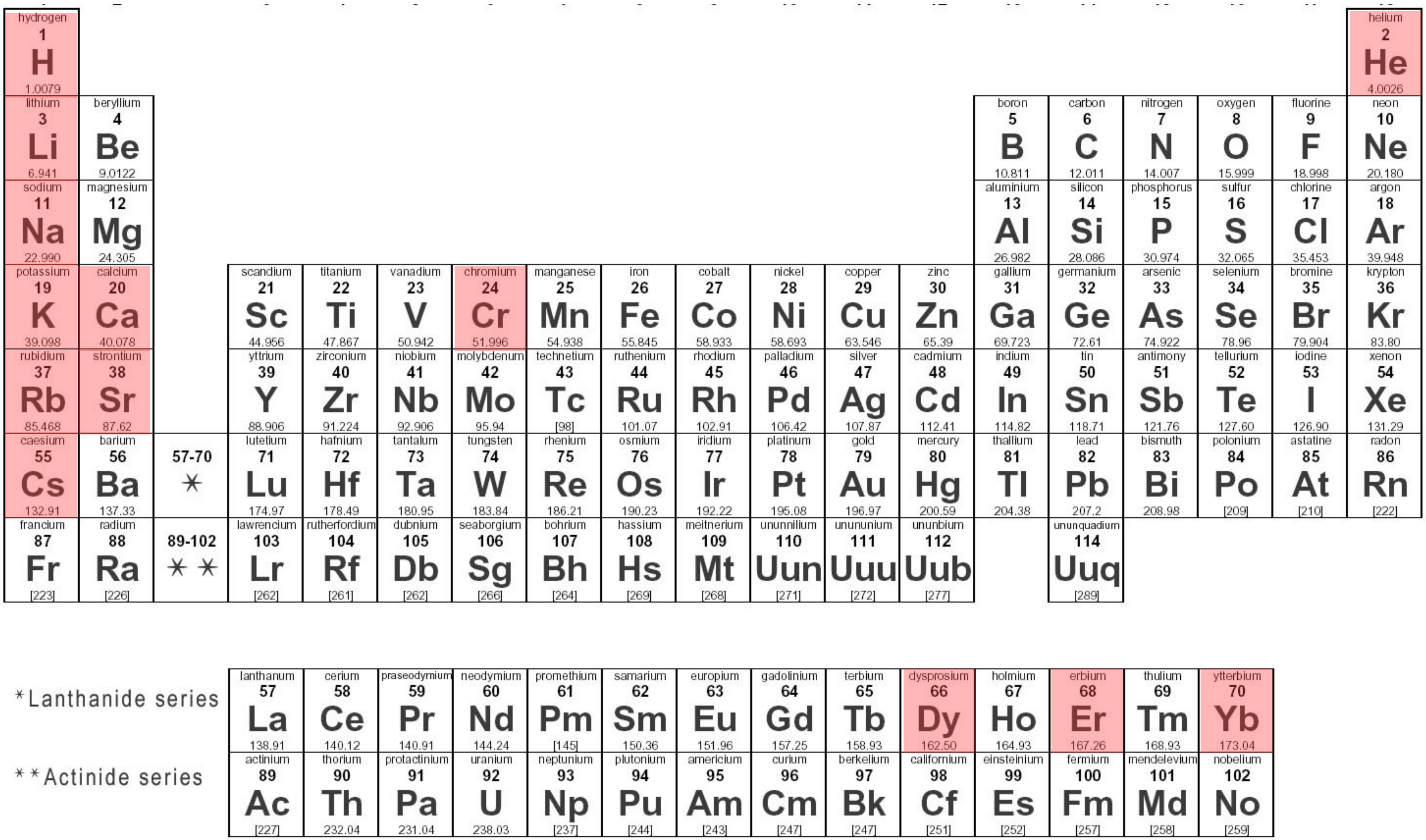


Laser cooling : ~µK



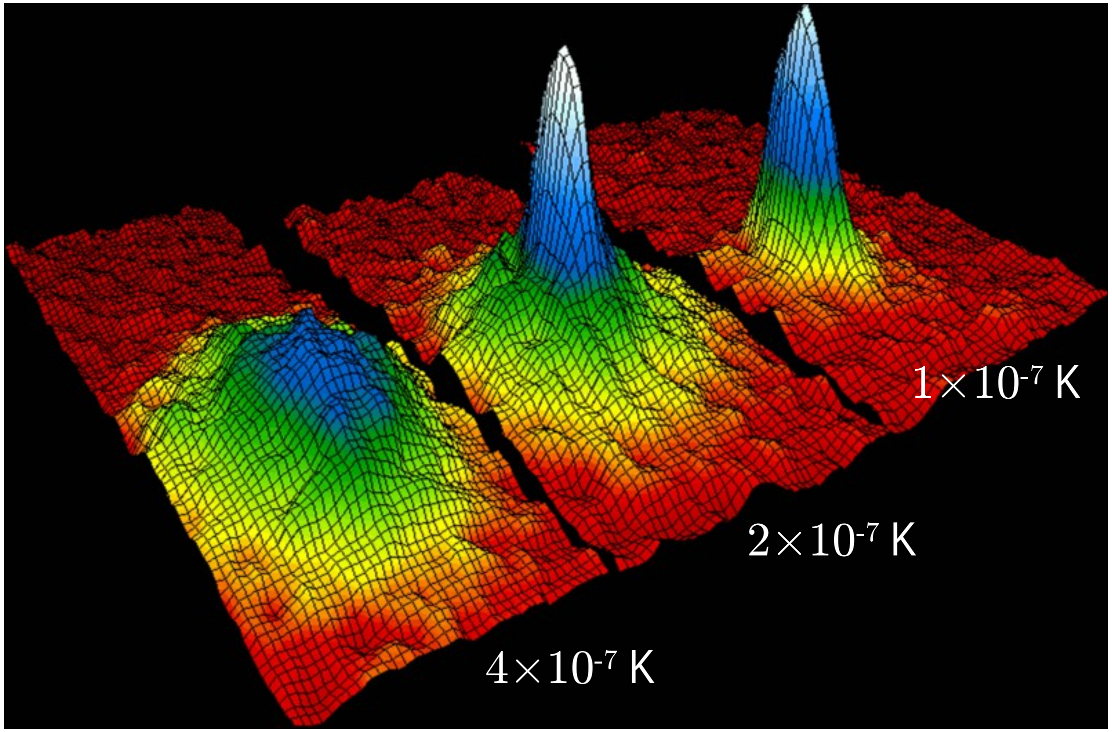
Evaporative cooling : ~nK

Ultracold atomic gases

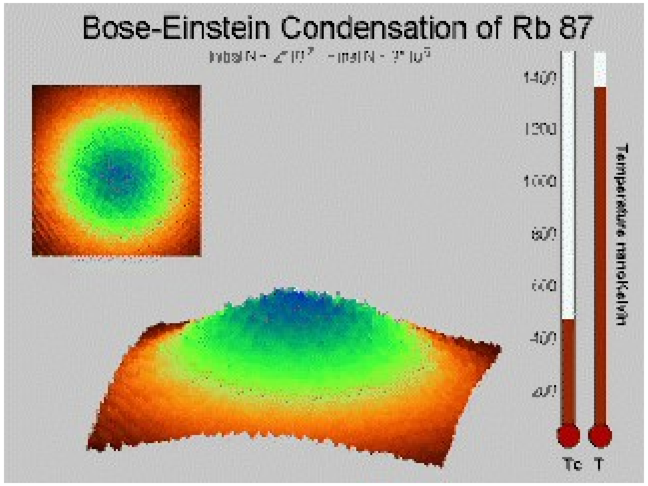


Bose-Einstein condensation

Experimental image for momentum distribution of Bose atoms (^{87}Rb)



Temperature and condensation



JILA, 1995

Similar images are obtained for cooper pairs and bounded pairs (molecules) of Fermi atoms

Comparison with condensed matter systems (electrons and superfluid helium)

	condensed matter	cold atoms
density (m^{-3})	10^{28-31} (dense)	10^{16-23} (dilute)
uniformity	uniform	not uniform (trapped)
microscopic dynamics	ps - ns (rapid)	μs - ms (slow)
conservation	open system	isolated system

- Perturbation theory is valid for quantitative comparison between microscopic theories and experiments
- Interaction strength is also controllable

Order parameter

Bosons

$$\langle \psi^\dagger(x)\psi(x')\rangle \xrightarrow{|x-x'|} \psi^*(x)\psi(x') \; : \text{ Bose-Einstein condensation}$$

Fermions

$$\langle \psi_\sigma^\dagger(x)\psi_{\sigma'}^\dagger(x)\psi_{\sigma'}(x')\psi_\sigma(x')\rangle \xrightarrow{|x-x'|} \Delta^*(x)\Delta(x') \; : \text{ Cooper pairing}$$

$$\psi(x) = |\psi(x)|e^{i\phi(x)} \; : \text{ spontaneous symmetry breaking}$$

for $U(1)$-phase shift

$$\rho(x) = |\psi(x)|^2 \; : \text{ particle density}$$

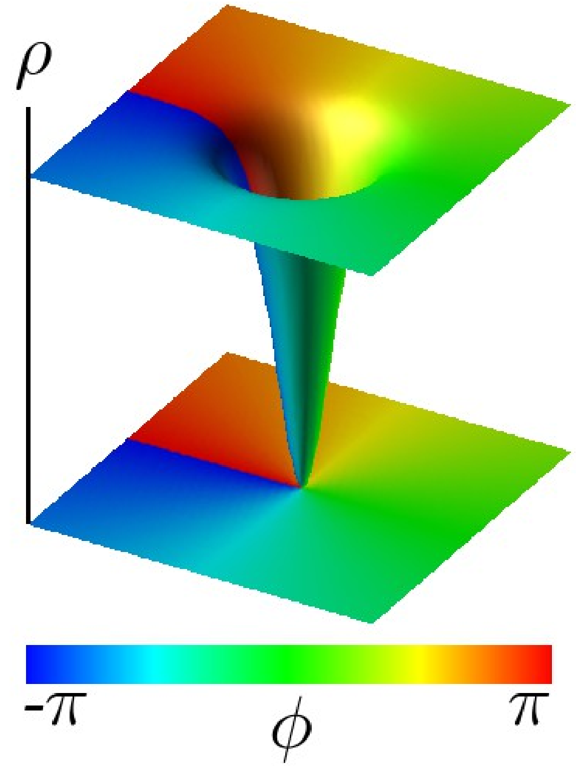
$$\boldsymbol{v}(x) = \frac{\hbar}{M}\nabla\phi(x) \; : \text{ superfluid velocity}$$

Quantized vortex

$\psi(x) = |\psi(x)|e^{i\phi(x)}$

$\rho(x) = |\psi(x)|^2$: particle density

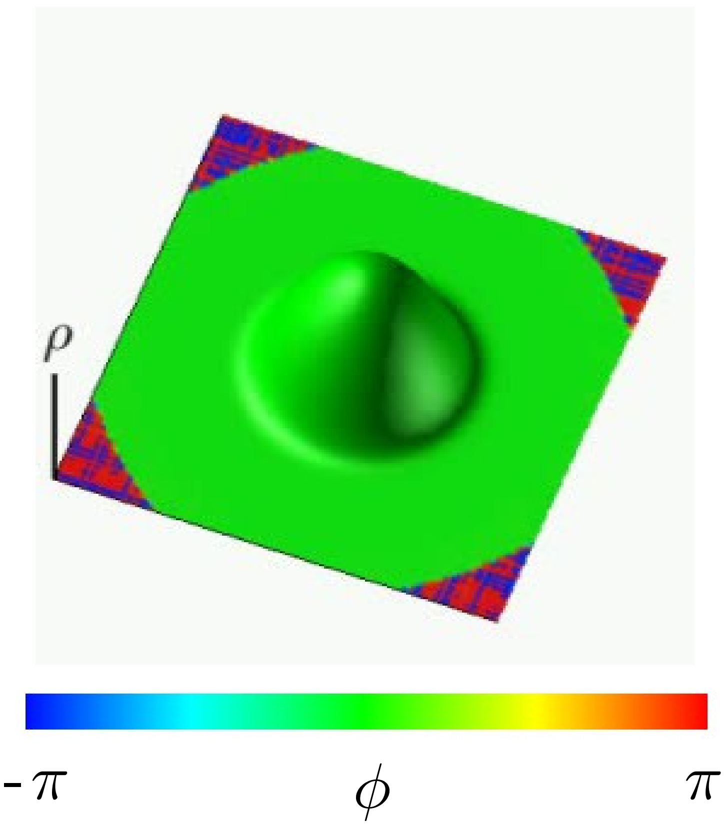
$\boldsymbol{v}(x) = \dfrac{\hbar}{M}\nabla\phi(x)$: superfluid velocity



Vortex appears as topological defect for $U(1)$ phase shift (density $\rho(x)$ vanishes)

Vortex lattice formation

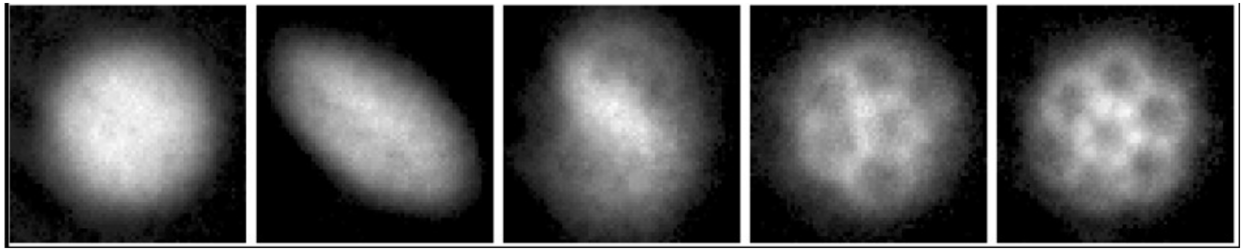
Simulation of the non-linear Schrödinger equation

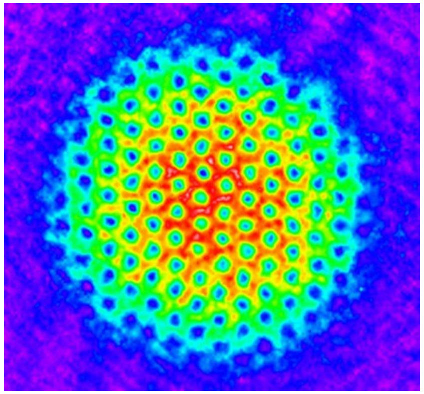


- Phase vortices appear in the outside of the condensate
- Condensate is squashed
- Vortices excite the surface of the condensate
- Vortices enter the condensate
- Triangular vortex lattice is formed

Vortex lattice formation

Vortex lattice in ^{87}Rb BEC



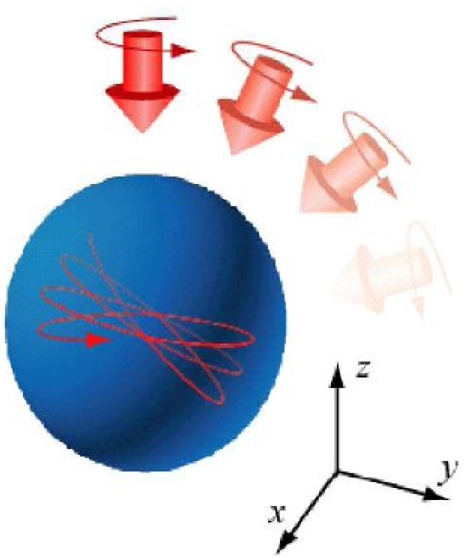


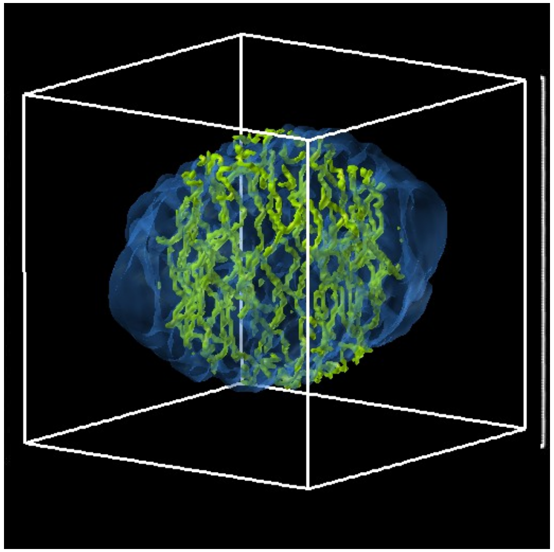
K. W. Madison et al. PRL **86**, 4443 (2001)

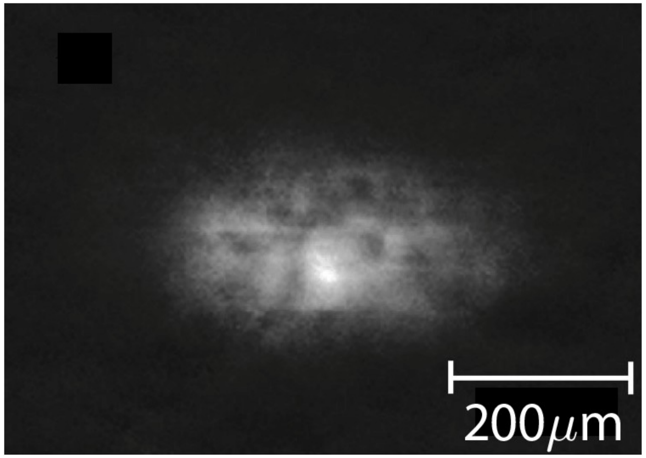
Quantum turbulence

Precession rotation of condensation

PRA **76**, 045603 (2007)







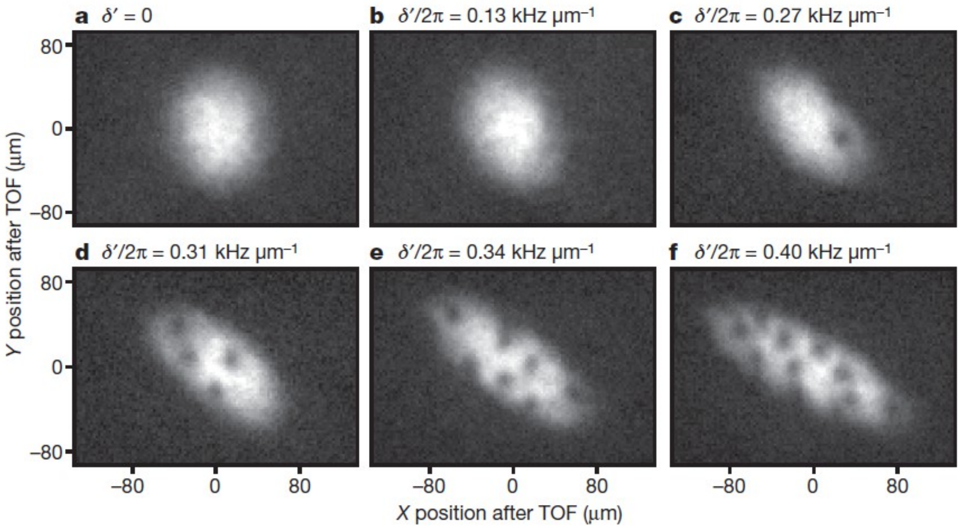
PRL **103**, 045301 (2009)

Vortices are not lattice but tangled (quantum turbulence)

Other method to excite vortices

Artificial magnetic (gauge) field

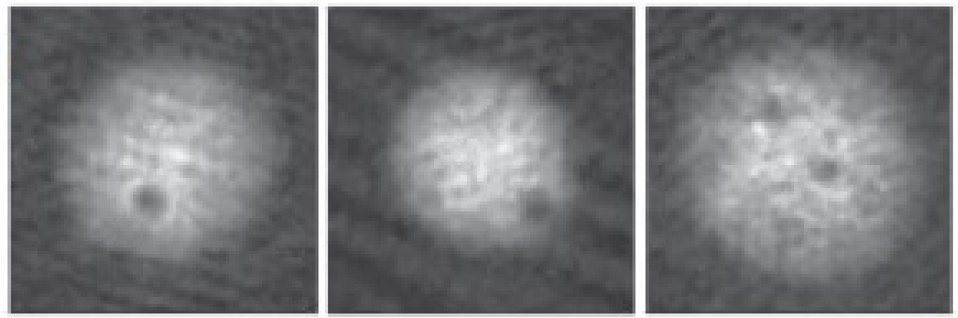
Nature **462**, 628 (2009)



No centrifugal force
→Large number of vortices can be excited
→Toward quantum Hall state

rapid temerature quench

Nature **455**, 948 (2008)



- Phase imprinting
- Interaction with Laguerre-Gaussian beam

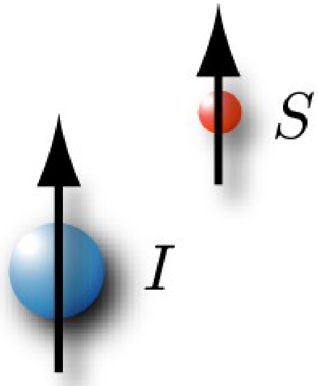
Internal degrees of freedom

- Multi-component condensate
- Spinor condensate

type of particle trap	spin degrees of freedom	type of condensate
magnetic trap	frozed	scalar condensate
optical trap	alived	spinor condensate

Hyperfine coupling of nuclear and electron spins :

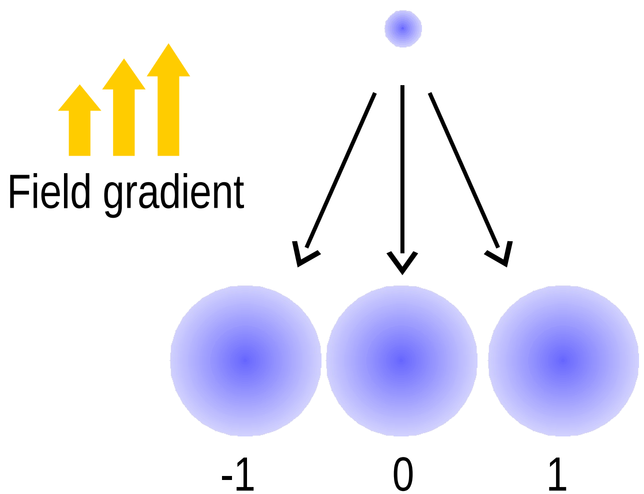
$F = I$ (nuclear spin) $+$ S (electron spin) $+$ L (electron orbital)



^{87}Rb, ^{23}Na ^{7}Li, ^{41}K	$F = 1, 2$
^{85}Rb	$F = 2, 3$
^{133}Cs	$F = 3, 4$
^{52}Cr	$S = 3, I = 0$

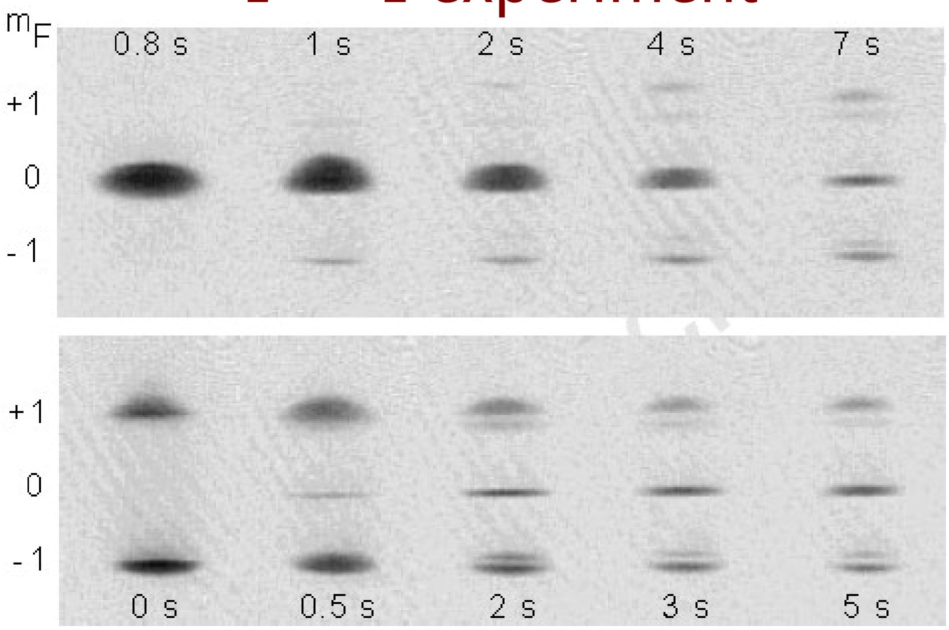
Spinor condensate

^{87}Rb $(I = 3/2,\ S = 1/2,\ L = 0) \rightarrow F = 1,\ 2$



Stern-Gerlach experiment

$F = 1$ experiment



J. Stenger et al. Nature **396**, 345 (1998)

Mean-field theory for spinor condensates

order parameter : $\psi = \begin{pmatrix} \psi_F & \psi_{F-1} & \cdots & \psi_{-F} \end{pmatrix}$

spin-1 : $H = H_0 + \frac{2\pi\hbar^2}{M} \int d\boldsymbol{x} \left(a_0 \rho^2 + a_1 \boldsymbol{S}^2 \right)$

spin-2 : $H = H_0 + \frac{2\pi\hbar^2}{M} \int d\boldsymbol{x} \left(a_0 \rho^2 + a_1 \boldsymbol{S}^2 + a_2 |A_{20}|^2 \right)$

1-body part

$$H_0 = \int d\boldsymbol{x} \left[\sum_{m=-F}^{F} \left\{ \frac{\hbar^2}{2M} |\nabla \psi_m|^2 + (q_1 m + q_2 m^2 + V) |\psi_m|^2 \right\} \right]$$

q_1 : linear-Zeeman coefficient

q_2 : quadratic-Zeeman coefficient

V : 1-body external trapping potential

Mean-field theory for spinor condensates

order parameter : $\psi = \begin{pmatrix} \psi_F & \psi_{F-1} & \cdots & \psi_{-F} \end{pmatrix}$

spin-1 : $H = H_0 + \dfrac{2\pi\hbar^2}{M}\displaystyle\int d\boldsymbol{x}\,(a_0\rho^2 + a_1\boldsymbol{S}^2)$

spin-2 : $H = H_0 + \dfrac{2\pi\hbar^2}{M}\displaystyle\int d\boldsymbol{x}\,(a_0\rho^2 + a_1\boldsymbol{S}^2 + a_2|A_{20}|^2)$

$a_{0,1,2}$: s-wave scattering length

$\rho = \psi^\dagger\psi$: density

$\boldsymbol{S} = \psi^\dagger\hat{\boldsymbol{S}}\psi$: spin density

$A_{20} = \displaystyle\sum_{m=-F}^{F}(-1)^m\psi_m\psi_{-m}$: singlet-pair amplitude
(expectation value of unitary operator)

Ground states for spin-1 condensates without Zeeman field

$$H = \int d\boldsymbol{x} \sum_{m=-1}^{1} \left(\frac{\hbar^2}{2M} |\nabla \psi_m|^2 + V |\psi_m|^2 \right) + \frac{1}{2} (g_0 \rho^2 + g_1 \boldsymbol{S}^2)$$

$$g_{0,1,2} = \frac{4\pi\hbar^2 a_{0,1,2}}{M}$$

$g_1 > 0$: polar (^{23}Na BEC)

$$e^{i\phi} e^{-i\boldsymbol{n}\cdot\hat{\boldsymbol{S}}\alpha} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$|\boldsymbol{S}| = 0$$

$g_1 < 0$: Ferromagnetic (^{87}Rb BEC)

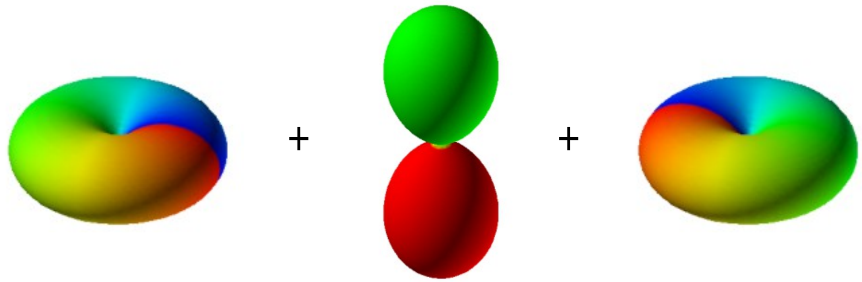
$$e^{i\phi} e^{-i\boldsymbol{n}\cdot\hat{\boldsymbol{S}}\alpha} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$|\boldsymbol{S}| = 1$$

Representation with spherical-harmonic function

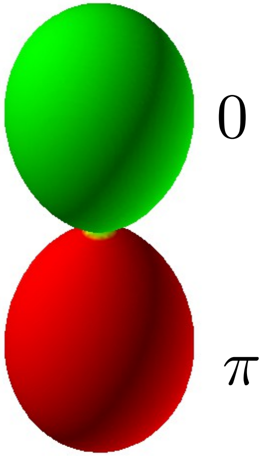
$Y_{1,1}$ $Y_{1,0}$ $Y_{1,-1}$

$$\sum_{m=-1}^{1} \psi_m Y_{1,m} =$$



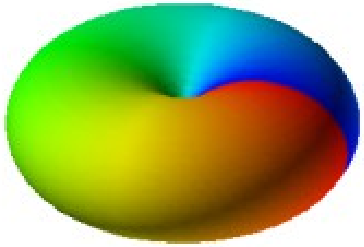
$\cos\theta$

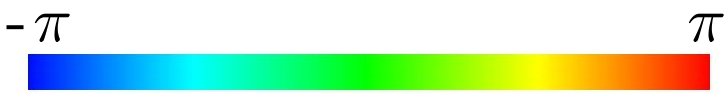
Polar (dumbbell)



$-e^{i\varphi}\sin\theta$

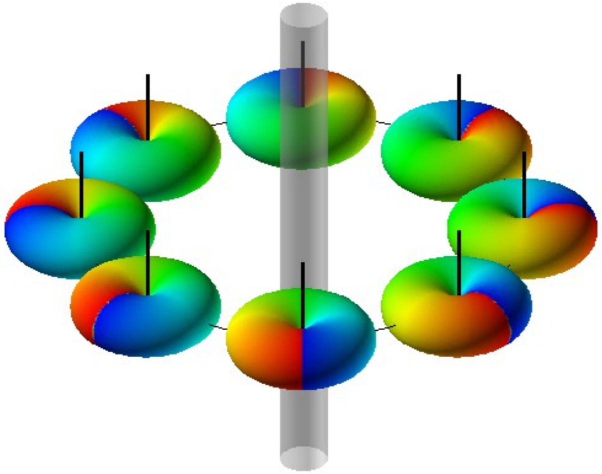
Ferromagnetic (torus)





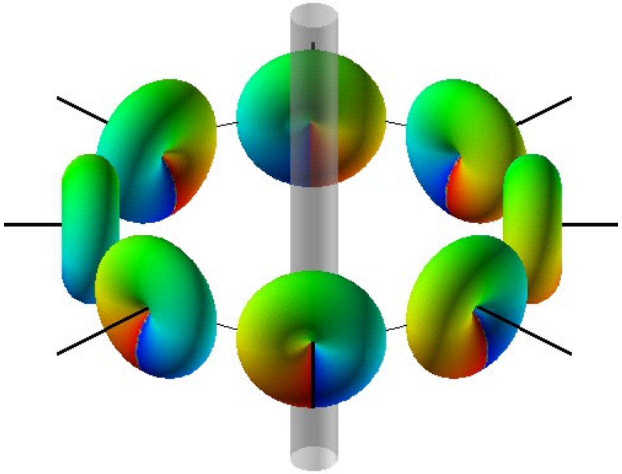
Vortex in Ferromagnetic phase

Phase vortex



Continuously transformed

Spin vortex



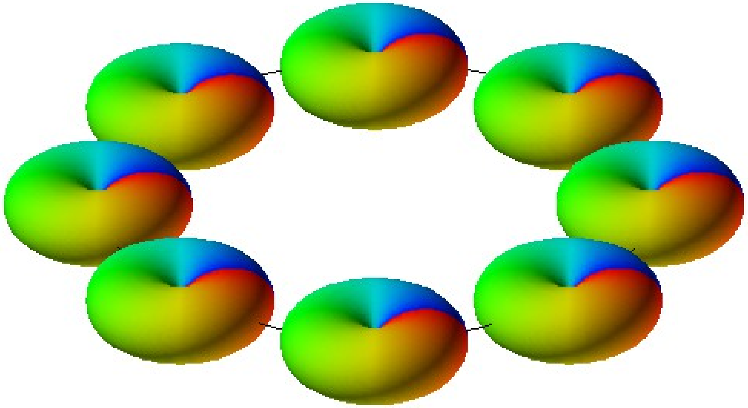
$$\psi = \begin{pmatrix} e^{i\phi} \\ 0 \\ 0 \end{pmatrix}$$

$$\psi = \frac{1}{2}\begin{pmatrix} 1 \\ \sqrt{2}e^{i\phi} \\ 1 \end{pmatrix}$$

Vortex in Ferromagnetic phase

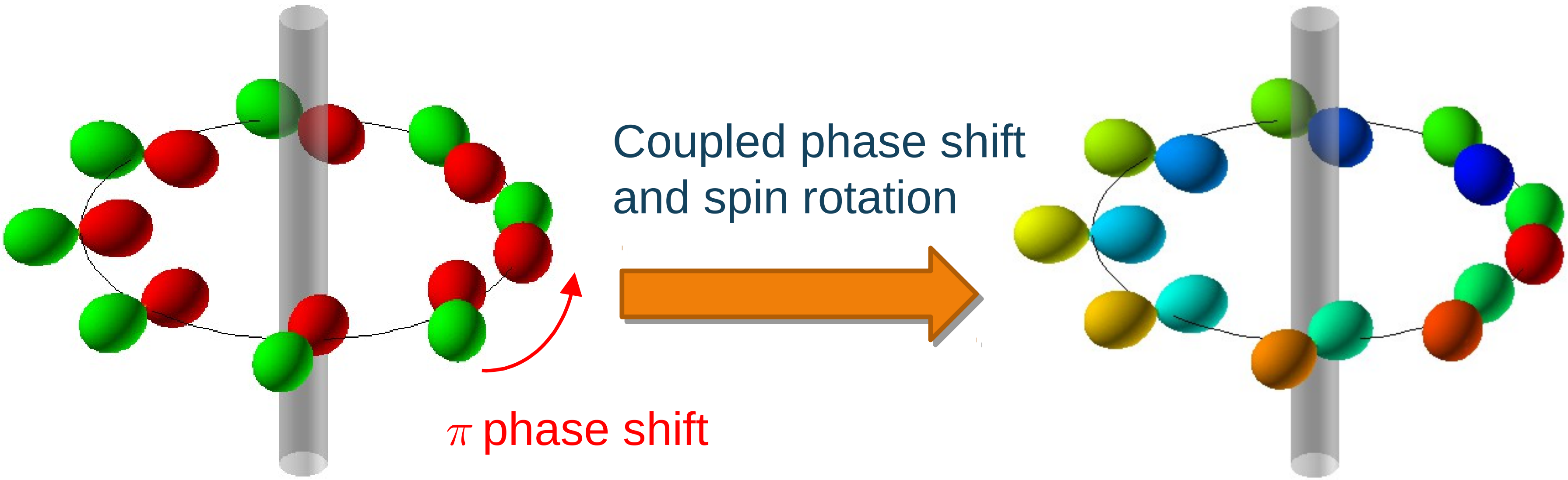
Double winding vortex is continuously transformed to uniform state (phase imprinting)

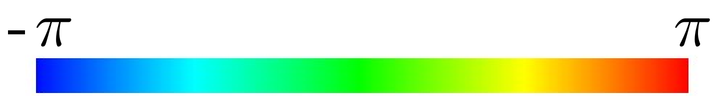
$\rightarrow \mathbb{Z}_2$ vortex



PRA **89**, 190403 (2002)
PRL **93**, 160406 (2004)

Vortex in polar phase



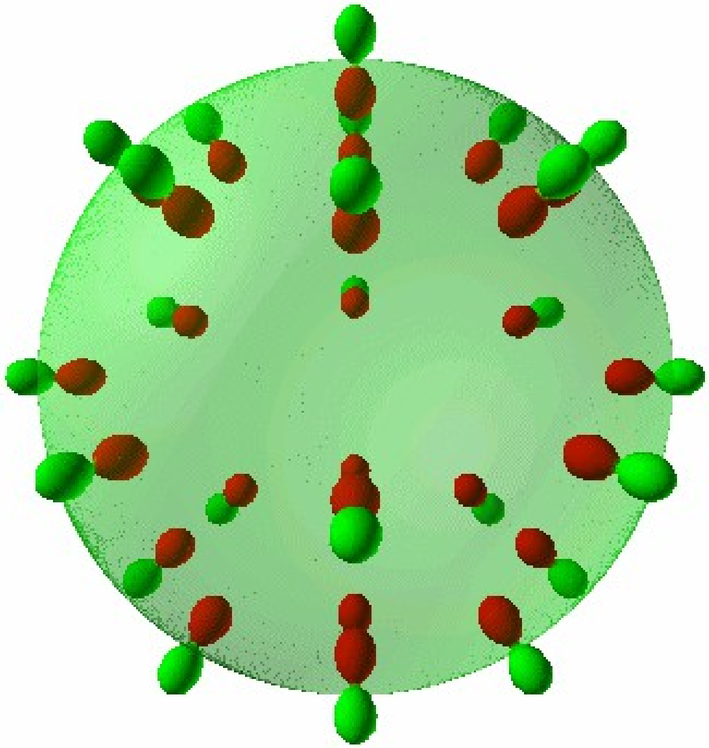


$$\psi = \frac{1}{\sqrt{2}} \begin{pmatrix} -e^{i\phi} \\ 0 \\ 1 \end{pmatrix}$$

Mass circulation around the vortex
is half of that of scalar condensate
(half-quantized vortex)

Monopole in polar phase

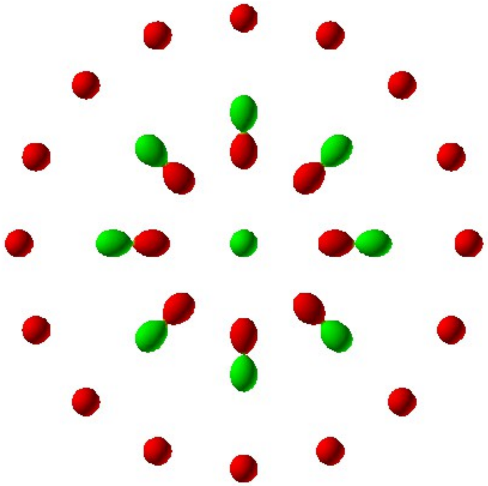
't Hooft-Polyakov monopole
in Polar phase



$$\psi = \frac{1}{\sqrt{2}} \begin{pmatrix} -e^{-i\phi} \sin\theta \\ \cos\theta \\ e^{i\phi} \sin\theta \end{pmatrix}$$

Monopole cannot exist in Ferromagnetic phase

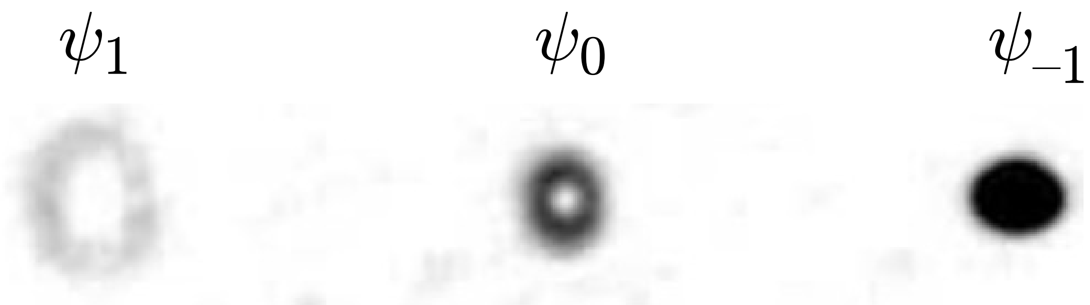
2D skyrmion in polar phase



$$\psi = \frac{1}{\sqrt{2}}\begin{pmatrix} -e^{-i\phi}\sin f(r) \\ \cos f(r) \\ e^{i\phi}\sin f(r) \end{pmatrix}$$

$$f(0) = 0 \qquad f(r\to\infty) = \pi$$

Experimentally created by Laguerre-Gaussian beam



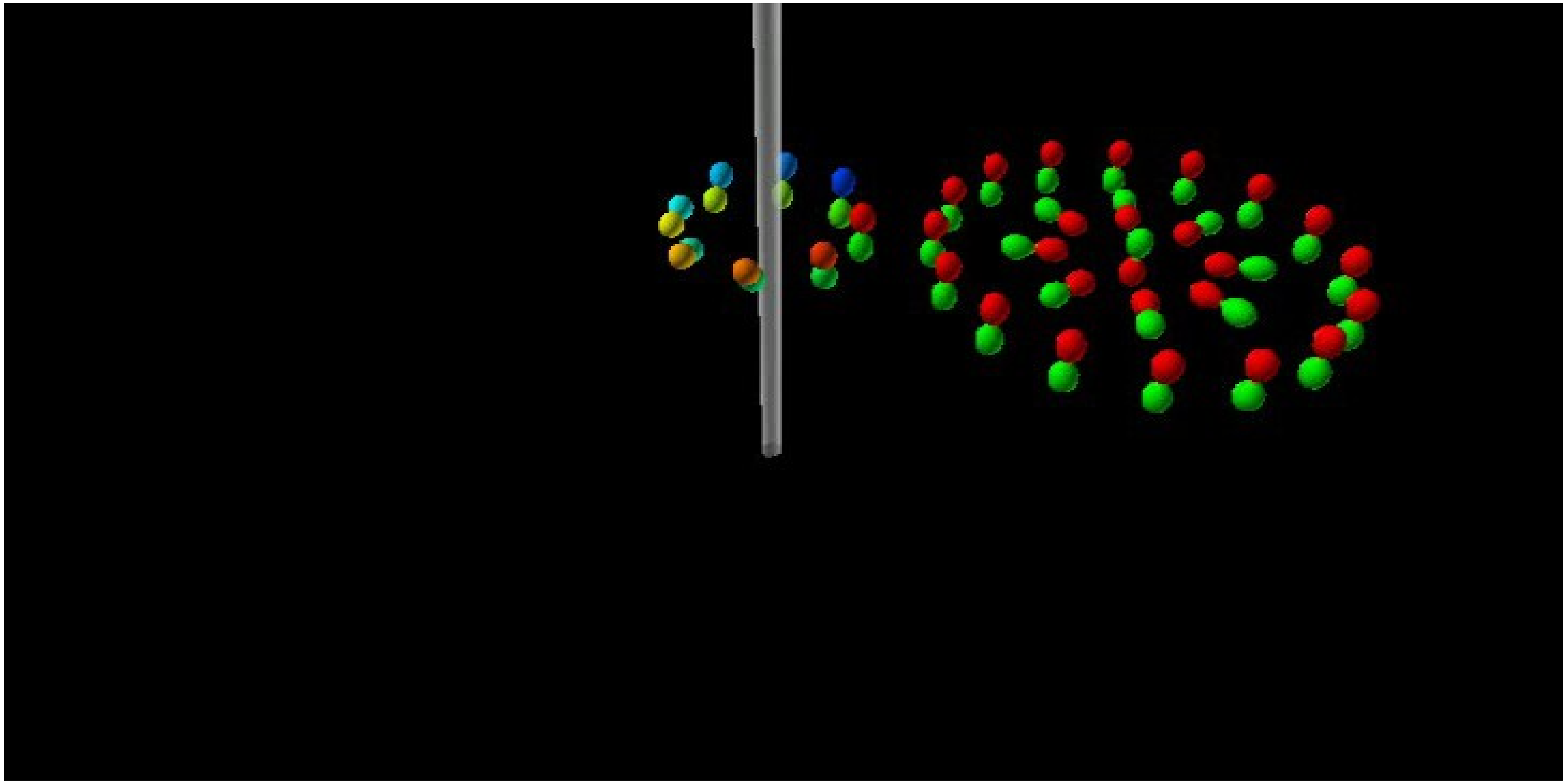
Phys. Rev. Lett. **103** 50401, (2009)

Vortex-skyrmion combined state (metastable)

$$\psi = \frac{1}{\sqrt{2}}\begin{pmatrix} -e^{-2i\phi}\sin f(r) \\ e^{-i\phi}\cos f(r) \\ \sin f(r) \end{pmatrix}$$

Interaction between 2D skyrmion and half-quantized vortex

Skyrmion rotates around the half-quantized vortex



S. Kobayashi, et al. Nucl. Phys. B **856**, 577 (2012)

Interaction between 2D skyrmion and half-quantized vortex

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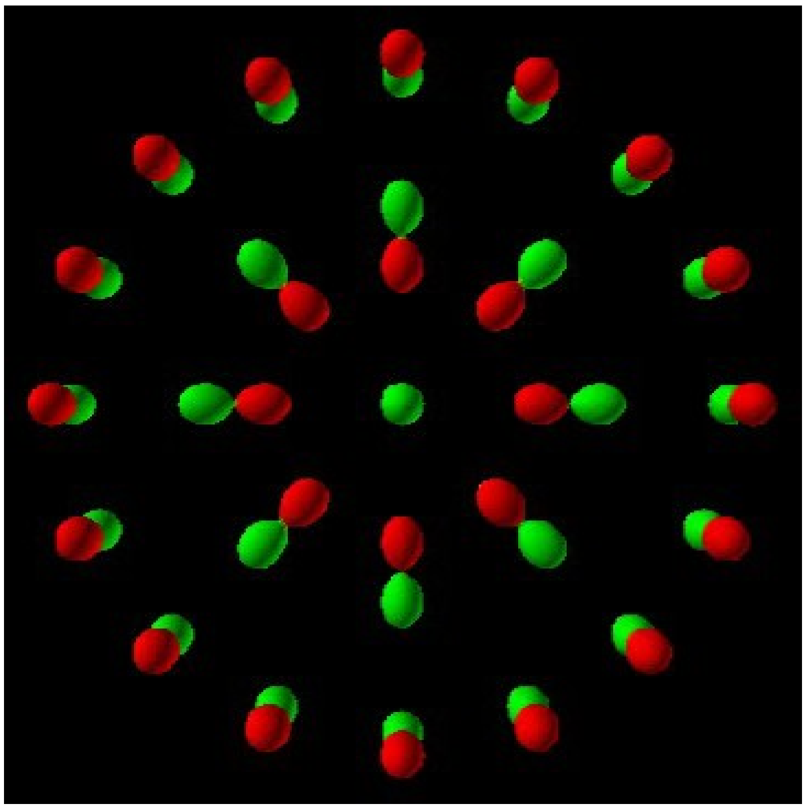


Skyrmion charge is inverted when skyrmion rotates around the half-quantized vortex

$\rightarrow \mathbb{Z}_2$ skyrmion

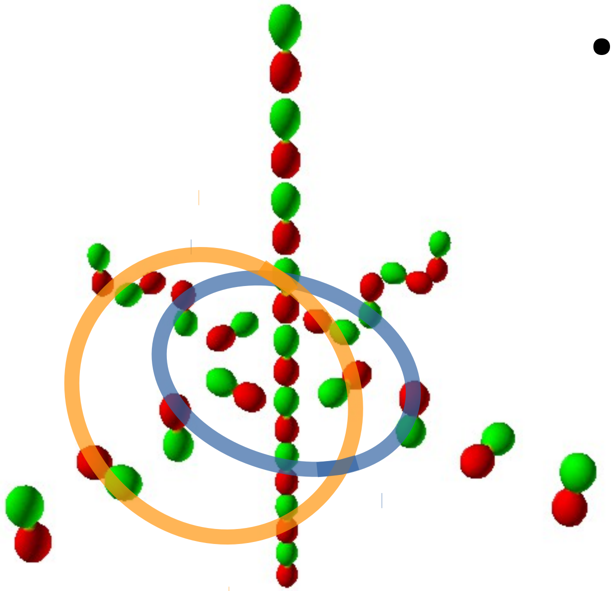
S. Kobayashi, et al. Nucl. Phys. B **856**, 577 (2012)

Short summary

Spinor condensates induce a variety of topological defects

	π_1	π_2	π_3
Polar	half-quantized vortex	monopole 2D $\mathbb{Z}_2$ skyrmion	Hopfion
Ferro	$\mathbb{Z}_2$ vortex		3D skyrmion

Hopfion

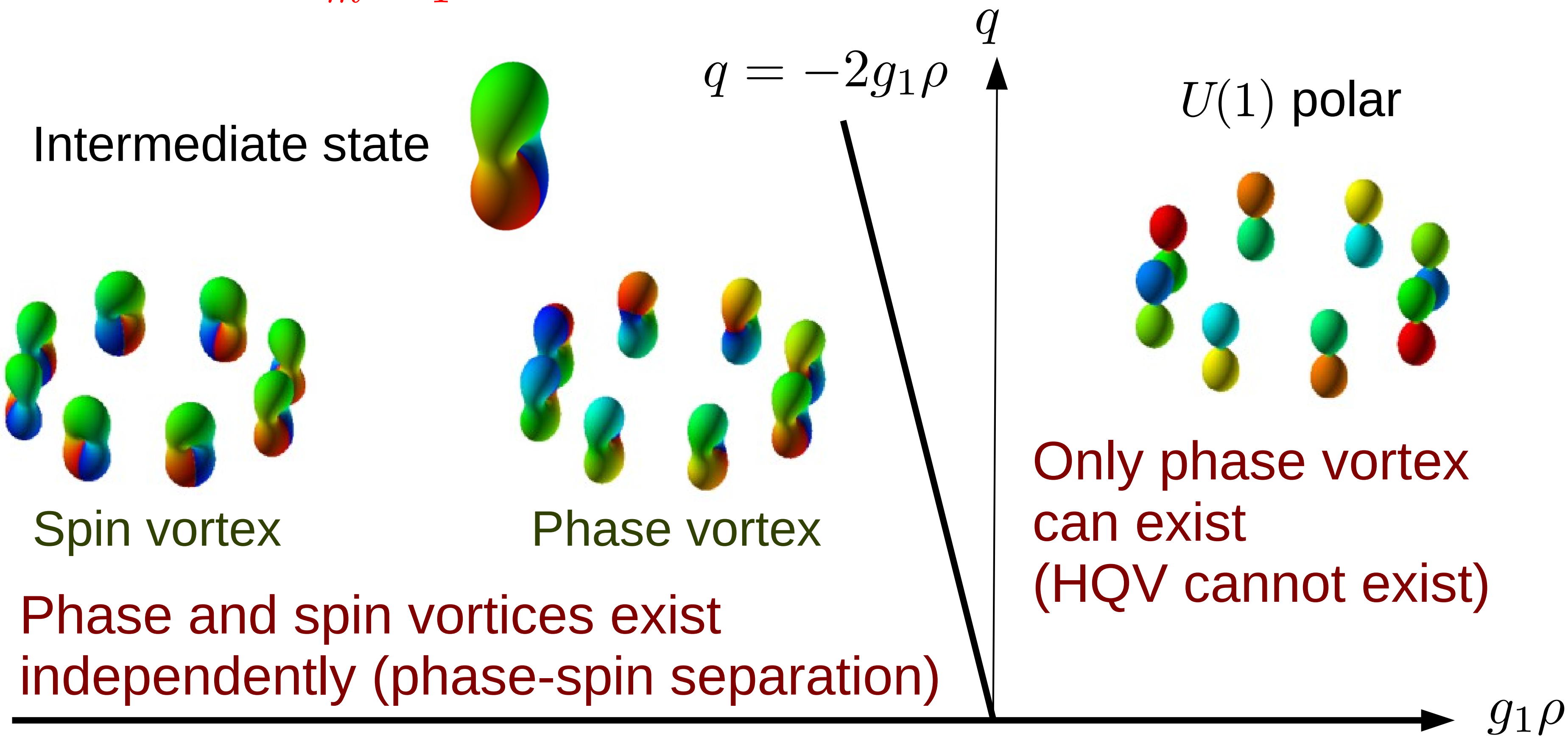


- Orbit of the same spin state has 1D ring
- Arbitrary 2 rings form the Hopf link

Phys. Rev. Lett. **100** 180403, (2008)

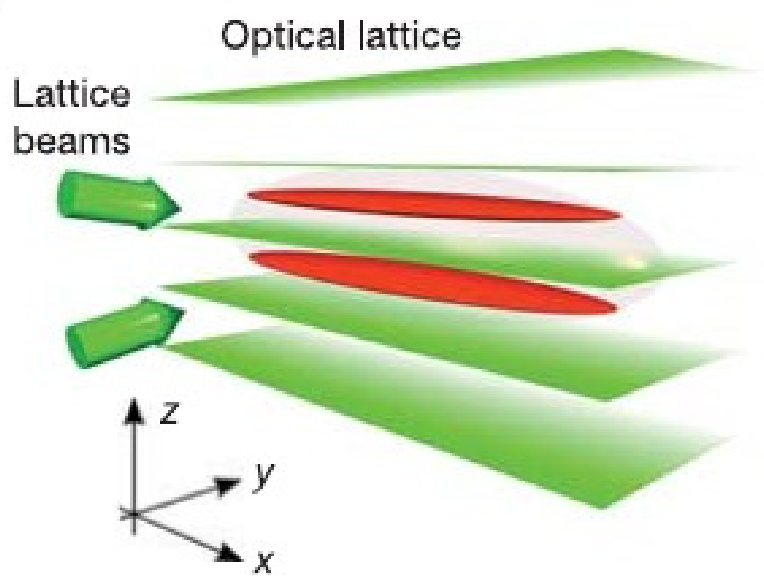
Spin-1 condensates under quadratic Zeeman field

$$H=\int d\boldsymbol{x}\sum_{m=-1}^{1}\left(\frac{\hbar^2}{2M}|\nabla\psi_m|^2+q_2 m^2|\psi_m|^2\right)+\frac{1}{2}(g_0\rho^2+g_1\boldsymbol{S}^2)$$



Kosterlitz-Thouless transition in 2D system

Relationship between properties of vortex and KT transition?



2D system can be created by using optical standing wave

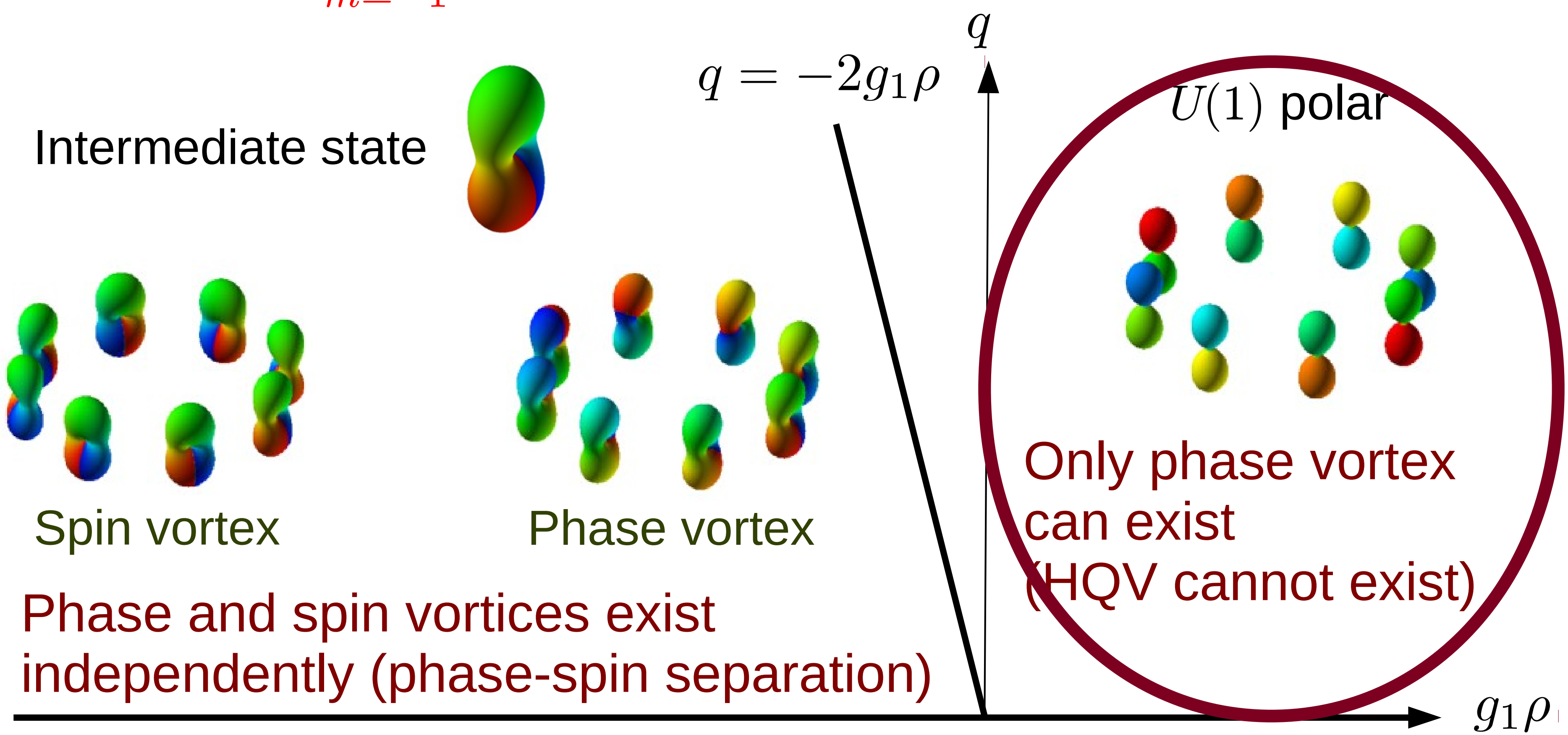
Monte-Carlo simulation

$$\langle f \rangle = \left(\prod_{m=-1}^{1} \int D\psi_m \, D\psi_m^* \right) \times f[\psi_m, \psi_m^*] e^{-H/T}$$

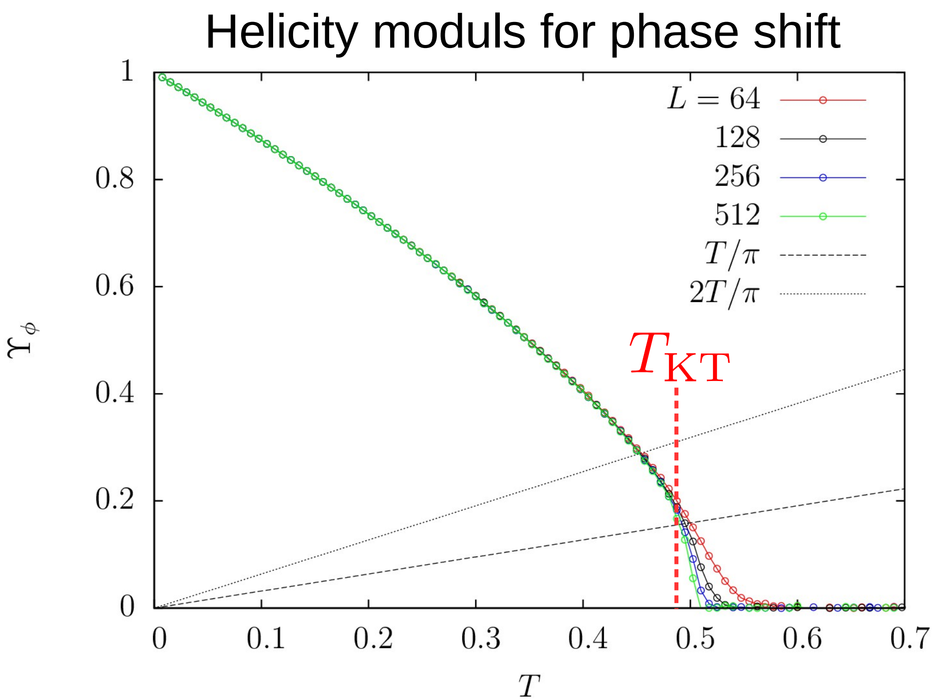
Important value :
helicity modules for phase shift and spin rotation

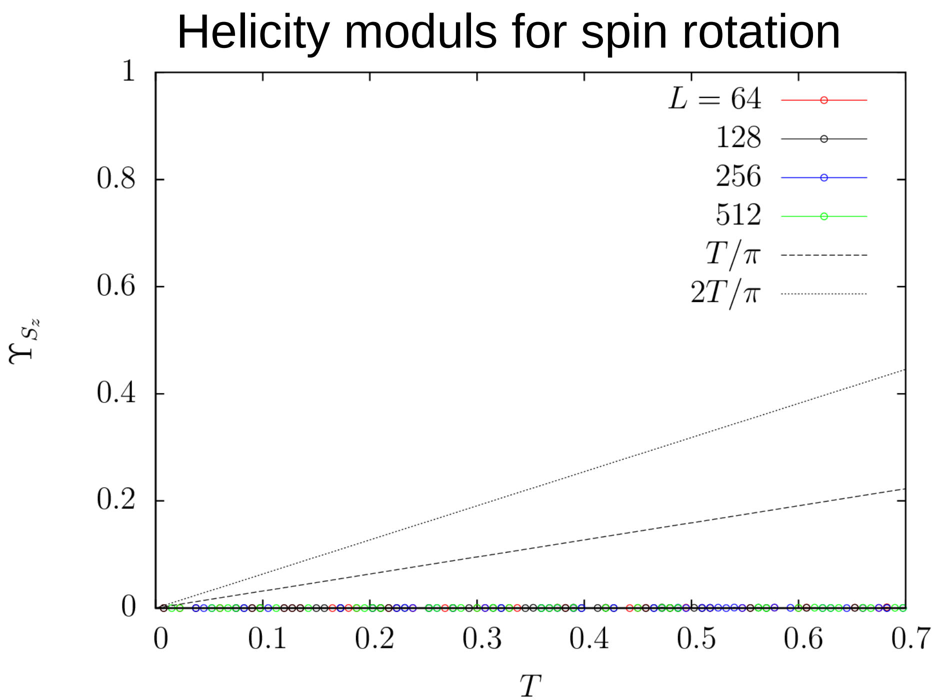
Kosterlitz-Thouless transition in 2D system

$$H = \int d\boldsymbol{x} \sum_{m=-1}^{1} \left(\frac{\hbar^2}{2M} |\nabla \psi_m|^2 + q_2 m^2 |\psi_m|^2 \right) + \frac{1}{2}(g_0 \rho^2 + g_1 \boldsymbol{S}^2)$$



Kosterlitz-Thouless transition in $U(1)$ polar state





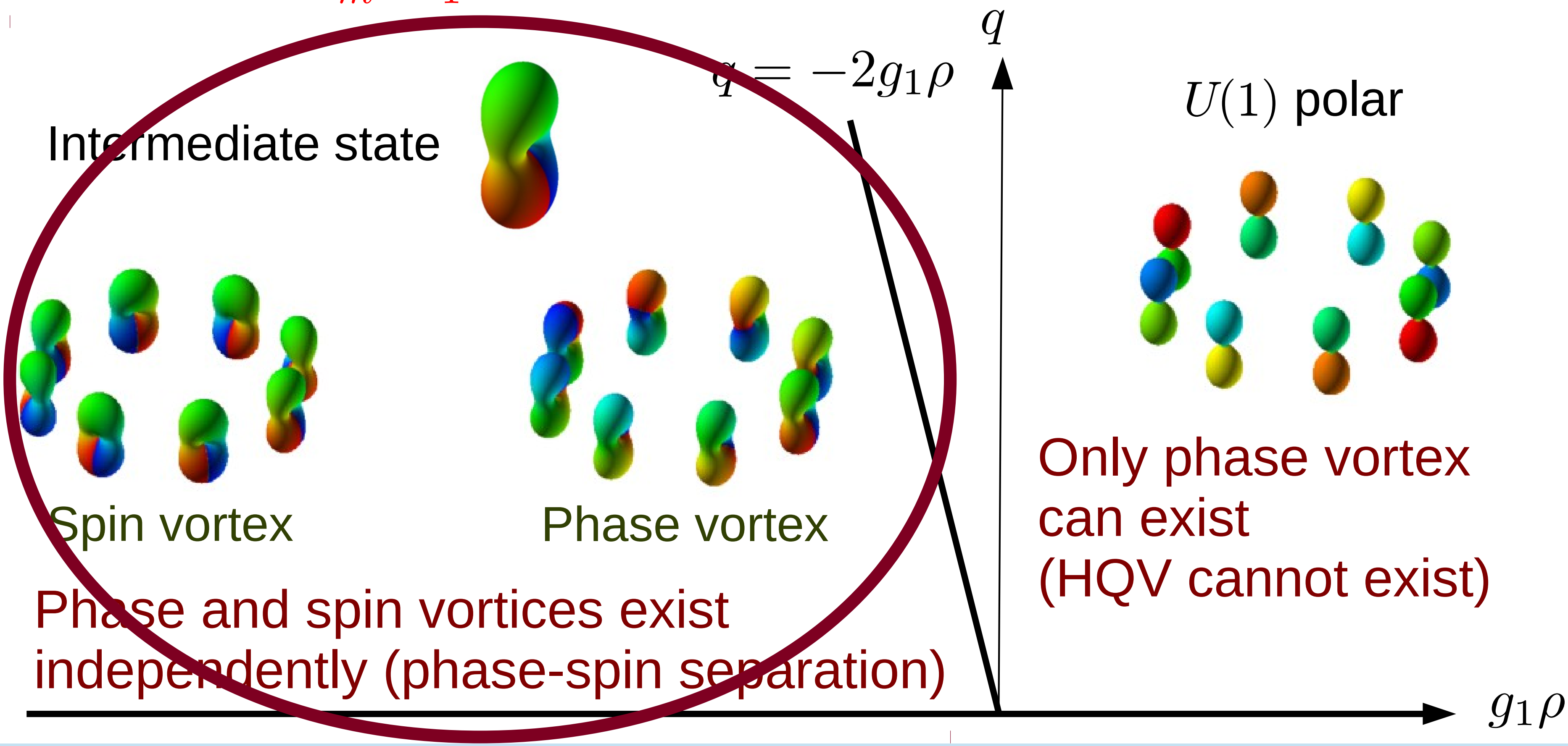
T_{KT} is obtained from Binder cumulant

KT transition is completely same as scale condensate :

$\Delta\Upsilon_\phi / T_{\mathrm{KT}} \simeq 1/\pi$

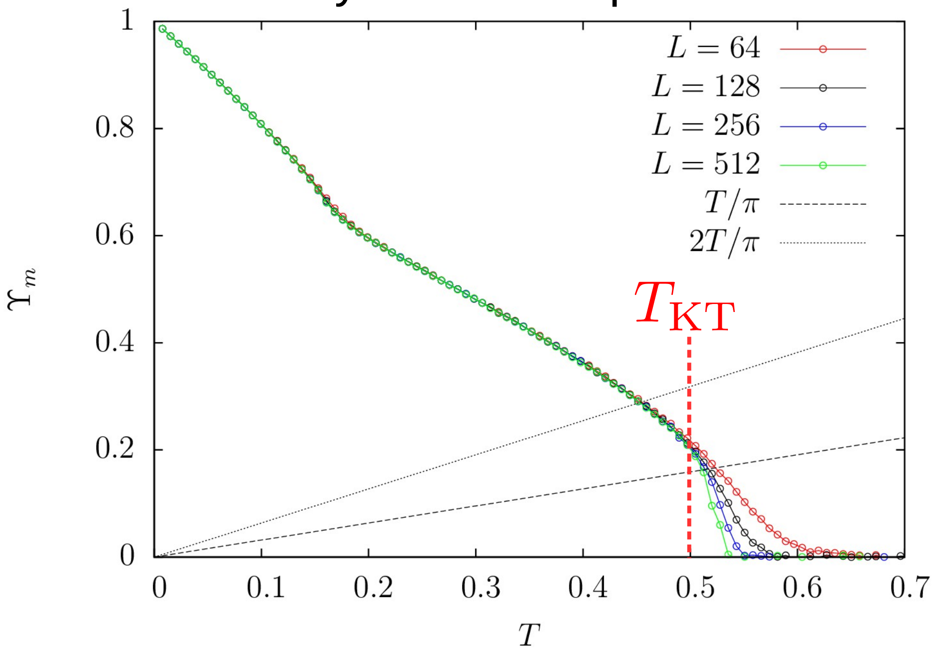
Kosterlitz-Thouless transition in 2D system

$$H=\int d\boldsymbol{x}\ \sum_{m=-1}^{1}\left(\frac{\hbar^2}{2M}|\nabla\psi_m|^2+q_2m^2|\psi_m|^2\right)+\frac{1}{2}(g_0\rho^2+g_1\boldsymbol{S}^2)$$

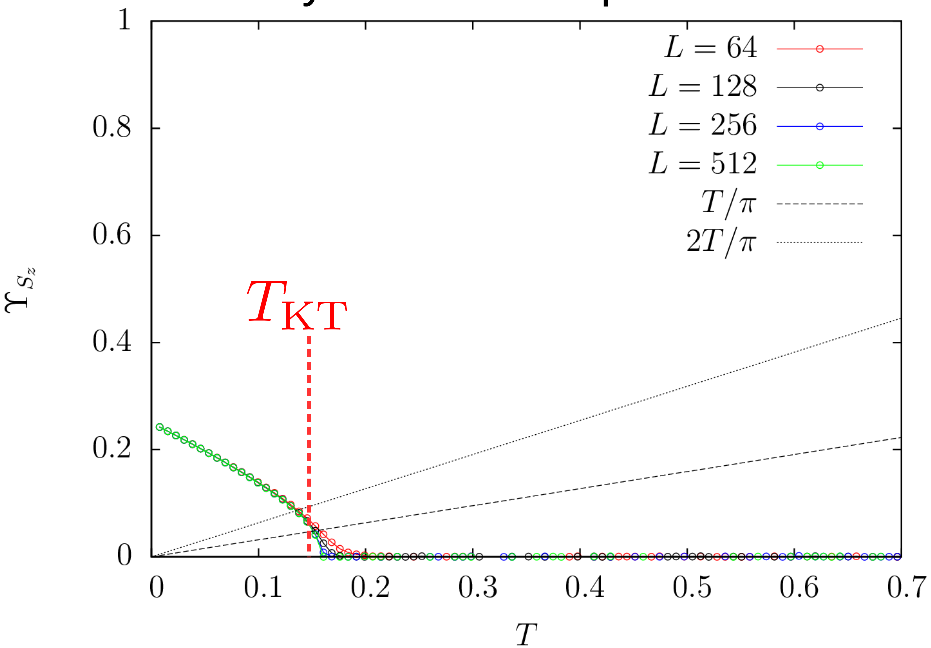


Kosterlitz-Thouless transition in intermediate state

Helicity moduls for phase shift



Helicity moduls for spin rotation



2-step KT transition occurs at different temperatures for phase shift and spin rotation due to phase-spin separation

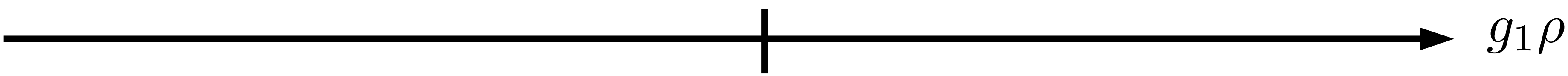
Universal jump is the same as that for the scalr condensate :
$\Delta\Upsilon_\phi / T_{\mathrm{KT}} \simeq 1/\pi$

For zero quadratic Zeeman field

$$H=\int d\boldsymbol{x}\sum_{m=-1}^{1}\frac{\hbar^2}{2M}|\nabla\psi_m|^2+\frac{1}{2}(g_0\rho^2+g_1\boldsymbol{S}^2)$$

Ferromagnetic : $\mathbb{Z}_2$ vortex

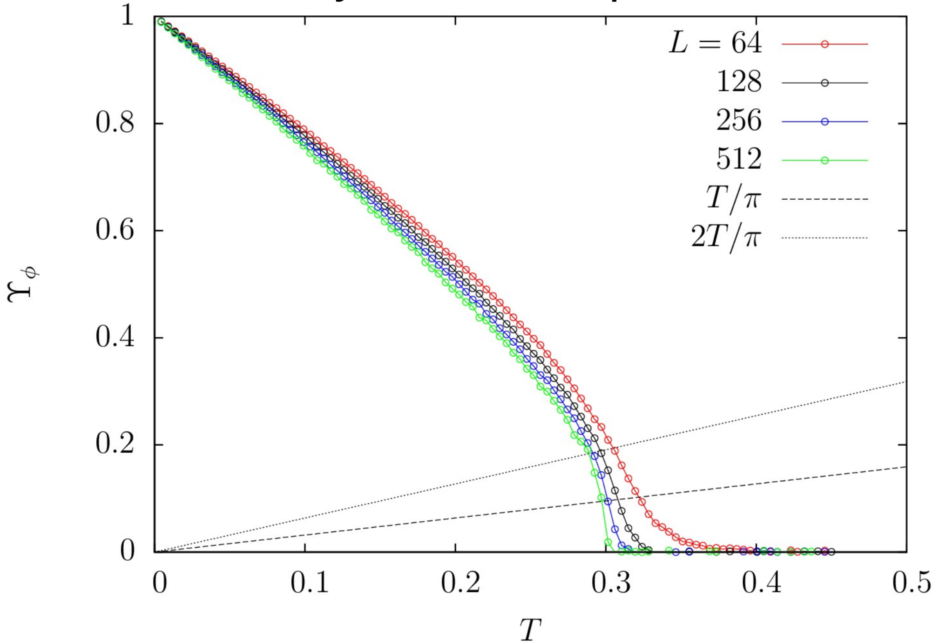
Polar : half-quantized vortex



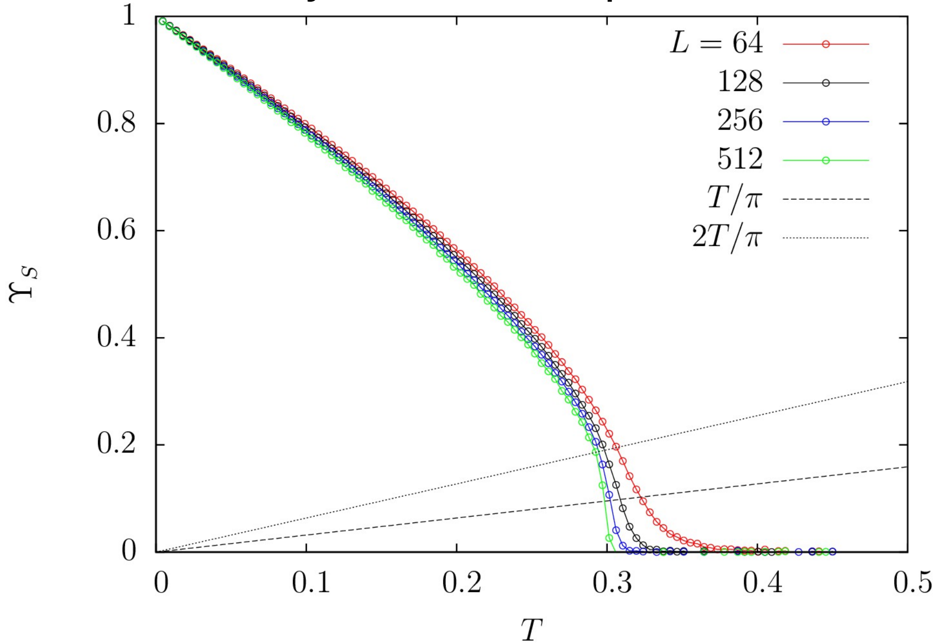
Properties of vortices changes discretely between $q>0$ and $q=0$

Kosterlitz-Thouless transition in ferromagnetic state

Helicity moduls for phase shift



Helicity moduls for spin rotation

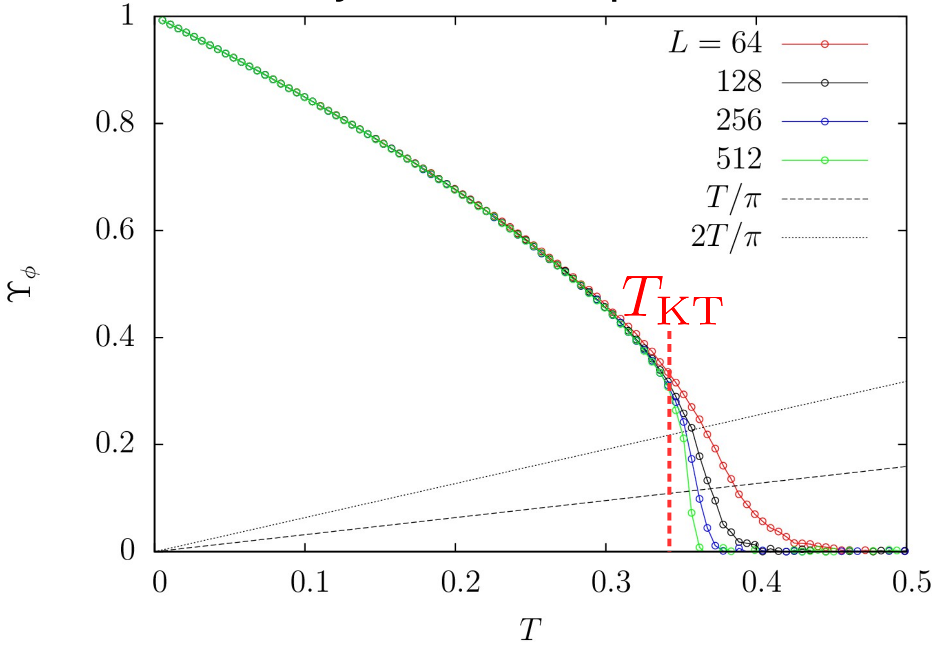


Due to strong finite-size effect, we cannot judge whether KT transition occurs in the thermodynamic limit

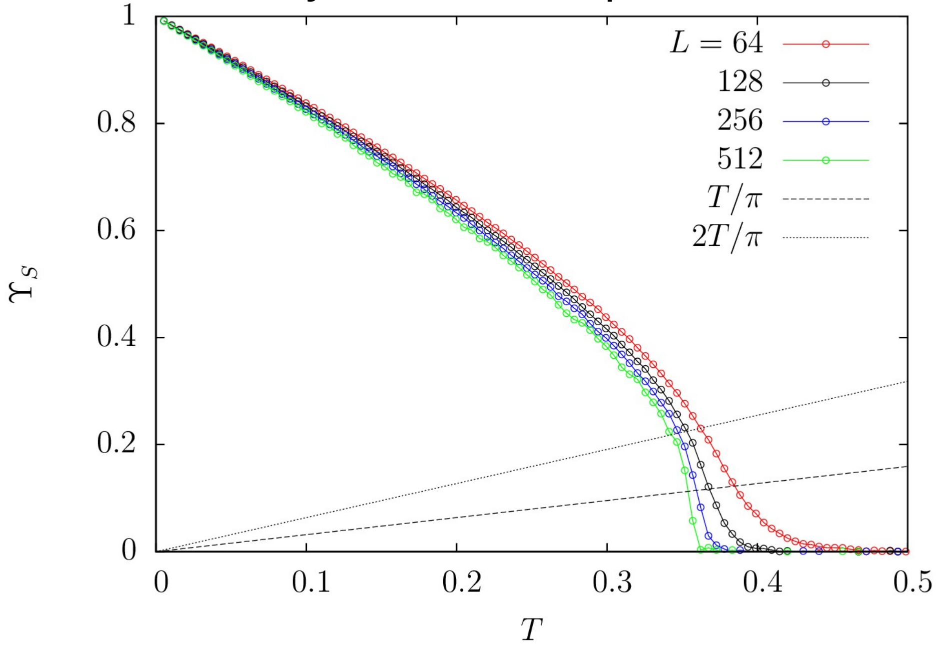
KT transition for $\mathbb{Z}_2$ vortex has not been solved yet (2D antiferromagnetic Heisenberg model).

Kosterlitz-Thouless transition in polar state

Helicity moduls for phase shift



Helicity moduls for spin rotation

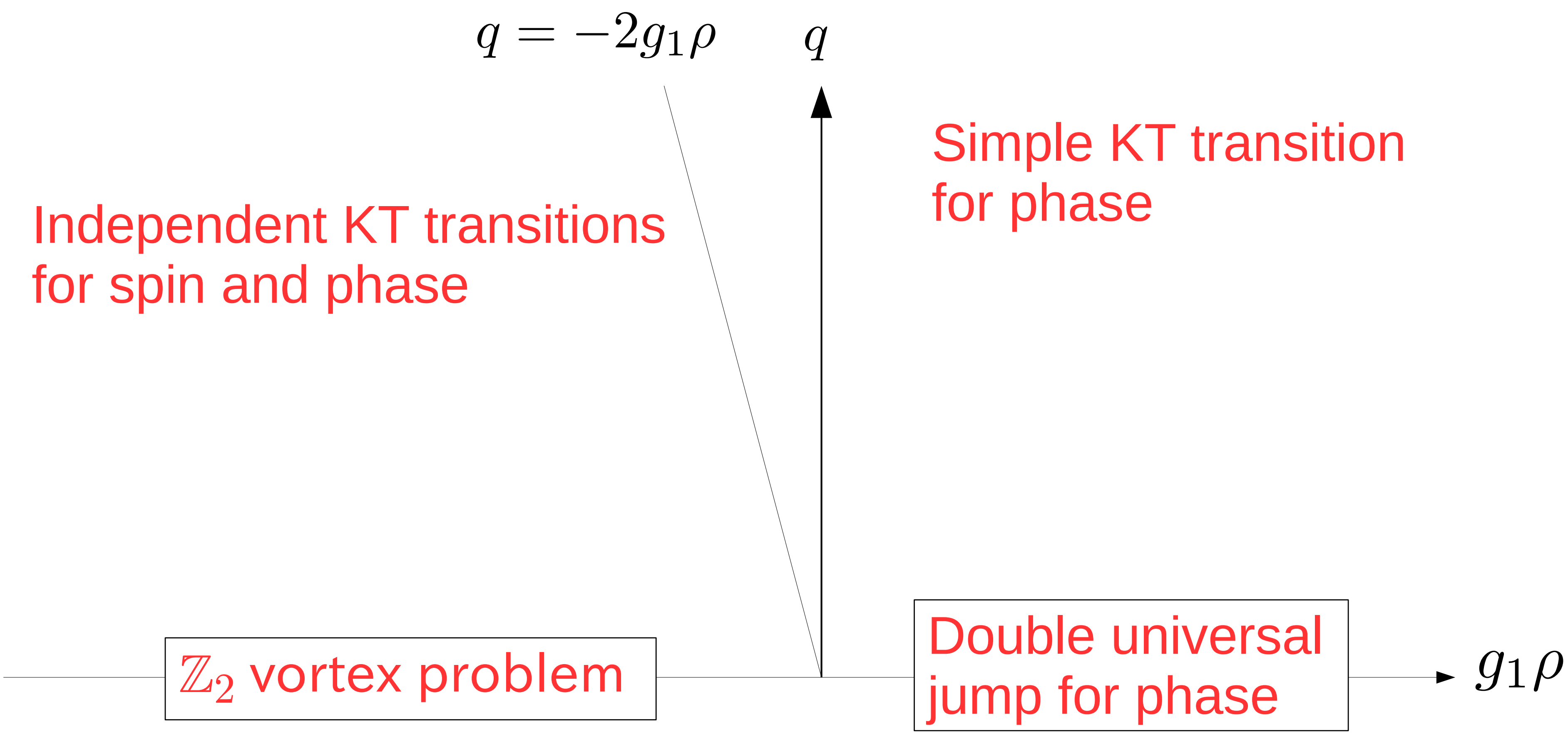


KT transition for phase shift is clear, but that for spin rotation in unclear due to strong finite-size effect (spin part has $\mathbb{Z}_2$ charge)

Universal jump is 2 times larger than that for the scalr condensate due to half circulation of vortex : $\Delta\Upsilon_\phi / T_{\mathrm{KT}} \simeq 2/\pi$

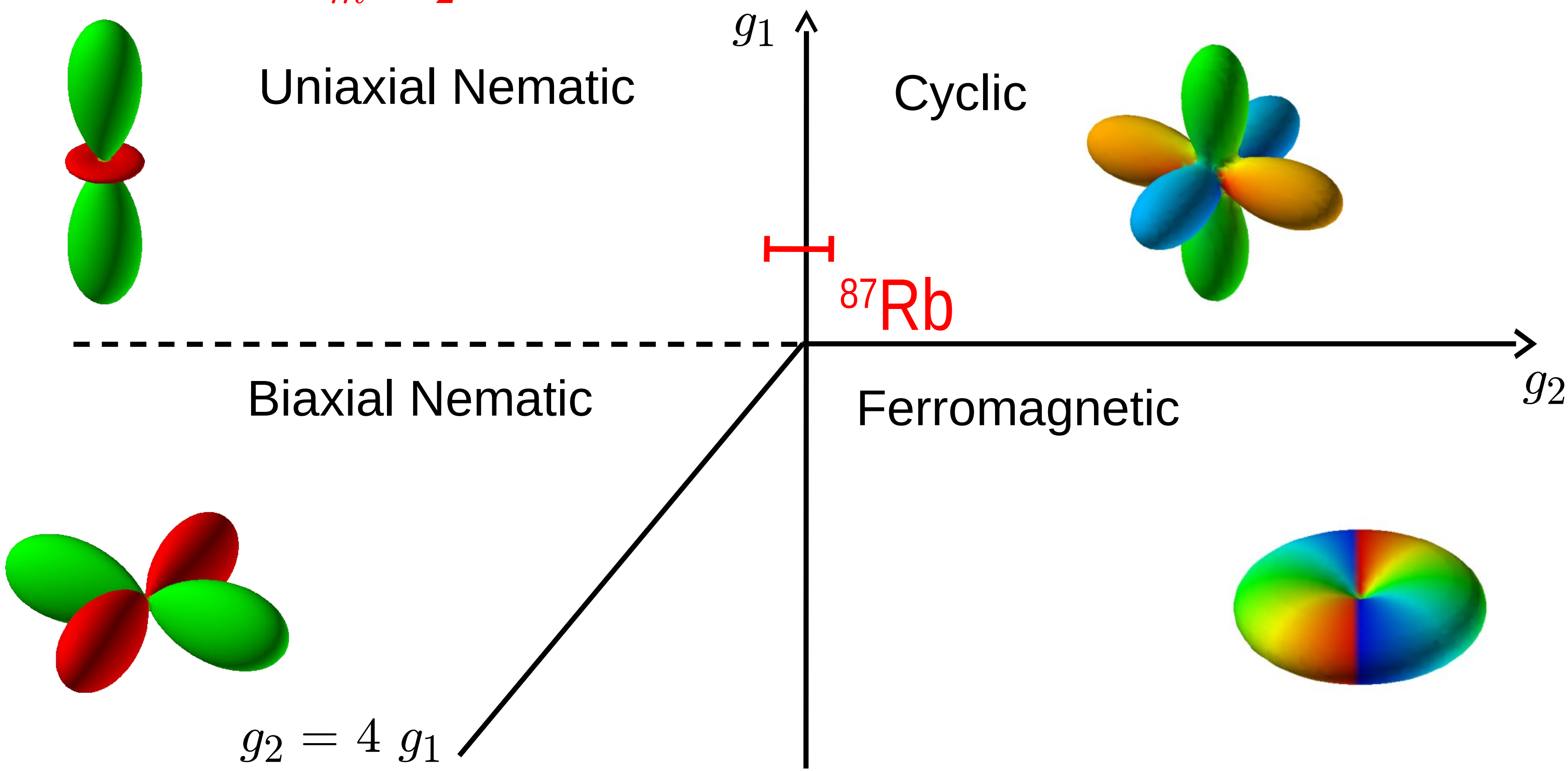
Short summary

Vortex properties can be seen in 2D KT transition

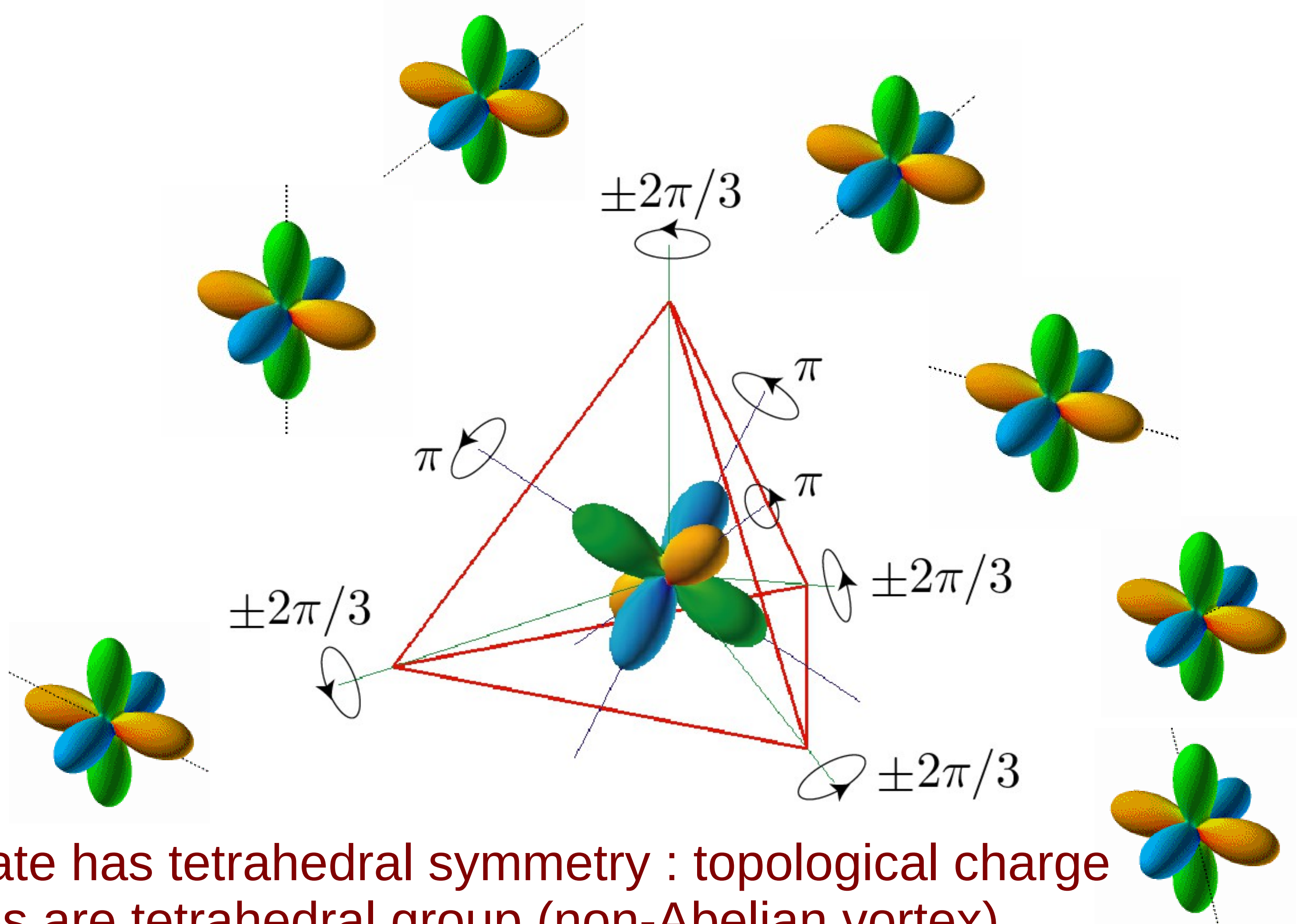


Ground states for spin-2 condensates

$$H = \int d\boldsymbol{x} \sum_{m=-2}^{2} \left(\frac{\hbar^2}{2M} |\nabla \psi_m|^2 \right) + \frac{1}{2}(g_0 \rho^2 + g_1 \boldsymbol{S}^2 + g_2 |A_{20}|^2)$$



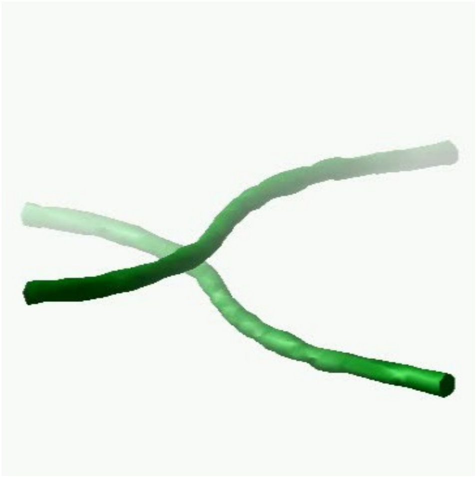
Vortices in cyclic state



Cyclic state has tetrahedral symmetry : topological charge of vortices are tetrahedral group (non-Abelian vortex)

Collision dynamics of vortices

Commutative topological charge



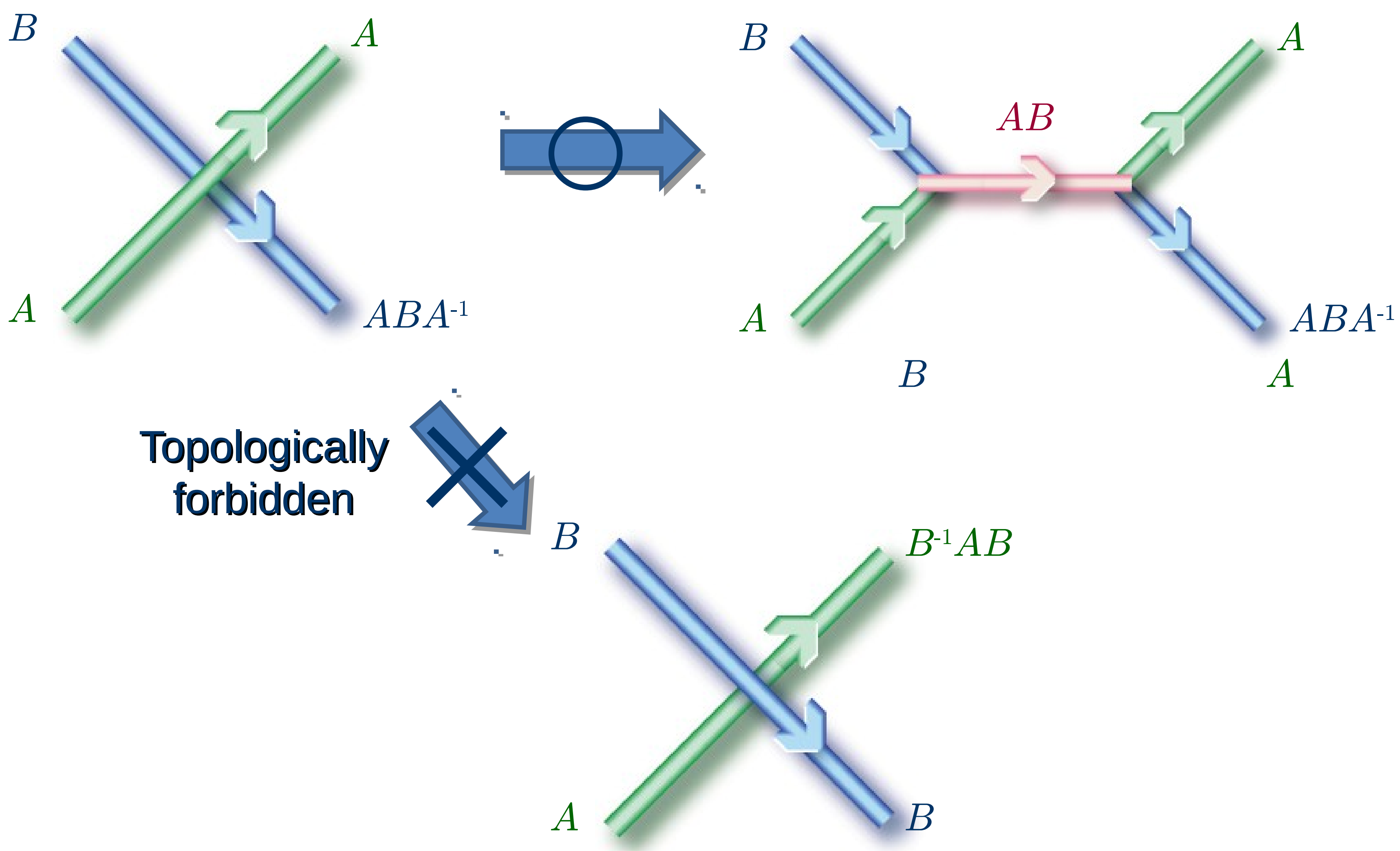
reconnection

passing through

Non-commutative topological charge

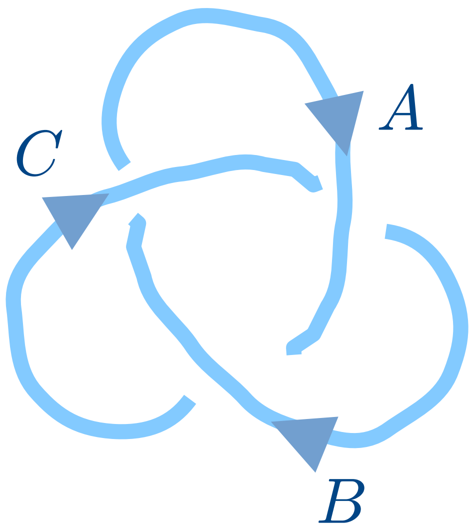
rung

Collision dynamics of vortices



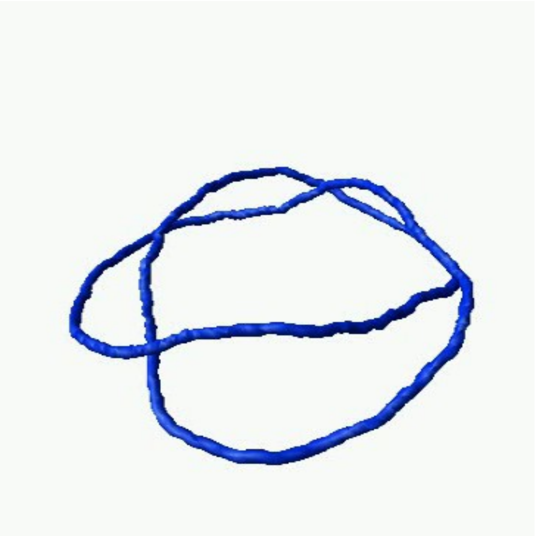
Knots for non-Abelian vortex

Trefoil with orientation
(parity broken)



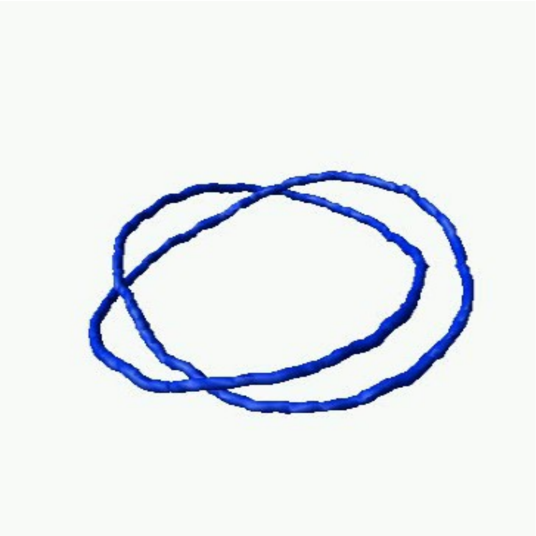
A, B, C are non-commutative

commutative



Knot is unraveled

non-commutative

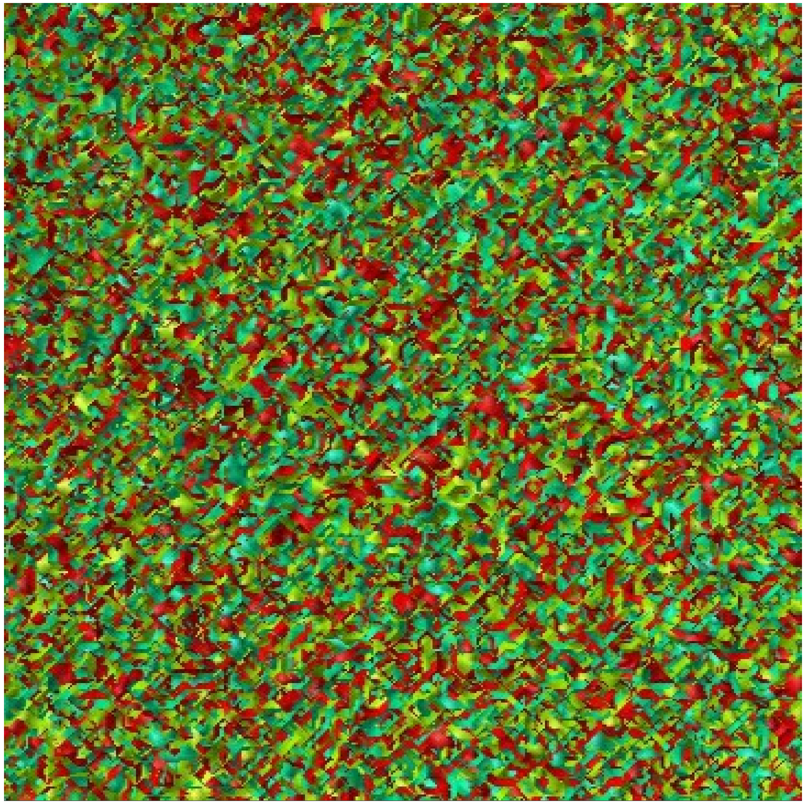


Knot is stable

- New kind of topological structure not classified by Homotopy and not suffered from Derrick's theorem
- Vortex knot breaks chiral symmetry with finite helicity

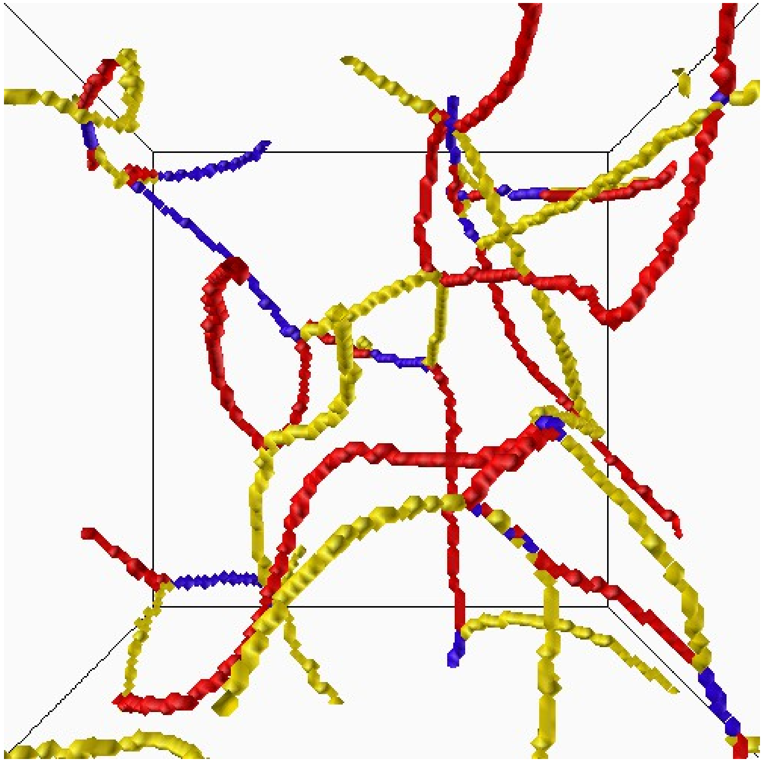
Ordering dynamics after temperature quench

Initial state : fully randomized state surrounded (high temperature) with low-energy uniform ground state (heat reservoir)



Ordering dynamics after temperature quench

Final state



Large scaled knotted vortex structure is remained with finite helicity
→spontaneous breaking of chiral symmetry with vortex knot

Short Summary

- Vortices becomes non-Abelian for spin-2 spinor condensate (cyclic state)
- Non-Abelian vortices enable new topological defect : vortex knots
- Vortex knots have finite helicity (chirality) and metastable not suffering from Derrick's theorem
- Large scaled vortex knotted structure (chirality) is expected after the temperature quench with spontaneous chiral symmetry breaking

Thank you very much for your attention